Workshop #1: Design methods for retaining walls

Assessment of active and passive earth pressure: what do we owe to Boussinesq and Caquot?

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Terrasol, Setec
Introduction

Values used for the design of both:

- gravity walls with limit equilibrium methods
- retaining walls with subgrade reaction methods
Introduction

Two main issues:

- what are the main contributions for the elaboration of these tables?
- How have these values been determined?
→ Strong criticism of the Rankine theory.
→ New equations are proposed but it is not possible to provide closed-form solutions.
## Active earth pressures: first values (horizontal component)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$f_0$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
<th>$f_6$</th>
<th>$f_7$</th>
<th>$f_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15°</td>
<td>0.17339</td>
<td>0.23065</td>
<td>0.27166</td>
<td>0.32819</td>
<td>0.41378</td>
<td>0.50152</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0°</td>
<td>0.17748</td>
<td>0.23744</td>
<td>0.28099</td>
<td>0.34133</td>
<td>0.43087</td>
<td>0.52699</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0.184</td>
<td>0.139</td>
<td>0.368</td>
</tr>
<tr>
<td>5°</td>
<td>0.080</td>
<td>0.108</td>
<td>0.199</td>
</tr>
<tr>
<td>10°</td>
<td>0.043</td>
<td>0.090</td>
<td>0.123</td>
</tr>
</tbody>
</table>

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**Réusal (1903)**
The stress state ensures the equilibrium of the system that is fully plastic.

The parameter \( k \) ensures that the stress state with its inclination \( \alpha \) is on the Mohr circle ('+' for active state and '−' for passive state).

A first method is proposed but the numerical values are still those calculated by Résal in 1903.
The calculation principles are clearly established with the distinction of two parts: the first one is related to the Rankine equilibrium while the second one is related to the Boussinesq equilibrium.

The equations are undetermined: for any stress inclination on the wall, it is possible to find an earth pressure on the wall that ensures the equilibrium with the Rankine state on the line OM$_1$.

Some values are proposed.
Active and passive earth pressure coefficients are calculated for various cases using a systematic procedure: the first tables proposed by Caquot and Kerisel are presented.

\[
\frac{dn}{d\omega} = 3t - \sin \omega \quad (1) \quad \frac{dt}{d\omega} = nm - \cos \omega \quad (2)
\]

\[
m = 2k - 1
\]

\[
m = 1 + 4\tan^2 \varphi \pm \frac{4}{\cos \varphi} \sqrt{\tan^2 \varphi - \tan^2 \alpha}
\]
Active and passive earth pressures for weighted ground conditions

Active earth pressures: the values are the same since 1948.

\[ K_{ay} = \rho k_a \]

\[ k_a = \frac{\cos^2(\lambda - \varphi)}{\cos(\lambda + \delta) \left( 1 + \frac{\sin(\varphi + \delta) \sin(\varphi - \beta)}{\cos(\lambda + \delta) \cos(\beta - \lambda)} \right)^2} \]

\[ \ln(\rho) = -\left( 2 - \frac{\tan^2 \beta + \tan^2 \delta}{2 \tan^2 \varphi} \right) \sqrt{\sin \varphi} \ln \left[ (1 - 0.9 \zeta^2 - 0.1 \zeta^4)(1 - 0.3 \zeta^3) \right] \]

\[ \rho > 1.0 \]

Coulomb-Poncelet coefficient
Active and passive earth pressures for weighted ground conditions

→ Active earth pressures: the values are the same since 1948.

\[ K_a = \rho k_a \]

\[ k_a = \frac{\cos^2(\lambda - \varphi)}{\cos(\lambda + \delta)} \left(1 + \frac{\sin(\varphi + \delta)\sin(\varphi - \beta)}{\cos(\lambda + \delta)\cos(\beta - \lambda)}\right) \]

Coulomb-Poncelet coefficient

→ Passive earth pressures: the values have varied.

Example:
\[ c=0, \ \varphi=30° \ \text{and} \ \delta/\varphi =-1 \]

Caquot, Kerisel and Absi:
\[ K_p=6,42 \ (1948), \ K_p=6,56 \ (1966), \ K_p=6,50 \ (1990) \]

Sokolowski (1965):
\[ K_p=6,55 \]

The differences are negligible for the practice but it may be interesting to better understand how these values were determined.
An example of numerical integration

\( \varphi = 30^\circ, \beta = 0, \lambda = 0, \delta/\varphi = -1 \) (case 1)

\[
\frac{dn}{d\omega} = 3t - \sin \omega \tag{1}
\]

\[
\frac{dt}{d\omega} = nm - \cos \omega \tag{2}
\]

Runge Kutta (RK4) numerical integration method
An example of numerical integration

\( \varphi = 30^\circ, \beta = 0, \lambda = 0, \delta / \varphi = 1 \) (case 1)

\[ \frac{dn}{d\omega} = 3t - \sin \omega \quad (1) \]

\[ \frac{dt}{d\omega} = nm - \cos \omega \quad (2) \]

Runge Kutta (RK4) numerical integration method
An example of numerical integration

\[ \frac{dn}{d\omega} = 3t - \sin \omega \] (1)

\[ \frac{dt}{d\omega} = nm - \cos \omega \] (2)

Runge Kutta (RK4) numerical integration method

Method presented:
\[ K_p = 6,548 \]

Caquot, Kerisel, Absi:
\[ K_p = 6,42 \, (1948), \, K_p = 6,56 \, (1966), \, K_p = 6,50 \, (1990) \]

Sokolowski (1965):
\[ K_p = 6,55 \]
Other examples

Active earth pressure – $\delta/\varphi$

<table>
<thead>
<tr>
<th>Relative inclination</th>
<th>-1</th>
<th>-2/3</th>
<th>0</th>
<th>2/3</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presented method</td>
<td>0.886</td>
<td>0.477</td>
<td>0.333</td>
<td>0.301</td>
<td>0.307</td>
</tr>
<tr>
<td>Caquot Kerisel</td>
<td>0.981</td>
<td>0.476</td>
<td>0.333</td>
<td>0.300</td>
<td>0.308</td>
</tr>
<tr>
<td>Coulomb</td>
<td>0.866</td>
<td>0.469</td>
<td>0.333</td>
<td>0.297</td>
<td>0.297</td>
</tr>
</tbody>
</table>

Passive earth pressure – $\delta/\varphi$

<table>
<thead>
<tr>
<th>Relative inclination</th>
<th>-1</th>
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<th>0</th>
<th>2/3</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presented method</td>
<td>6.55</td>
<td>5.26</td>
<td>3.0</td>
<td>1.46</td>
<td>---</td>
</tr>
<tr>
<td>Caquot Kerisel</td>
<td>6.50</td>
<td>5.30</td>
<td>3.0</td>
<td>1.46</td>
<td>---</td>
</tr>
</tbody>
</table>
Other examples

Active earth pressure – $\delta/\phi$

<table>
<thead>
<tr>
<th>Relative inclination</th>
<th>-1</th>
<th>-2/3</th>
<th>0</th>
<th>2/3</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presented method</td>
<td>0.792</td>
<td>0.533</td>
<td>0.405</td>
<td>0.362</td>
<td>0.356</td>
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<tr>
<td>Caquot Kerisel</td>
<td>0.869</td>
<td>0.533</td>
<td>0.405</td>
<td>0.358</td>
<td>0.355</td>
</tr>
<tr>
<td>Coulomb</td>
<td>0.766</td>
<td>0.533</td>
<td>0.403</td>
<td>0.349</td>
<td>0.334</td>
</tr>
</tbody>
</table>

Passive earth pressure – $\delta/\phi$

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<th>0</th>
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<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presented method</td>
<td>5.39</td>
<td>4.72</td>
<td>3.22</td>
<td>1.86</td>
<td>1.03</td>
</tr>
<tr>
<td>Caquot Kerisel</td>
<td>5.40</td>
<td>4.70</td>
<td>3.20</td>
<td>1.86</td>
<td>---</td>
</tr>
</tbody>
</table>
Failure mechanism – Slip lines

Active earth pressures

\[ r = e^{-\int \tan(\xi(\omega)) \, d\omega} \]

with:

\[ \xi(\omega) = \frac{\pi}{4} + \frac{\varphi}{2} - \frac{(\omega_u - \alpha)}{2} \quad \text{and} \quad \omega_u = \arcsin\left(\frac{\sin\alpha}{\sin\varphi}\right) \]
Failure mechanism – Slip lines

Active earth pressures

\[ r = e^{-\int \tan(\xi(\omega)) \, d\omega} \]

\[ \xi(\omega) = \frac{\pi}{4} + \frac{\psi}{2} - \frac{(\omega_a - \alpha)}{2} \text{ and } \omega_a = \arcsin\left(\frac{\sin\alpha}{\sin\psi}\right) \]

Passive earth pressures

\[ r = e^{-\int \tan(\xi(\omega)) \, d\omega} \]

\[ \xi(\omega) = \frac{\pi}{4} - \frac{\psi}{2} + \frac{(\omega_a + \alpha)}{2} \text{ and } \omega_a = \arcsin\left(\frac{\sin\alpha}{\sin\psi}\right) \]
A last example – Comparison with the kinematical approach of the yield analysis

Presented method: $K_p = 7.77$

Caquot-Kerisel: $K_p = 8.0$

The difference is significant even if it remains very low and negligible for the practice.

$\varphi = 35^\circ$, $\beta = 0$, $\lambda = 0$, $\delta/\varphi = -2/3$

[Graph showing comparison between methods]
A kinematical approach provides: $K_p = 7.95$

The two mechanisms are very similar.

→ The «true» value should be between 7.77 and 7.95.
Conclusions

Around 175 years after Coulomb, Caquot and Kerisel developed another approach to assess active and passive earth pressures for weighted ground conditions from the contributions of Bousinesq, Résal and Ravizé.

Failure mechanisms can also be obtained and compared to those considered by Coulomb explaining why Coulomb approach is not on the safe side.

Today, Bousinesq, Caquot and Kerisel approaches can be compared to some other solutions obtained by the kinematical approach of the yield analysis.

New values of active and passive earth pressure coefficients might be thus obtained by comparing both Bousinesq, Caquot and Kerisel approaches and the kinematical approach of the yield analysis.
Thank you for your attention