cfms
COMITÉ FRANÇAIS DE MÉCANIQUE DES SOLS ET DE GÉOTECHNIQUE

ACADÉMIE DES SCIENCES INSTITUT DE FRANCE

ACADÉMIE DES TECHNOLOGIES POUR UN PROGRÈS RAISONNÉ, CHOISI ET PARTAGÉ

SIMSG ISSMGE

LES TRAVAUX PUBLICS FÉDÉRATION NATIONALE

Charles-Augustin COULOMB - A geotechnical tribute

Paris, september 25 & 26, 2023

Workshop #1: Design methods for retaining walls

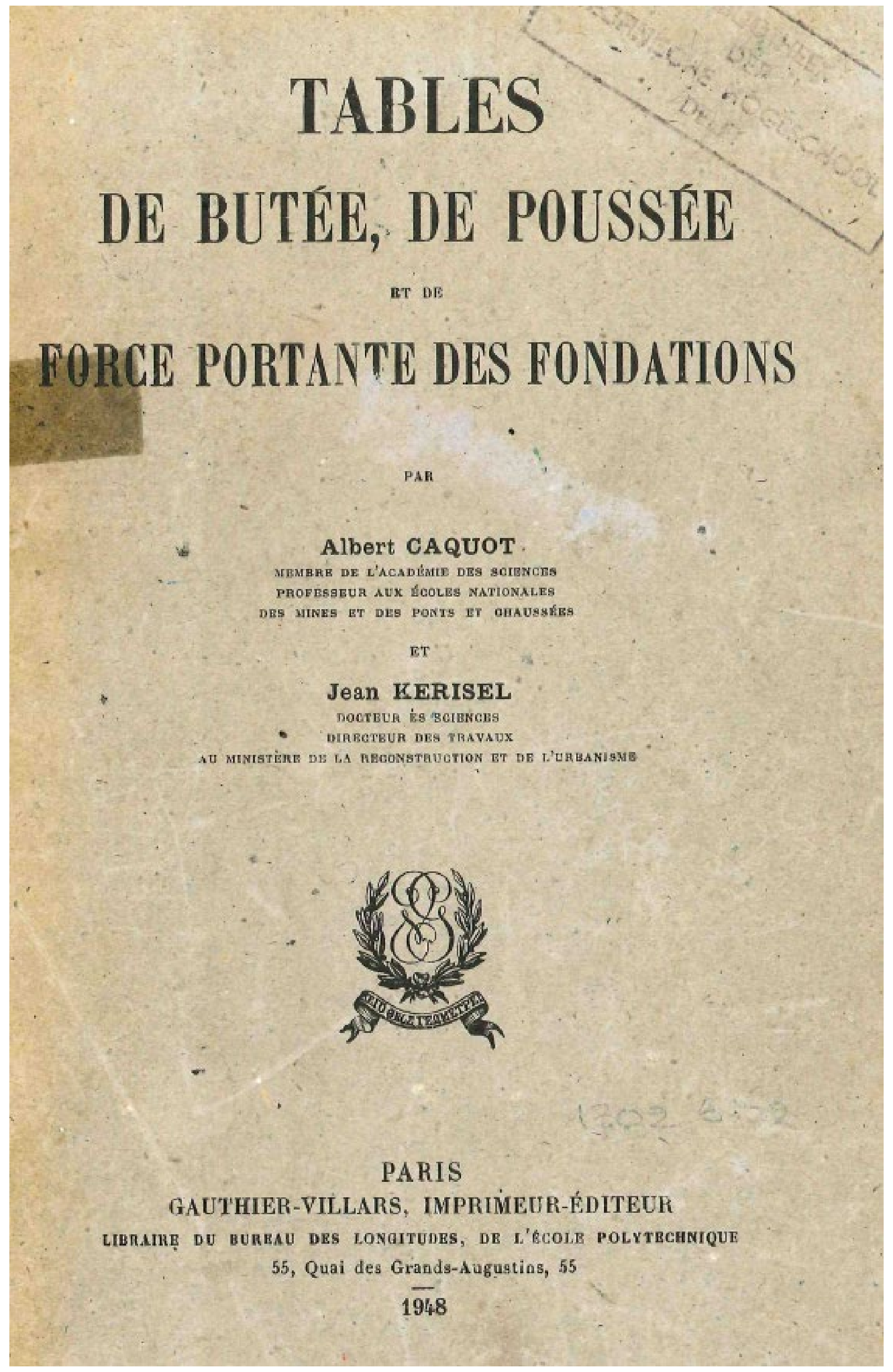
Assessment of active and passive earth pressure: what do we owe to Boussinesq and Caquot?

S.Burlon

Terrasol, Setec

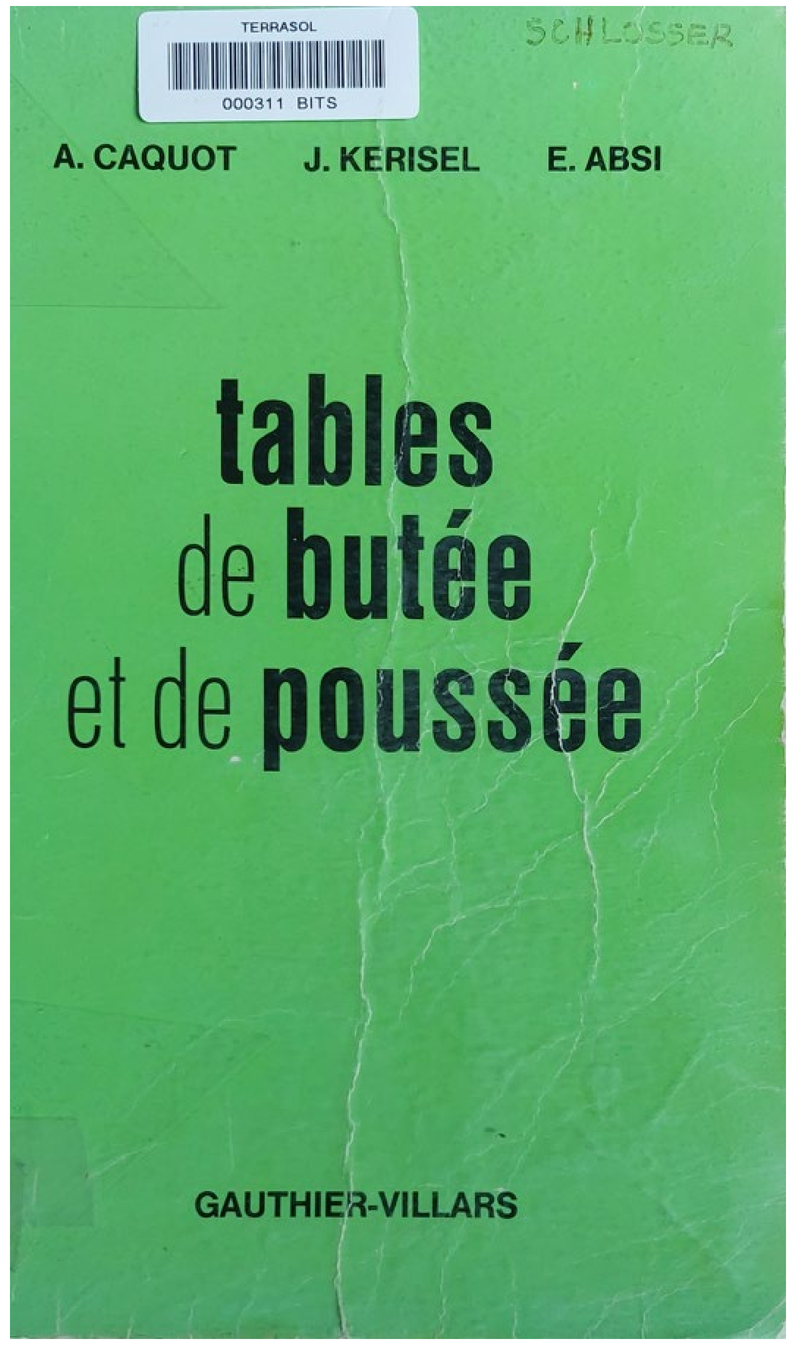
BUREAU VERITAS Shaping a World of Trust | Geos | SoilCloud | SOLETANCHE BACHY | spie batignolles / fondations | SSI 2U | terrasol setec

Introduction

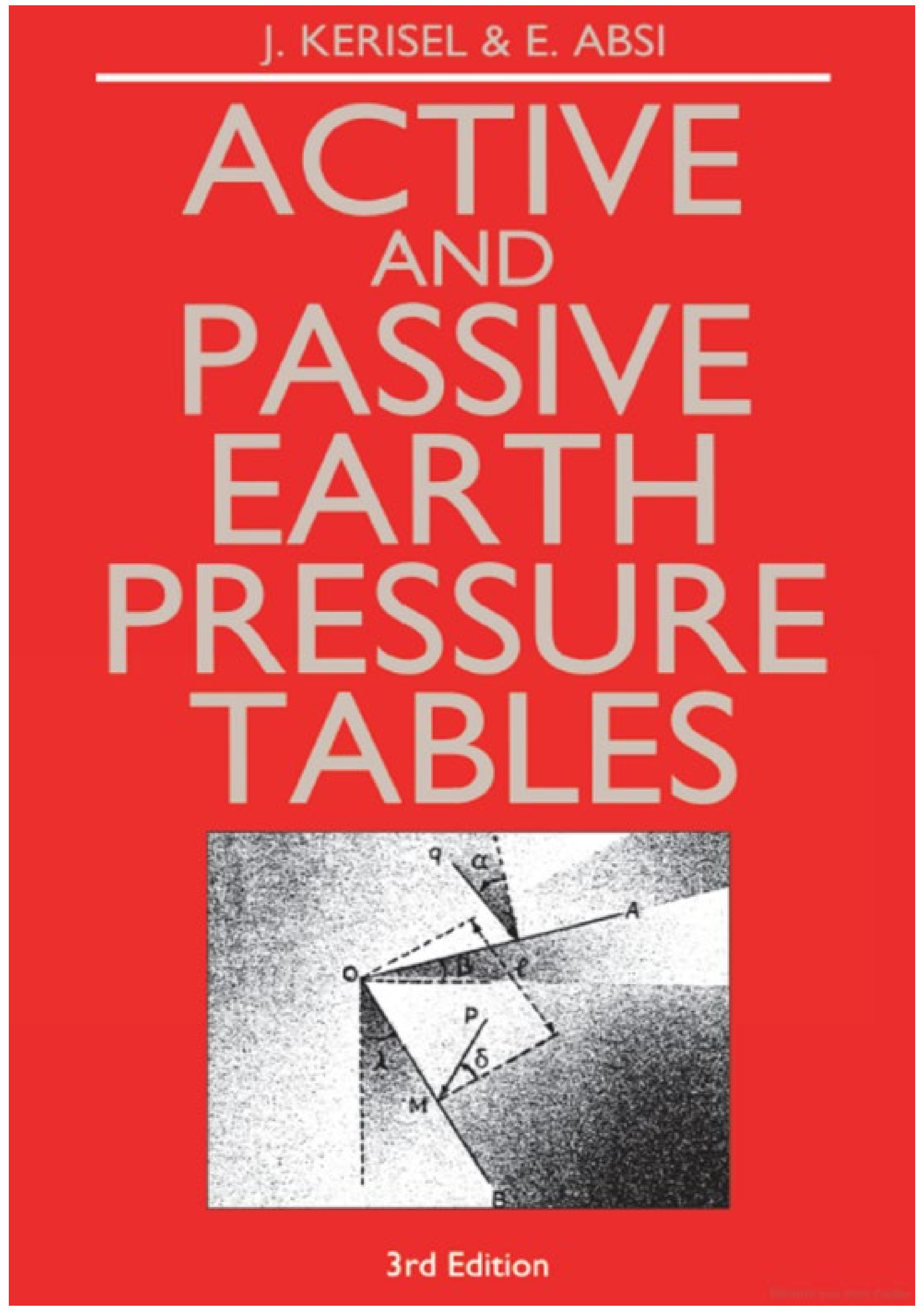


1948
1st edition

1966



1973
2nd edition



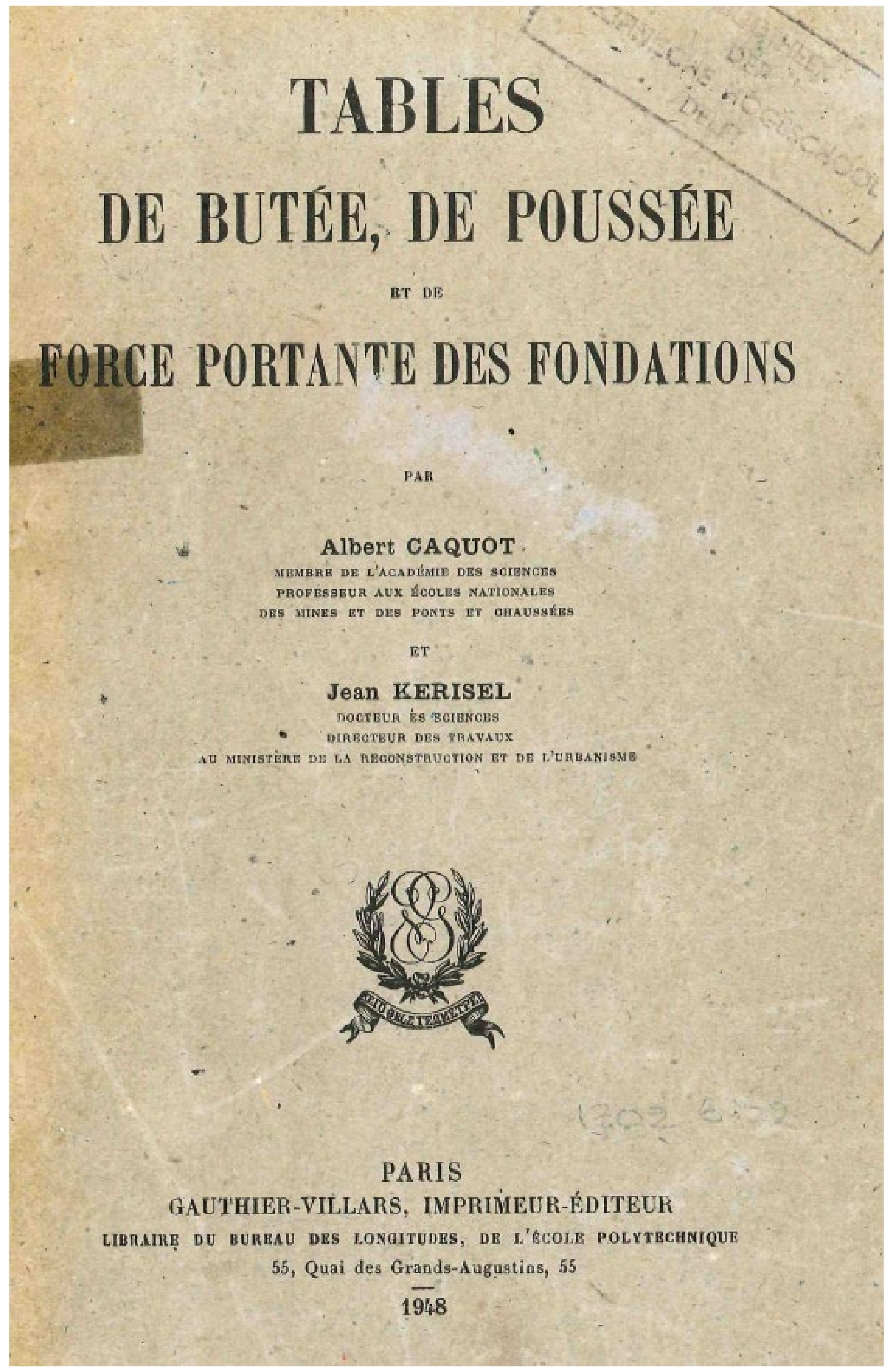
1990
3rd edition

Values used for the design of both:

→ gravity walls with limit equilibrium methods

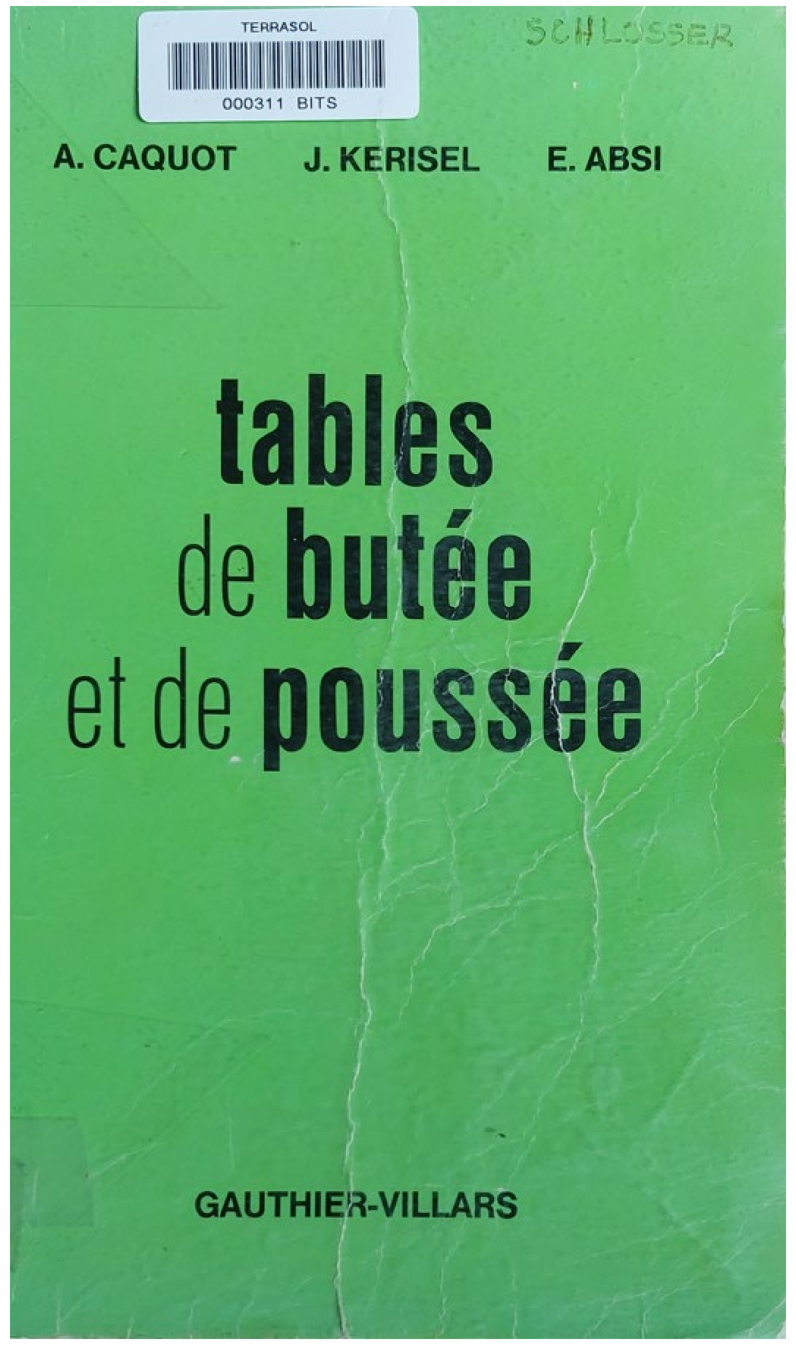
→ retaining walls with subgrade reaction methods

Introduction

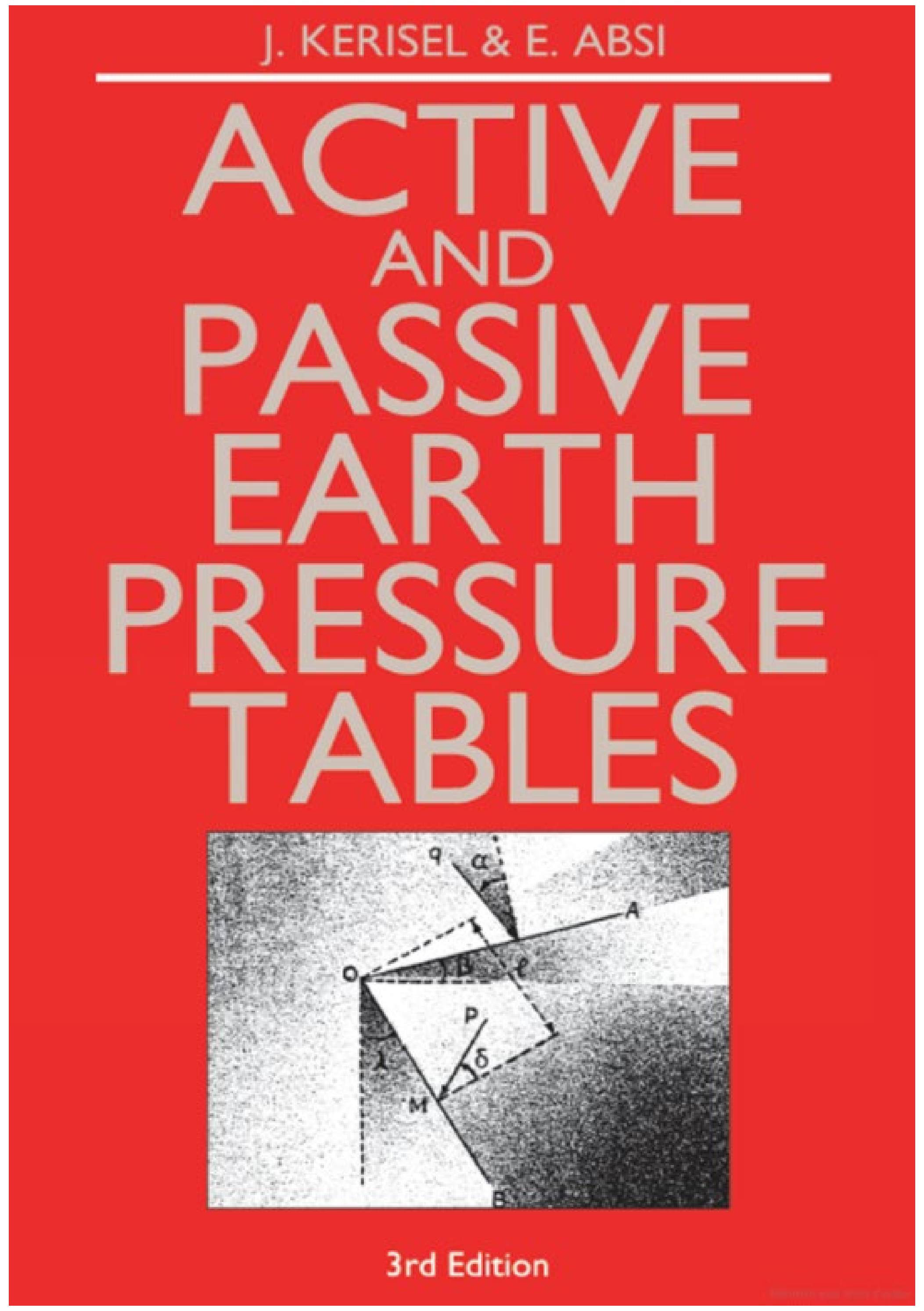


1948
1st edition

1966



1973
2nd edition



1990
3rd edition

Two main issues:

→ what are the main contributions for the elaboration of these tables?

→ How have these values been determined?

Boussinesq (1874)

ANNALES

DES

PONTS ET CHAUSSÉES.

MÉMOIRES ET DOCUMENTS.

5e SÉRIE.

TOME VIII.

1874

2e SEMESTRE.

1874

ESSAI THÉORIQUE

SUR

L'ÉQUILIBRE DES MASSIFS PULVÉRULENTS,

COMPARÉ A CELUI DE MASSIFS SOLIDES,

ET

SUR LA POUSSÉE DES TERRES SANS COHÉSION;

PAR

M. J. BOUSSINESQ,

PROFESSEUR A LA FACULTÉ DES SCIENCES DE LILLE.

BRUXELLES,

F. HAYEZ, IMPRIMEUR DE L'ACADÉMIE ROYALE DE BELGIQUE.

1876

1876

Introduction (1874)

Dans son mémoire, remarquable à plusieurs titres, *On the stability of loose Earth* (*), le regretté M. Macquorn-Rankine a donné une méthode ingénieuse pour l'étude des distributions planes des pressions à l'intérieur de masses pesantes quelconques en équilibre, et il a essayé, au moyen d'une hypothèse qui ramène analytiquement la question au problème classique du refroidissement d'une barre homogène, de l'employer dans la recherche des pressions produites aux divers points d'un massif sablonneux dont le profil supérieur est courbe.

Conclusion (1874)

La formule (μ) montre combien est inexacte l'hypothèse de M. Rankine, puisque l'expression qu'elle fournit pour X dépend, non pas seulement de H, comme il arriverait si elle pouvait être de la forme F(H), mais encore de *y* (*). Peut-être trouvera-t-on un jour quelque ordre de phénomènes auquel l'hypothèse considérée sera plus applicable, et qui réalisera ainsi cette curieuse analogie d'une distribution de pressions avec le mouvement de la chaleur dans une barre.

→ Strong criticism of the Rankine theory.

→ New equations are proposed but it is not possible to provide closed-form solutions.

Résal (1903)

ENCYCLOPÉDIE
DES
TRAVAUX PUBLICS
Fondée par M.-C. LECHALAS, Inspecteur général des Ponts et Chaussées

COURS DE L'ÉCOLE DES PONTS & CHAUSSÉES

POUSSÉE DES TERRES

STABILITÉ
DES
MURS DE SOUTÈNEMENT

PAR

JEAN RESAL
INGÉNIEUR EN CHEF, PROFESSEUR A L'ÉCOLE DES PONTS ET CHAUSSÉES

PARIS
LIBRAIRIE POLYTECHNIQUE, CH. BÉRANGER, ÉDITEUR
Successeur de BAUDRY & C[ie]
15, RUE DES SAINTS-PÈRES, 15
Même Maison à Liège, rue de la Régence, 21

1903

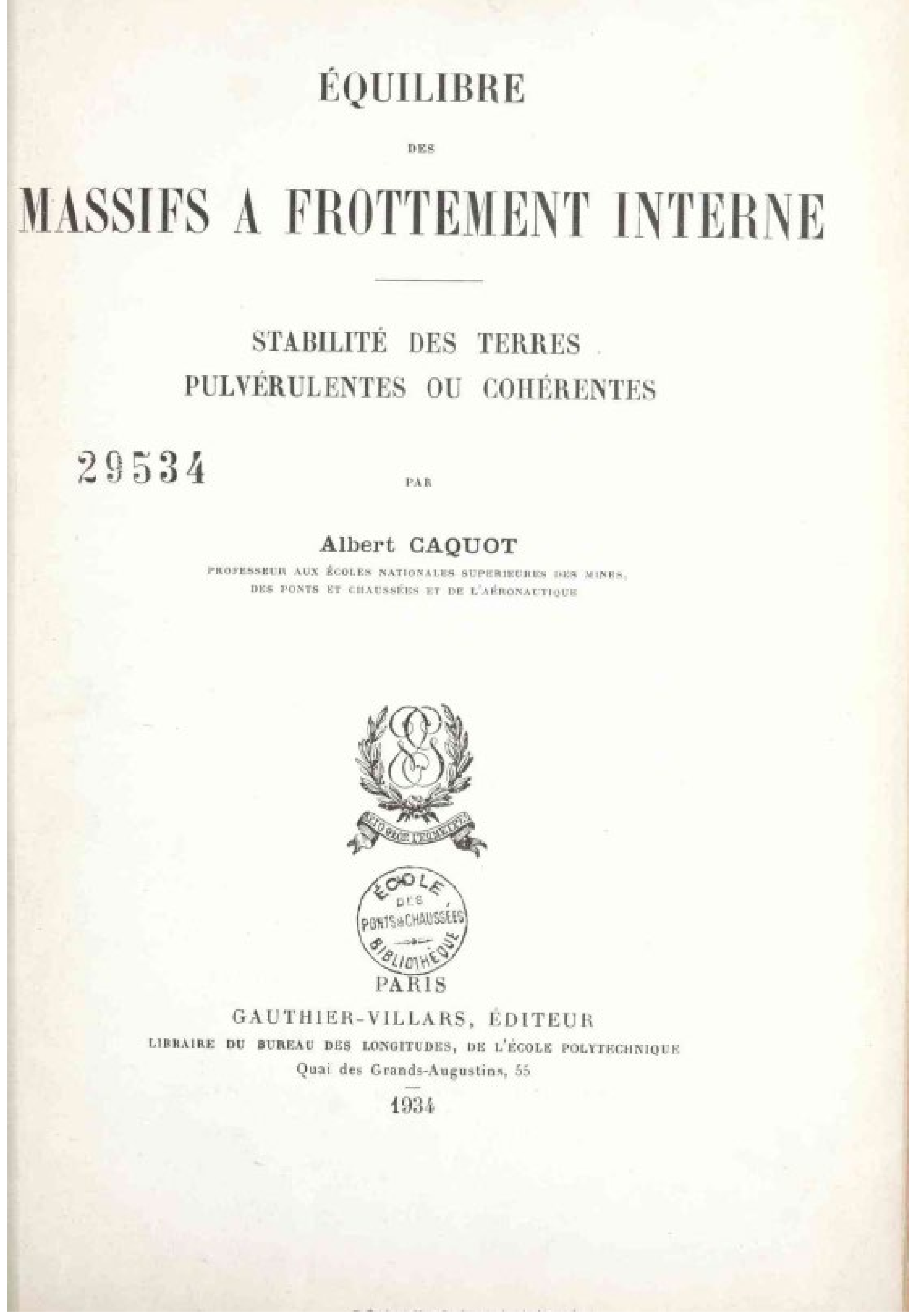
1903

Active earth pressures: first values (horizontal component)

Coefficients de poussée

	$i = 5°$						$i = 0°$					
$\varphi =$	45°	40°	35°	30°	25°	20°	45°	40°	35°	30°	25°	20°
$f(i) =$	0,17339	0,22005	0,27462	0,33849	0,41378	0,50152	0,17158	0,21744	0,27099	0,33333	0,40587	0,49029
$\rho =$	21°27'55"	23°35'5"	25°37'50"	27°28'50"	29°3'	30°7'5"	22°30'	25°	27°30'	30°	32°30	35°
$\gamma =$	23°32'5"	26°24'55"	29°22'10"	32°31'10"	35°57'	39°52'55"	22°30'	25°	27°30'	30°	32°30'	35°
$\alpha =$												
45°	0,202	0,258	0,322	0,397	0,486	0,589	0,172	0,217	0,271	0,333	0,406	0,490
40°	0,198	0,251	0,314	0,387	0,473	0,573	0,172	0,217	0,271	0,333	0,406	0,490
35°	0,194	0,246	0,307	0,378	0,463	0,561	0,172	0,217	0,271	0,333	0,406	0,490
30°	0,190	0,241	0,301	0,371	0,453	0,549	0,172	0,217	0,271	0,333	0,404	0,487
25°	0,186	0,237	0,294	0,361	0,441	0,533	0,172	0,217	0,269	0,330	0,396	0,479
20°	0,181	0,230	0,285	0,348	0,426	0,516	0,170	0,215	0,262	0,320	0,385	0,467
15°	0,173	0,219	0,270	0,331	0.408	0,498	0,163	0,203	0,251	0,307	0,372	0,454
10°	0,163	0,203	0,253	0,312	0,389	0,479	0,154	0,192	0,237	0,292	0,357	0,440
5°	0,150	0,189	0,236	0,296	0,369	0,460	0,142	0,179	0,222	0,276	0,341	0,424
0°	0,137	0,174	0,219	0,277	0,349	0,440	0,130	0,164	0,206	0,259	0,324	0,407
— 5°	0,124	0,159	0,201	0,258	0,328	0,416	0,118	0,150	0,189	0,241	0,308	0,386
— 10°	0,110	0,142	0,183	0,237	0,305	0,390	0,105	0,135	0,172	0,222	0,283	0,363
— 15°	0,095	0,125	0,163	0,216	0,280	0,362	0,091	0,120	0,154	0,202	0,260	0,337
— 20°	0,080	0,108	0.143	0,193	0,255	0,333	0,076	0,103	0,135	0,181	0,236	0,309
— 25°	0,063	0,090	0,123	0,170	0,229	0,300	0,060	0,086	0,116	0,159	0,212	0,280

Caquot (1934)

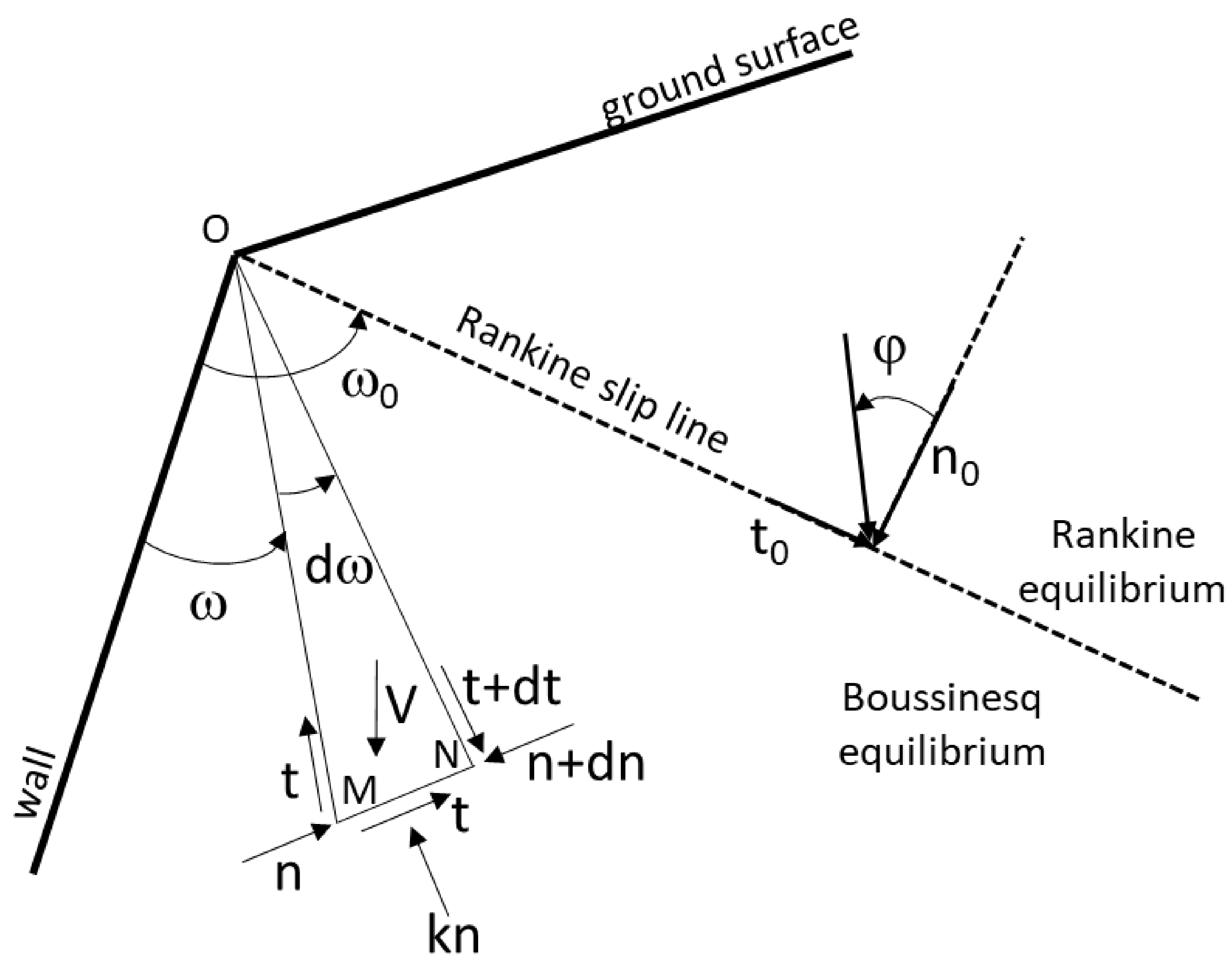


1934

$$\frac{dn}{d\omega} = 3t - sin\omega \ (1) \qquad \frac{dt}{d\omega} = nm - cos\omega \ (2)$$

$$m = 2k - 1$$

$$m = 1 + 4tan^2\varphi \pm \frac{4}{cos\varphi}\sqrt{tan^2\varphi - tan^2\alpha}$$

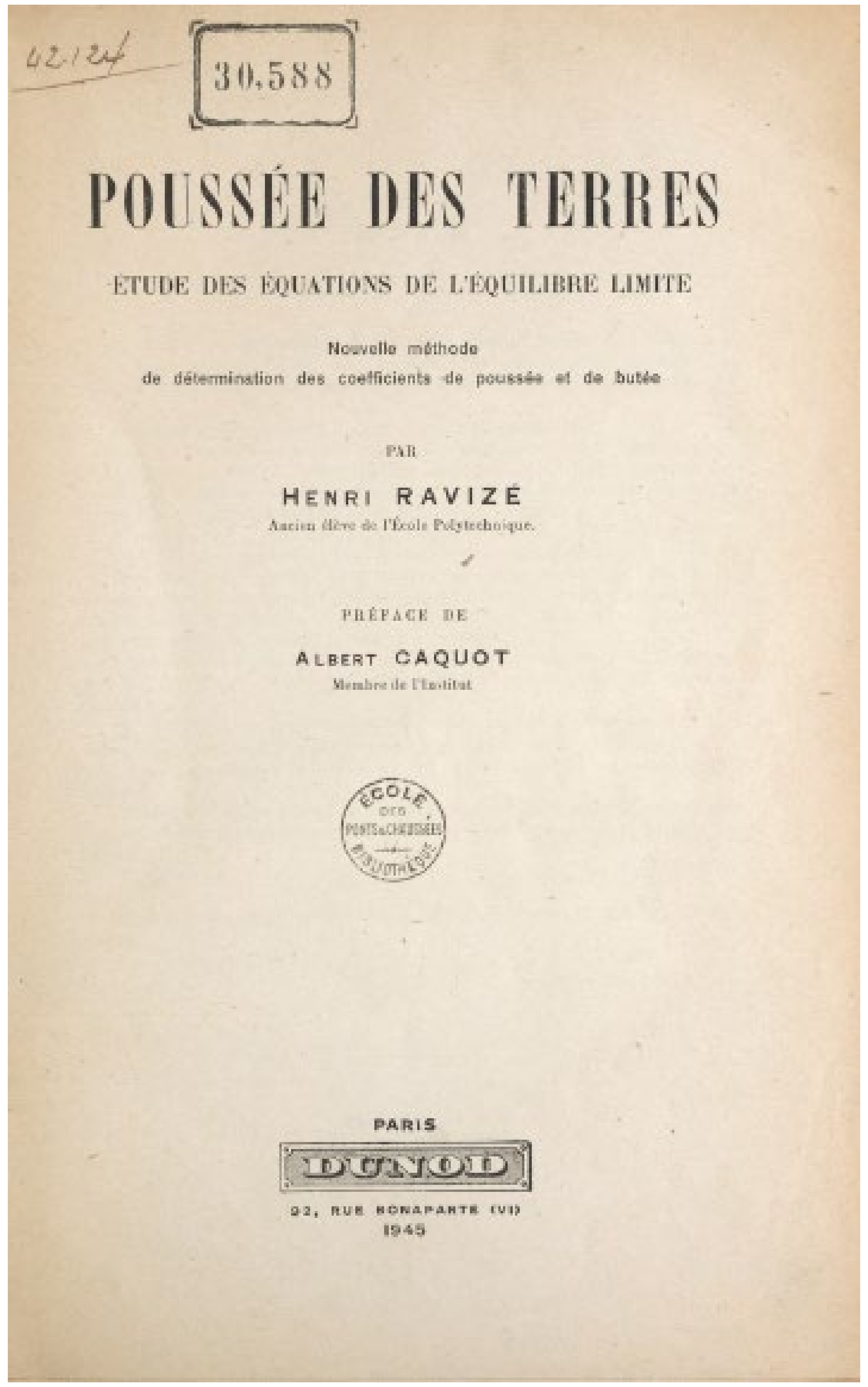


The stress state ensures the equilibrium of the system that is fully plastic.

The parameter k ensures that the stress state with its inclination α is on the Mohr circle (‘+’ for active state and ‘–’ for passive state)

A first method is proposed but the numerical values are still those calculated by Résal in 1903.

Ravizé (1945)

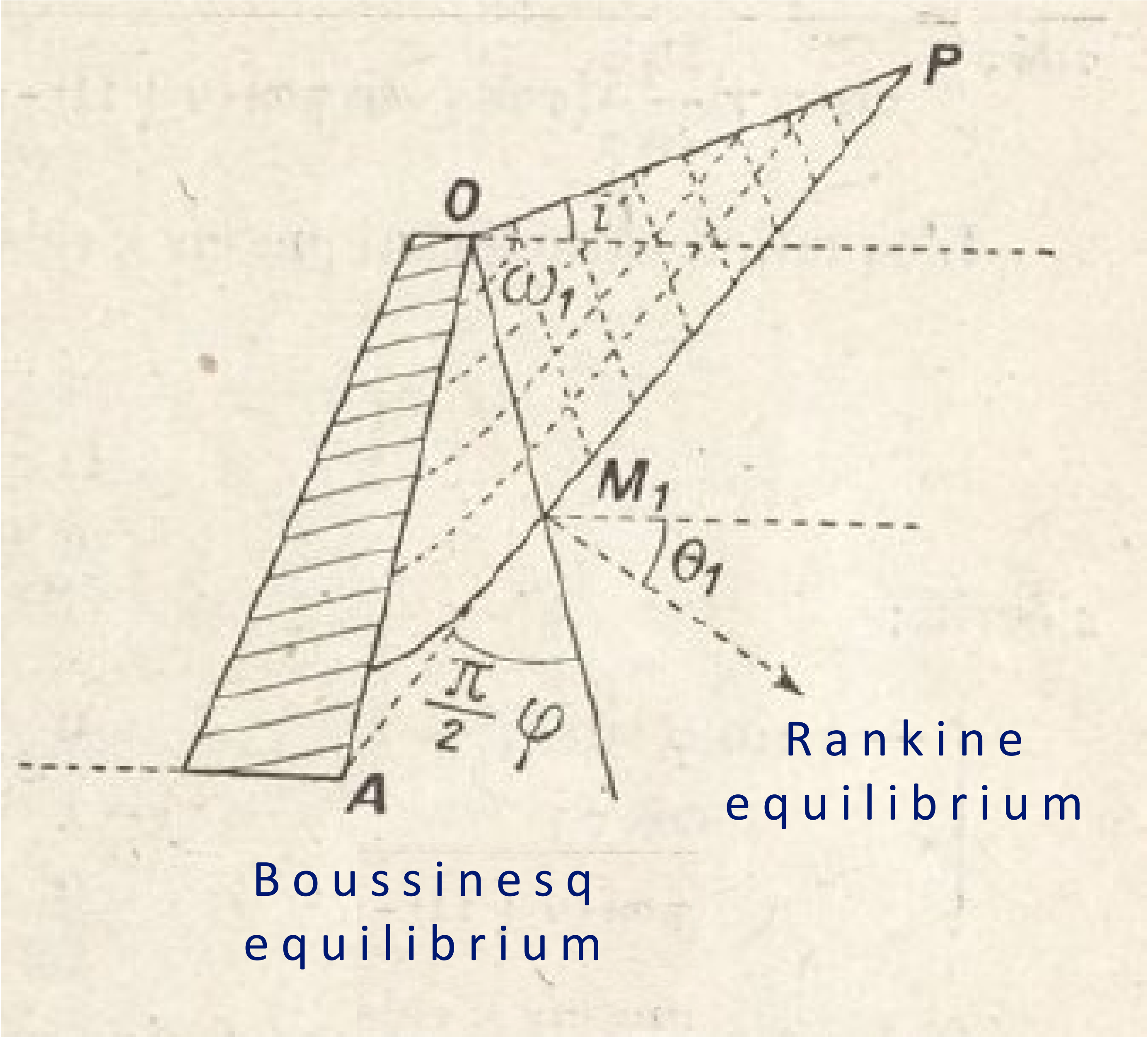

42124
30,588
POUSSÉE DES TERRES
ÉTUDE DES ÉQUATIONS DE L'ÉQUILIBRE LIMITE
Nouvelle méthode
de détermination des coefficients de poussée et de butée
PAR
HENRI RAVIZÉ
Ancien élève de l'École Polytechnique.
PRÉFACE DE
ALBERT CAQUOT
Membre de l'Institut
PARIS
DUNOD
92, RUE BONAPARTE (VI)
1945

1945

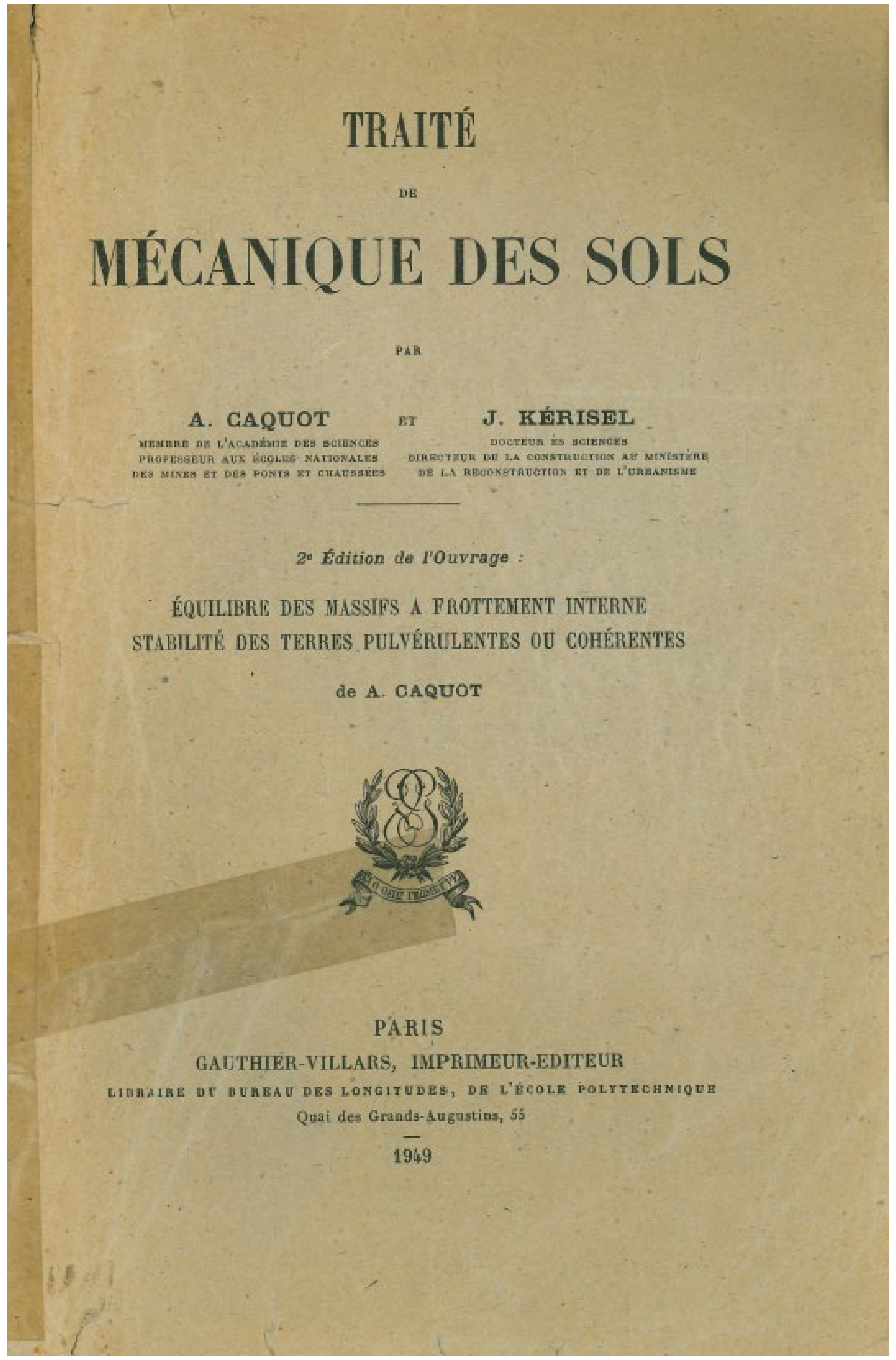
The calculation principles are clearly established with the distinction of two parts: the first one is related to the Rankine equilibrium while the second one is related to the Boussinesq equilibrium.

The equations are undetermined: for any stress inclination on the wall, it is possible to find an earth pressure on the wall that ensures the equilibrium with the Rankine state on the line OM_1.

Some values are proposed.



Caquot and Kerisel (1948 and 1949)



TRAITÉ

DE

MÉCANIQUE DES SOLS

PAR

A. CAQUOT ET J. KÉRISEL

MEMBRE DE L'ACADÉMIE DES SCIENCES
PROFESSEUR AUX ÉCOLES NATIONALES
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DOCTEUR ÈS SCIENCES
DIRECTEUR DE LA CONSTRUCTION AU MINISTÈRE
DE LA RECONSTRUCTION ET DE L'URBANISME

2e Édition de l'Ouvrage :

ÉQUILIBRE DES MASSIFS A FROTTEMENT INTERNE
STABILITÉ DES TERRES PULVÉRULENTES OU COHÉRENTES

de A. CAQUOT

PARIS
GAUTHIER-VILLARS, IMPRIMEUR-ÉDITEUR
LIBRAIRE DU BUREAU DES LONGITUDES, DE L'ÉCOLE POLYTECHNIQUE
Quai des Grands-Augustins, 55

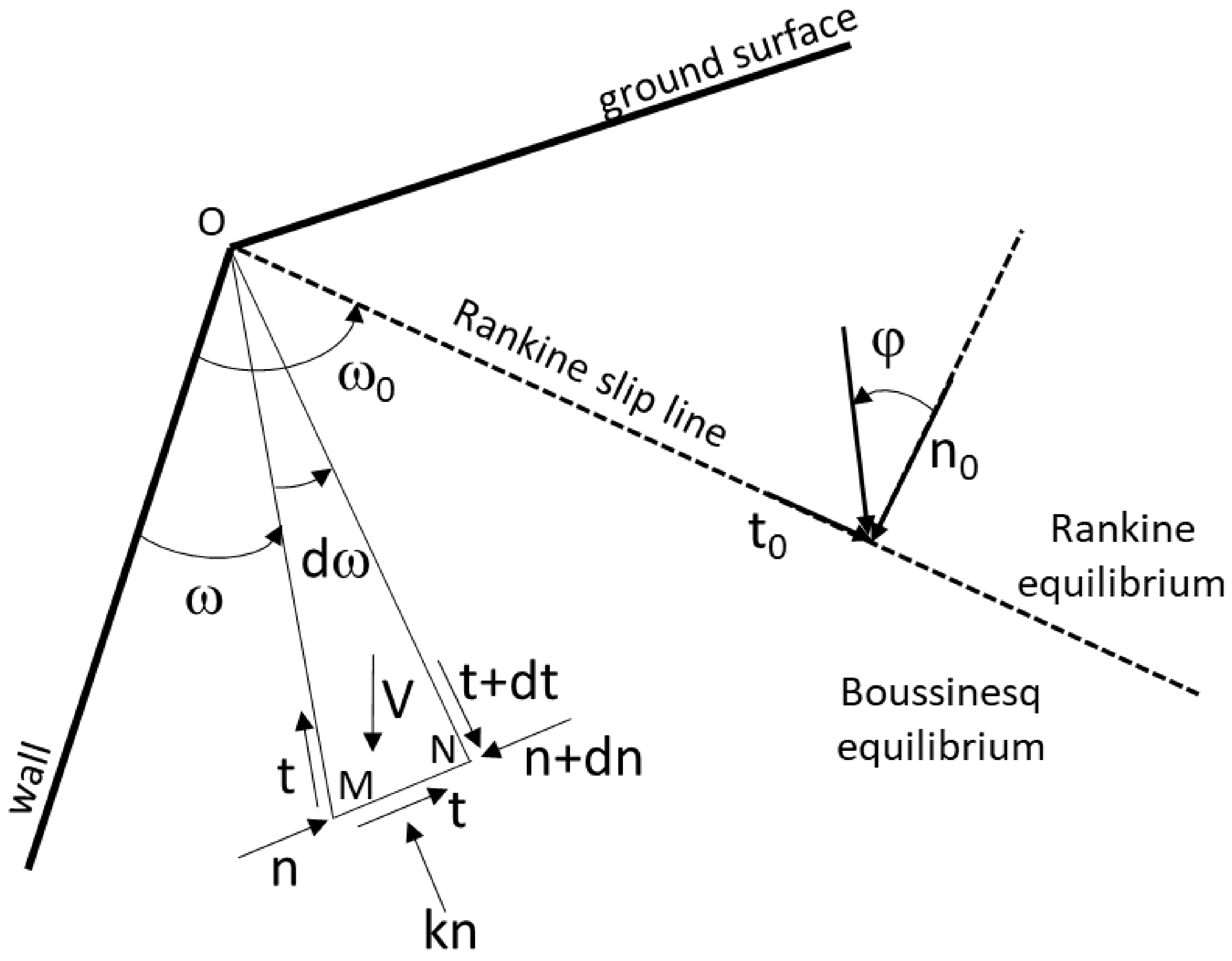
1949

1949

$$\frac{dn}{d\omega} = 3t - sin\omega \ (1) \qquad \frac{dt}{d\omega} = nm - cos\omega \ (2)$$

$$m = 2k - 1$$

$$m = 1 + 4tan^2\varphi \pm \frac{4}{cos\varphi}\sqrt{tan^2\varphi - tan^2\alpha}$$



Active and passive earth pressure coefficients are calculated for various cases using a systematic procedure: the first tables proposed by Caquot and Kerisel are presented.

Active and passive earth pressures for weighted ground conditions

→ Active earth pressures: the values are the same since 1948.

$$K_{a\gamma} = \rho k_a$$

$$k_a = \frac{\cos^2(\lambda - \varphi)}{\cos(\lambda + \delta)\left(1 + \sqrt{\frac{\sin(\varphi + \delta)\sin(\varphi - \beta)}{\cos(\lambda + \delta)\cos(\beta - \lambda)}}\right)^2}$$

Coulomb-Poncelet coefficient

$$\ln(\rho) = -\left(2 - \frac{\tan^2\beta + \tan^2\delta}{2\tan^2\varphi}\right)\sqrt{\sin\varphi}\,\ln[(1 - 0{,}9\zeta^2 - 0.1\zeta^4)(1 - 0{,}3\zeta^3)]$$

$$\rho > 1{,}0$$

Active and passive earth pressures for weighted ground conditions

→ Active earth pressures: the values are the same since 1948.

$$K_{a\gamma} = \rho k_a$$

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Coulomb-Poncelet coefficient

$$\ln(\rho) = -\left(2 - \frac{\tan^2\beta + \tan^2\delta}{2\tan^2\varphi}\right)\sqrt{\sin\varphi}\,\ln[(1 - 0{,}9\zeta^2 - 0.1\zeta^4)(1 - 0{,}3\zeta^3)]$$

$$\rho > 1{,}0$$

→ Passive earth pressures: the values have varied.

Example:

c=0, φ=30° and δ/φ =-1

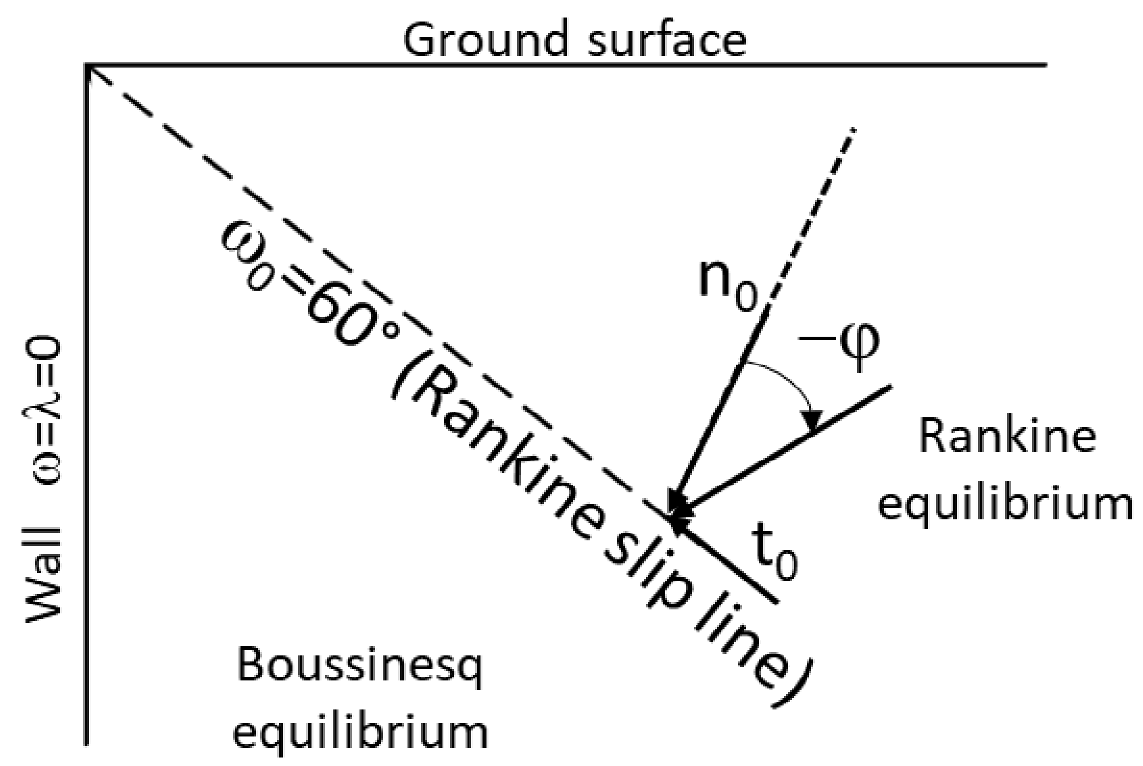
Caquot, Kerisel and Absi:
K_p=6,42 (1948), K_p=6,56 (1966),
K_p=6,50 (1990)

Sokolowski (1965):
K_p=6,55

The differences are negligible for the practice but it may be interesting to better understand how these values were determined.

An example of numerical integration

φ=30°, β=0, λ=0, δ/φ=-1   (case 1)



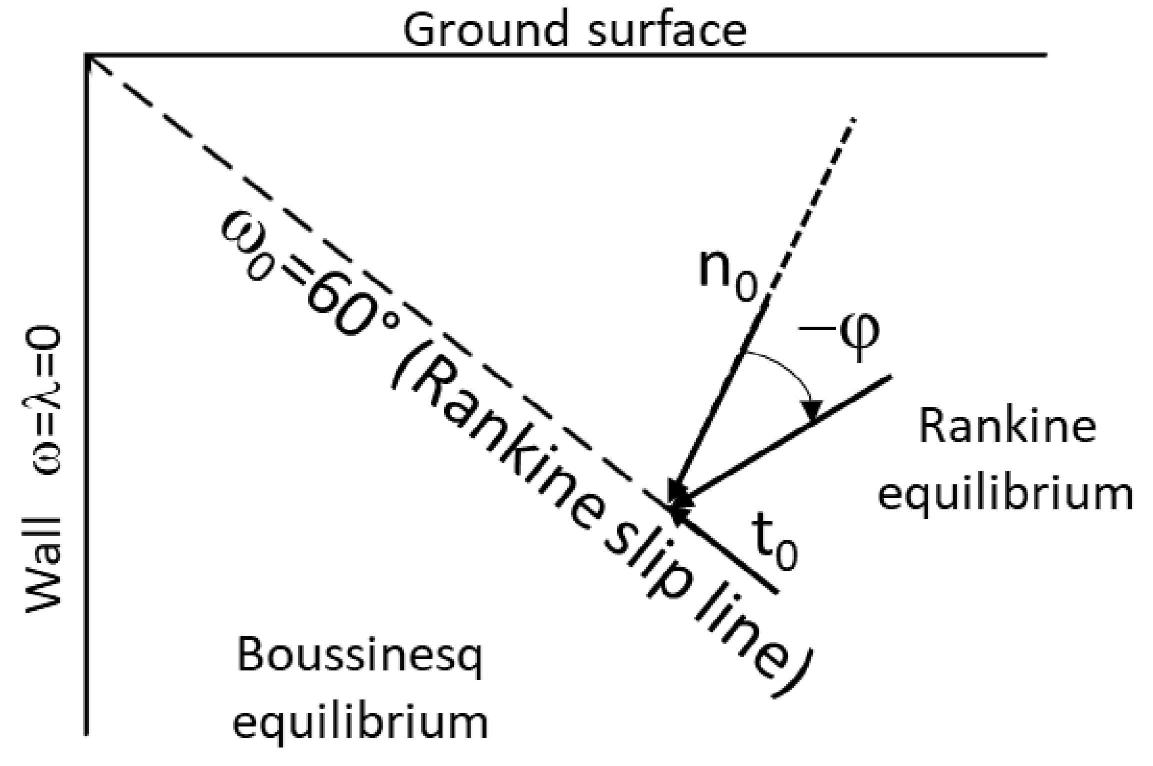
$$\frac{dn}{d\omega} = 3t - sin\omega \ (1)$$

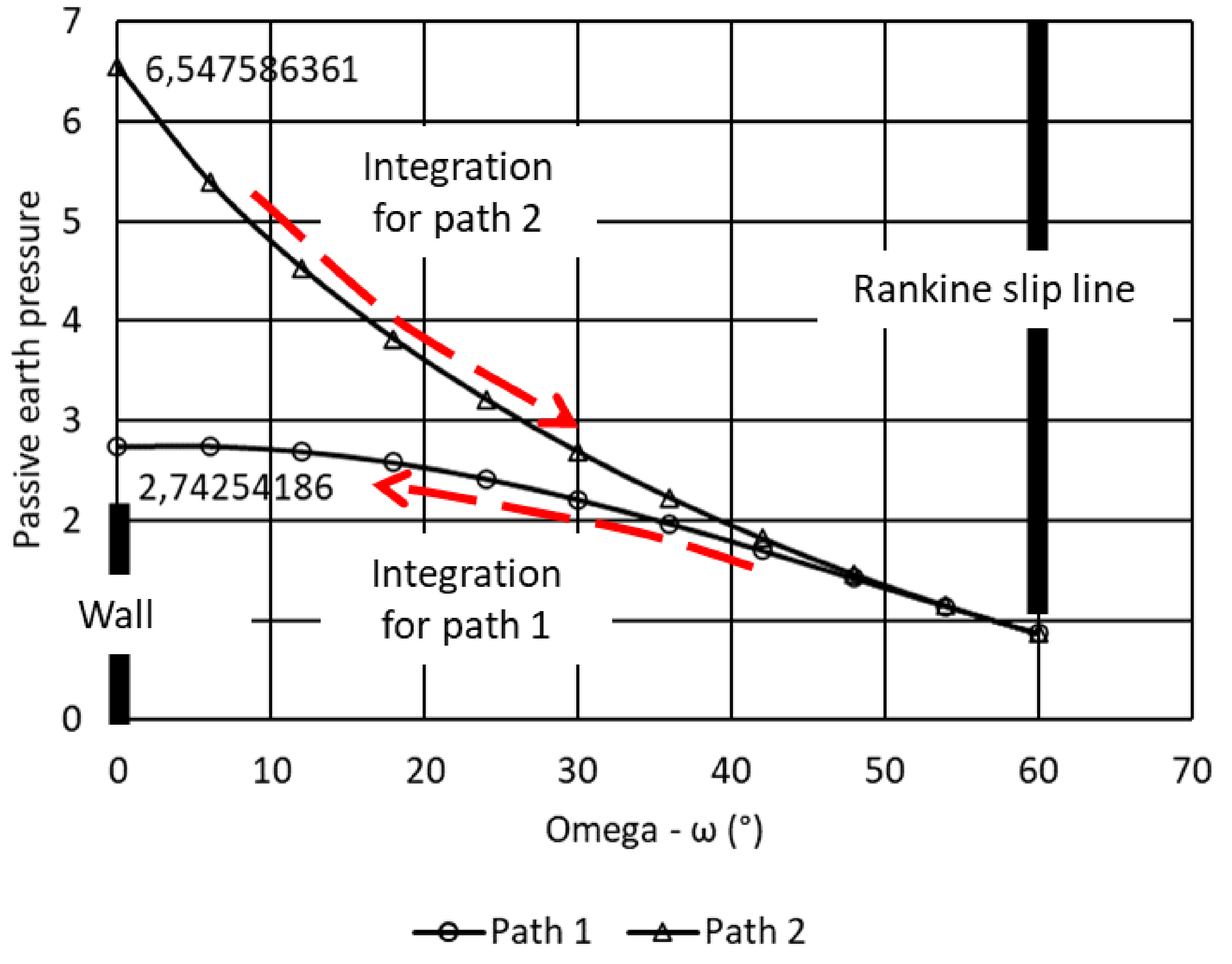
$$\frac{dt}{d\omega} = nm - cos\omega \ (2)$$

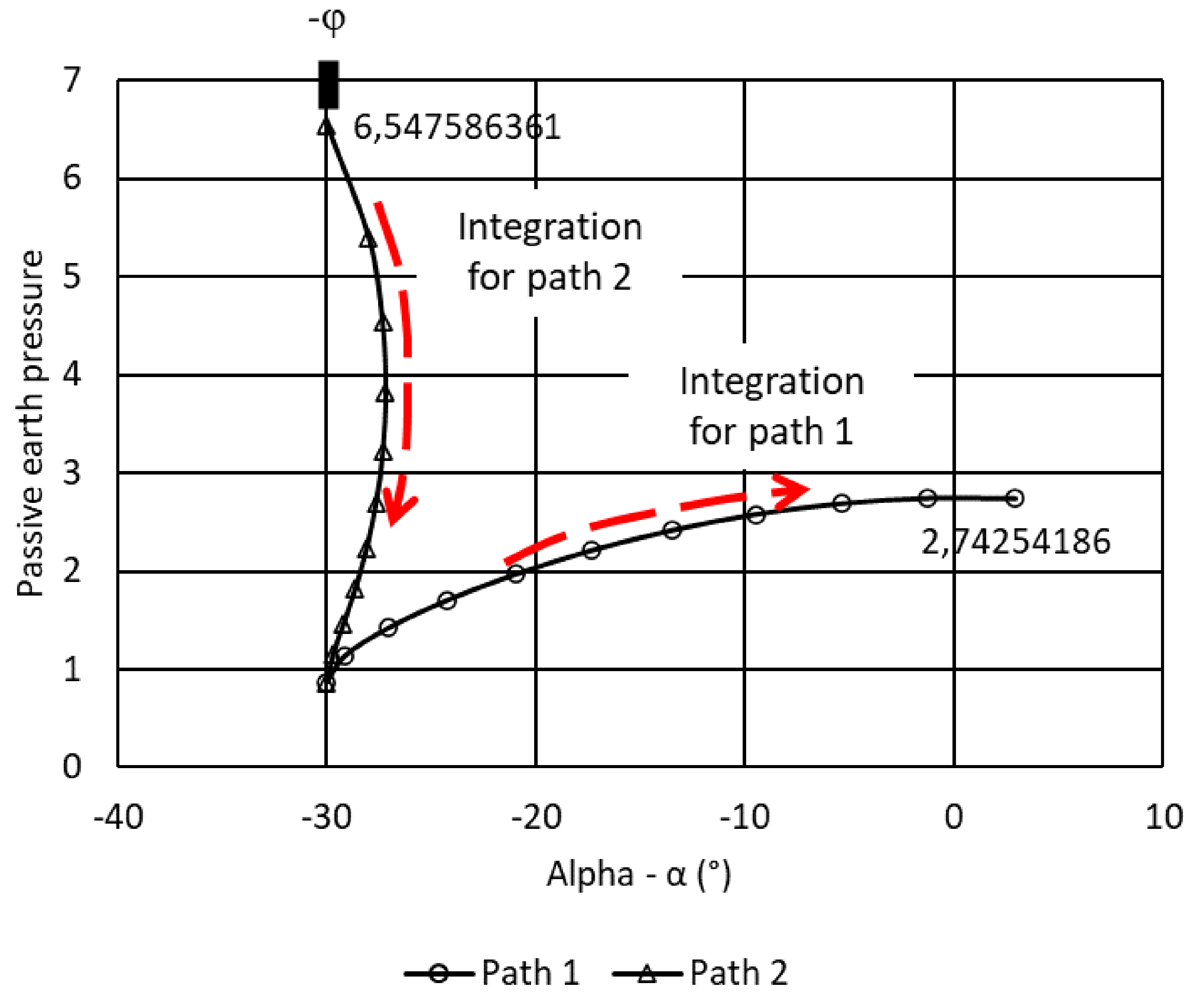
Runge Kutta (RK4) numerical integration method

An example of numerical integration

φ=30°, β=0, λ=0, δ/φ=-1   (case 1)







$$\frac{dn}{d\omega} = 3t - sin\omega \ (1)$$

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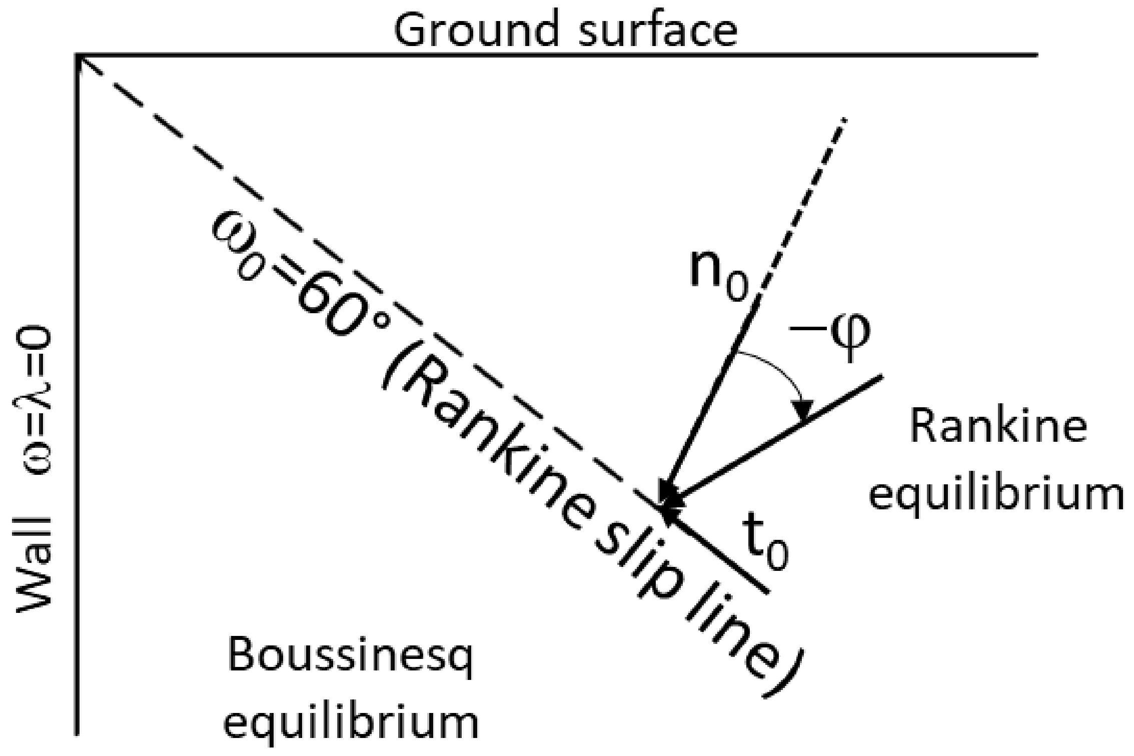
Runge Kutta (RK4) numerical integration method

An example of numerical integration

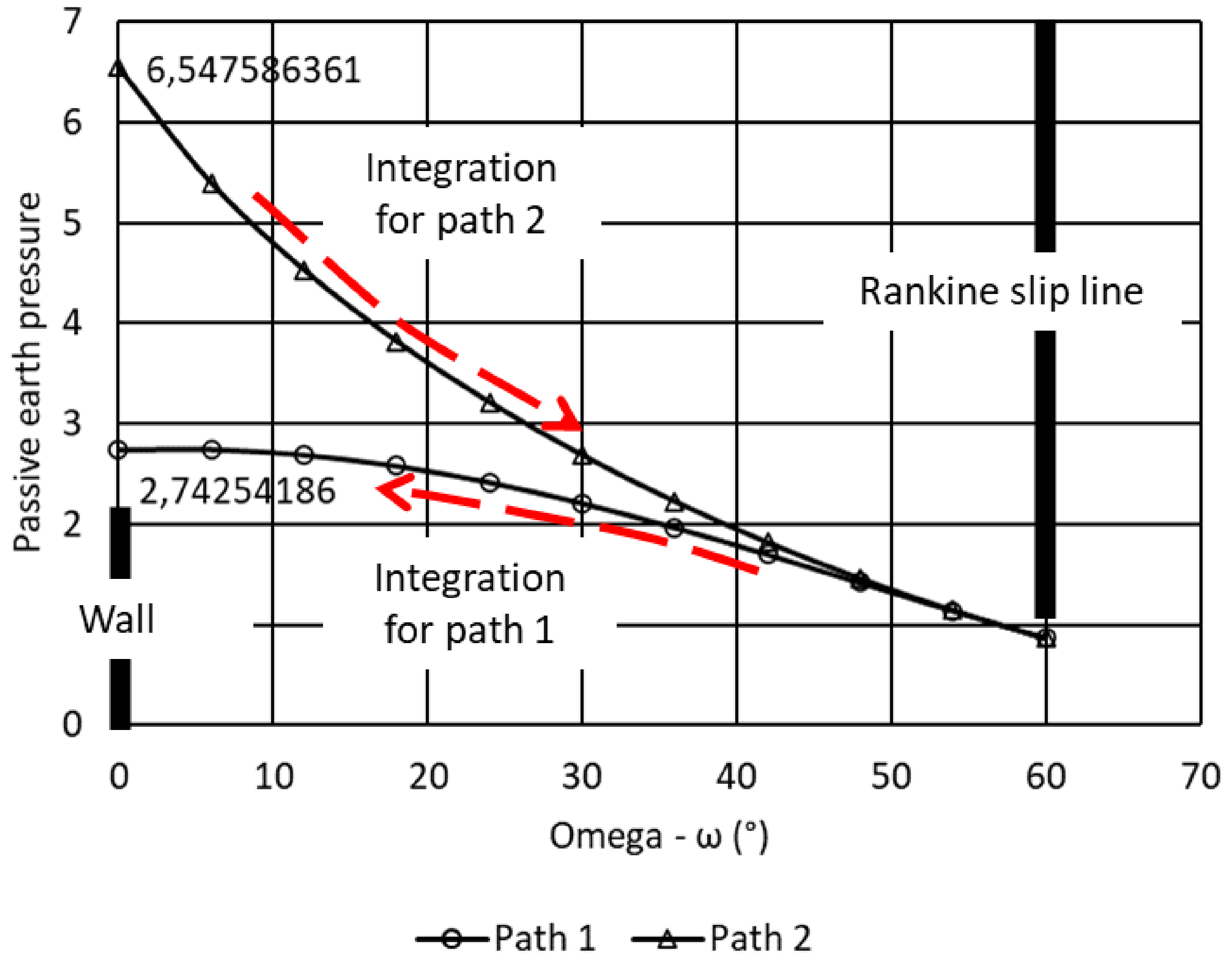
φ=30°, β=0, λ=0, δ/φ=-1 (case 1)

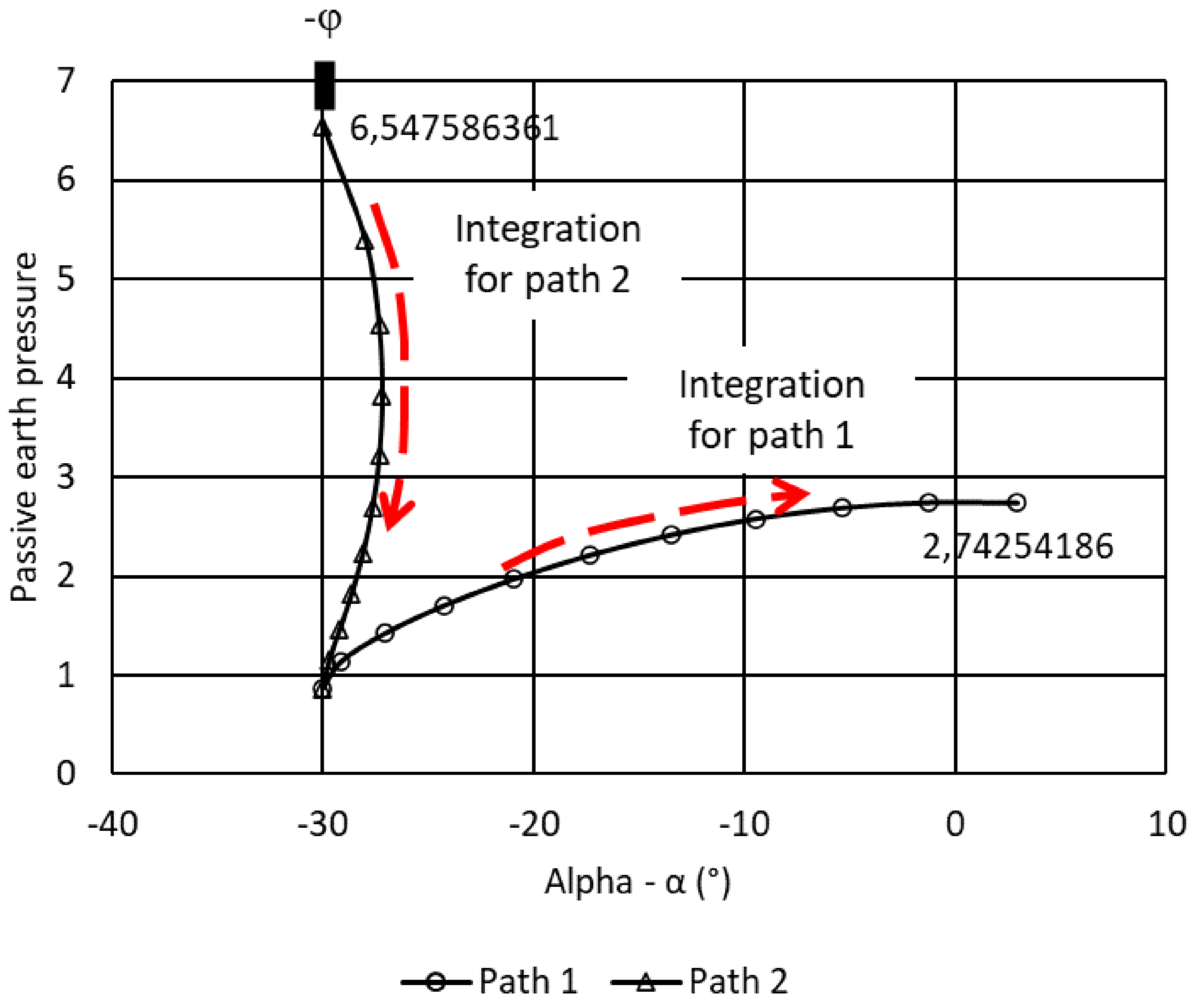


$$\frac{dn}{d\omega} = 3t - sin\omega \ (1)$$

$$\frac{dt}{d\omega} = nm - cos\omega \ (2)$$

Runge Kutta (RK4) numerical integration method





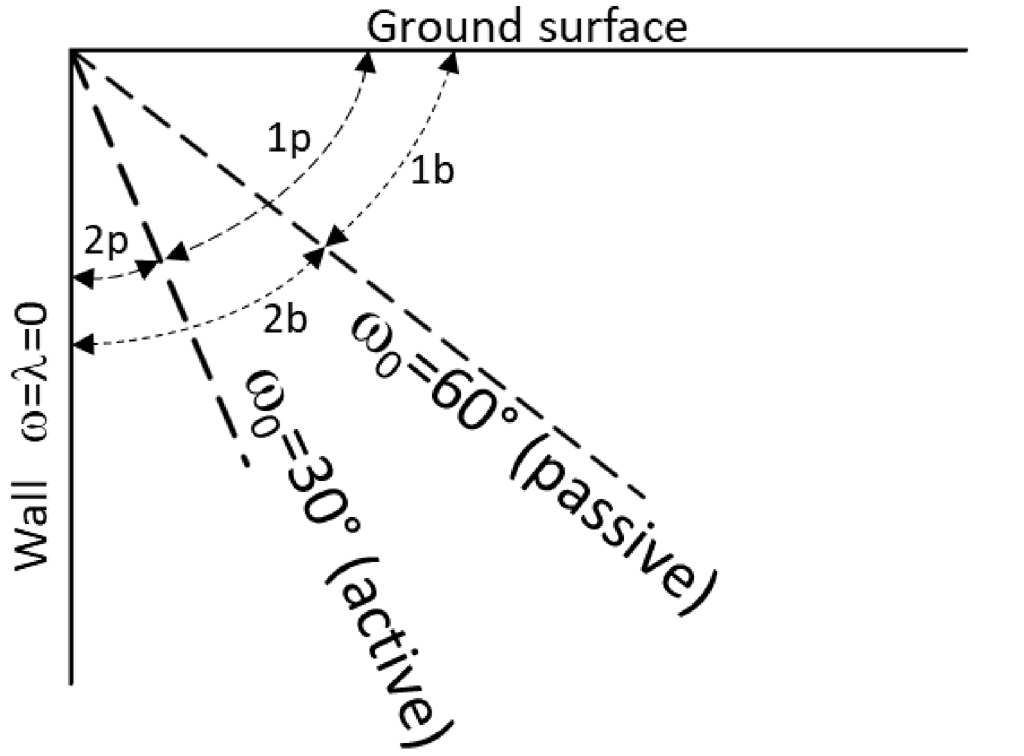
Method presented: K_p=6,548

Caquot, Kerisel, Absi: K_p=6,42 (1948), K_p=6,56 (1966), K_p=6,50 (1990)

Sokolowski (1965): K_p=6,55

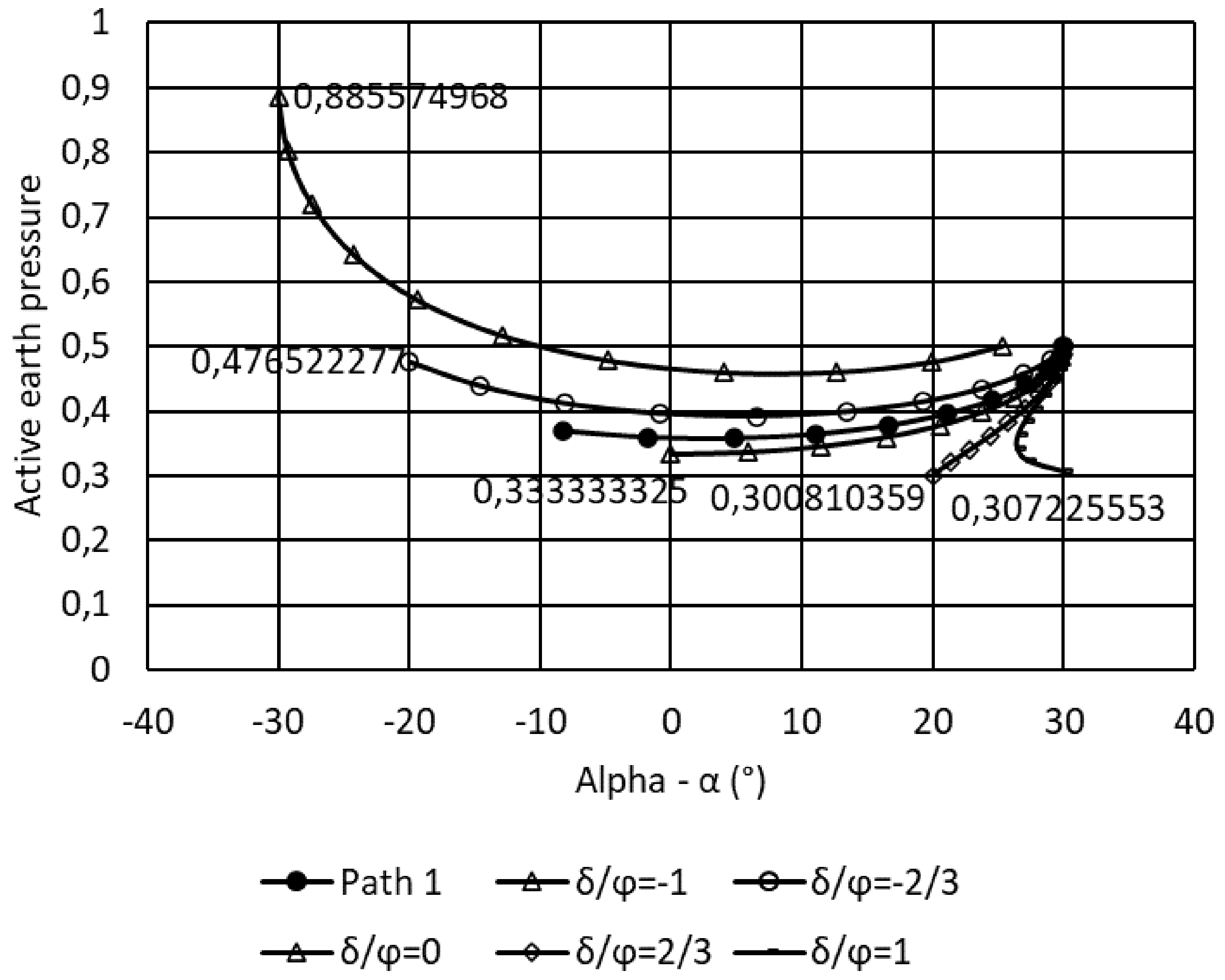
Other examples

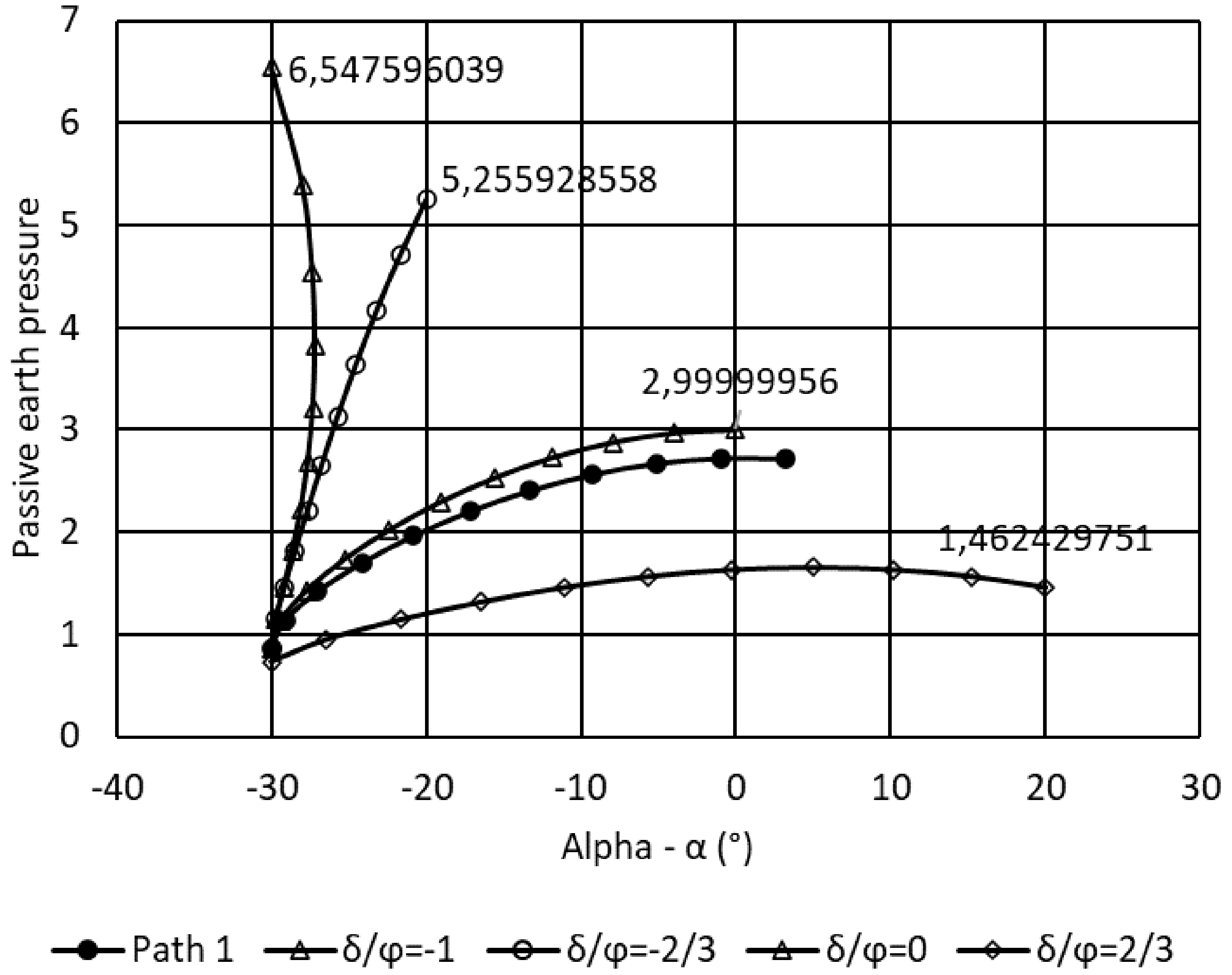
φ=30°, β=0, λ=0 (case 1)



1p : Rankine equilibrium for active state
1b : Rankine equilibrium for passive state

2p : Boussinesq equilibrium for active state
2b : Boussinesq equilibrium for passive state





Active earth pressure – δ/φ

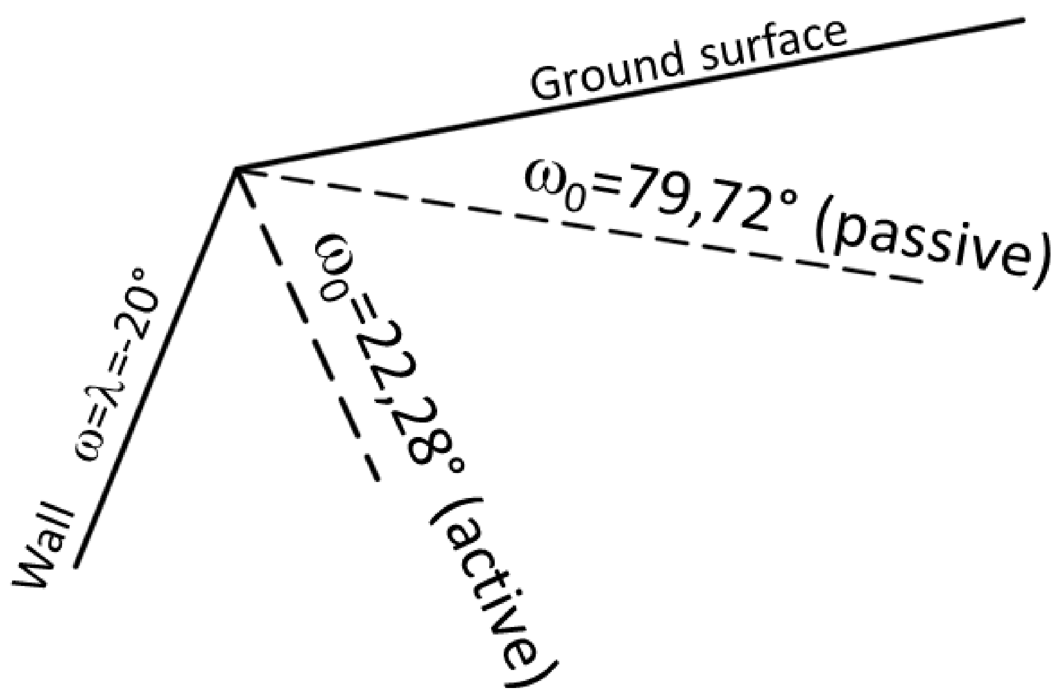
Relative inclination	-1	-2/3	0	2/3	1
Presented method	0,886	0,477	0,333	0,301	0,307
Caquot Kerisel	0,981	0,476	0,333	0,300	0,308
Coulomb	0,866	0,469	0,333	0,297	0,297

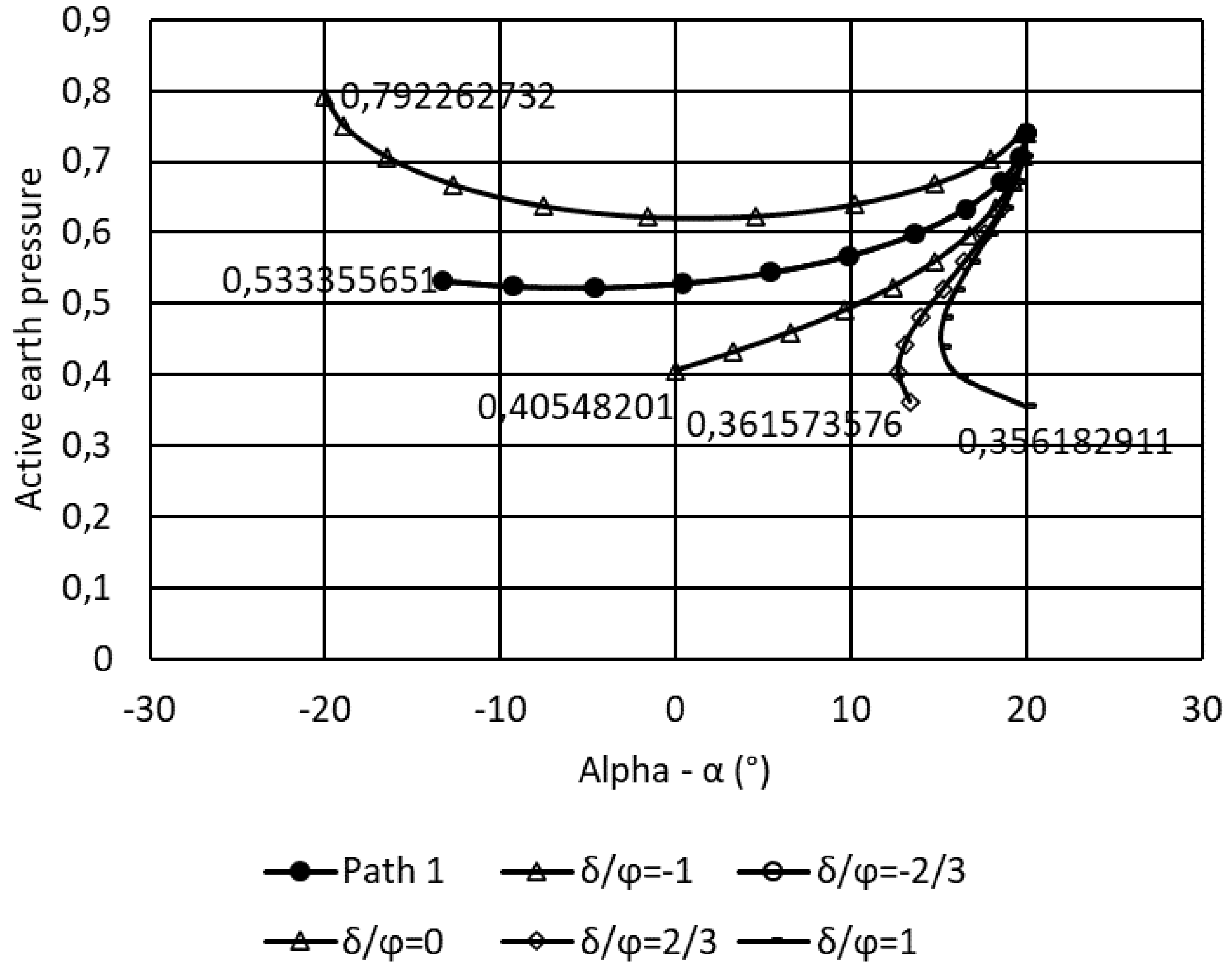
Passive earth pressure – δ/φ

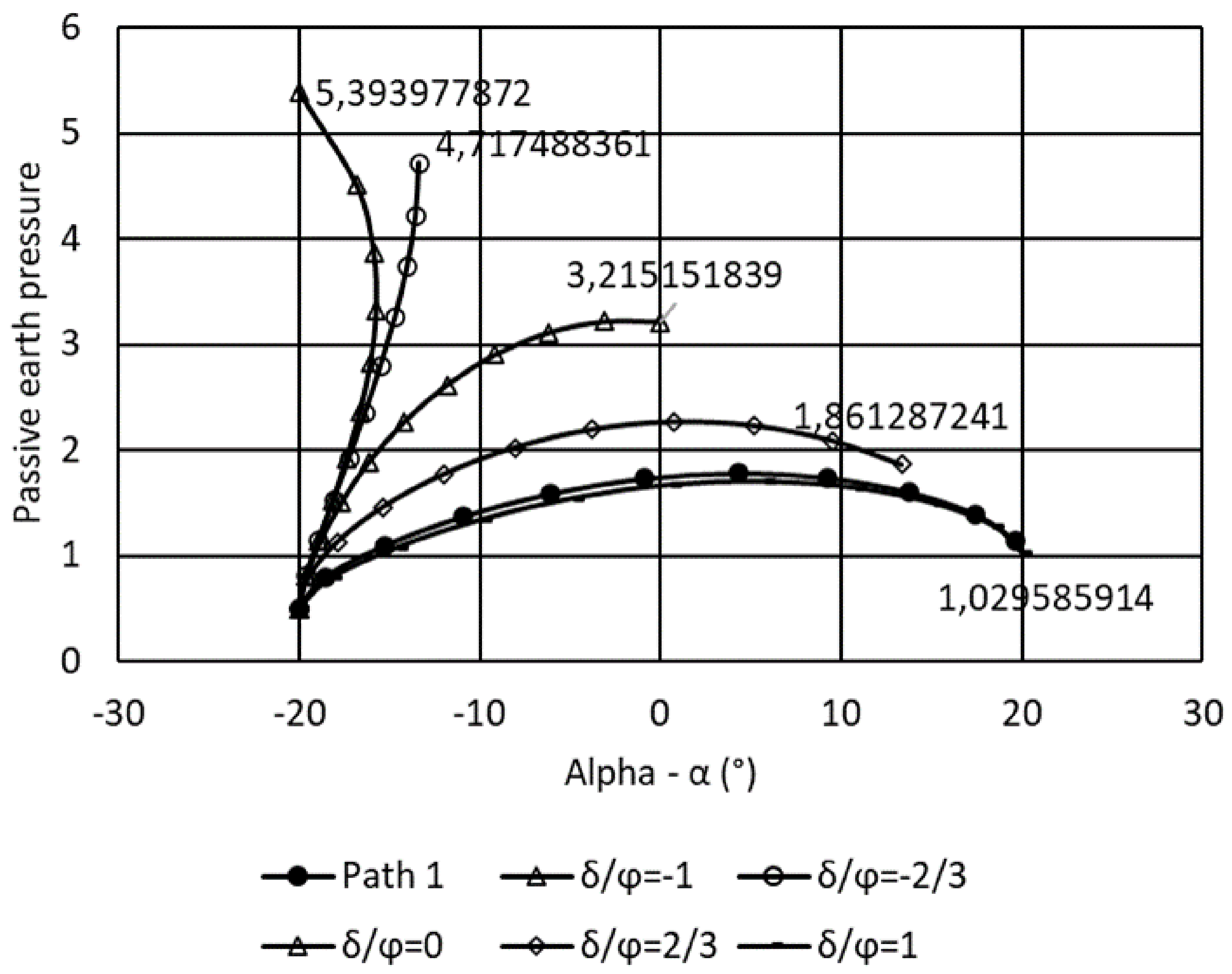
Relative inclination	-1	-2/3	0	2/3	1
Presented method	6,55	5,26	3,0	1,46	---
Caquot Kerisel	6,50	5,30	3,0	1,46	---

Other examples

φ=20°, β=12°, λ=-20° (case 2)







Active earth pressure – δ/φ

Relative inclination	-1	-2/3	0	2/3	1
Presented method	0,792	0,533	0,405	0,362	0,356
Caquot Kerisel	0,869	0,533	0,405	0,358	0,355
Coulomb	0,766	0,533	0,403	0,349	0,334

Passive earth pressure – δ/φ

Relative inclination	-1	-2/3	0	2/3	1
Presented method	5,39	4,72	3,22	1,86	1,03
Caquot Kerisel	5,40	4,70	3,20	1,86	---

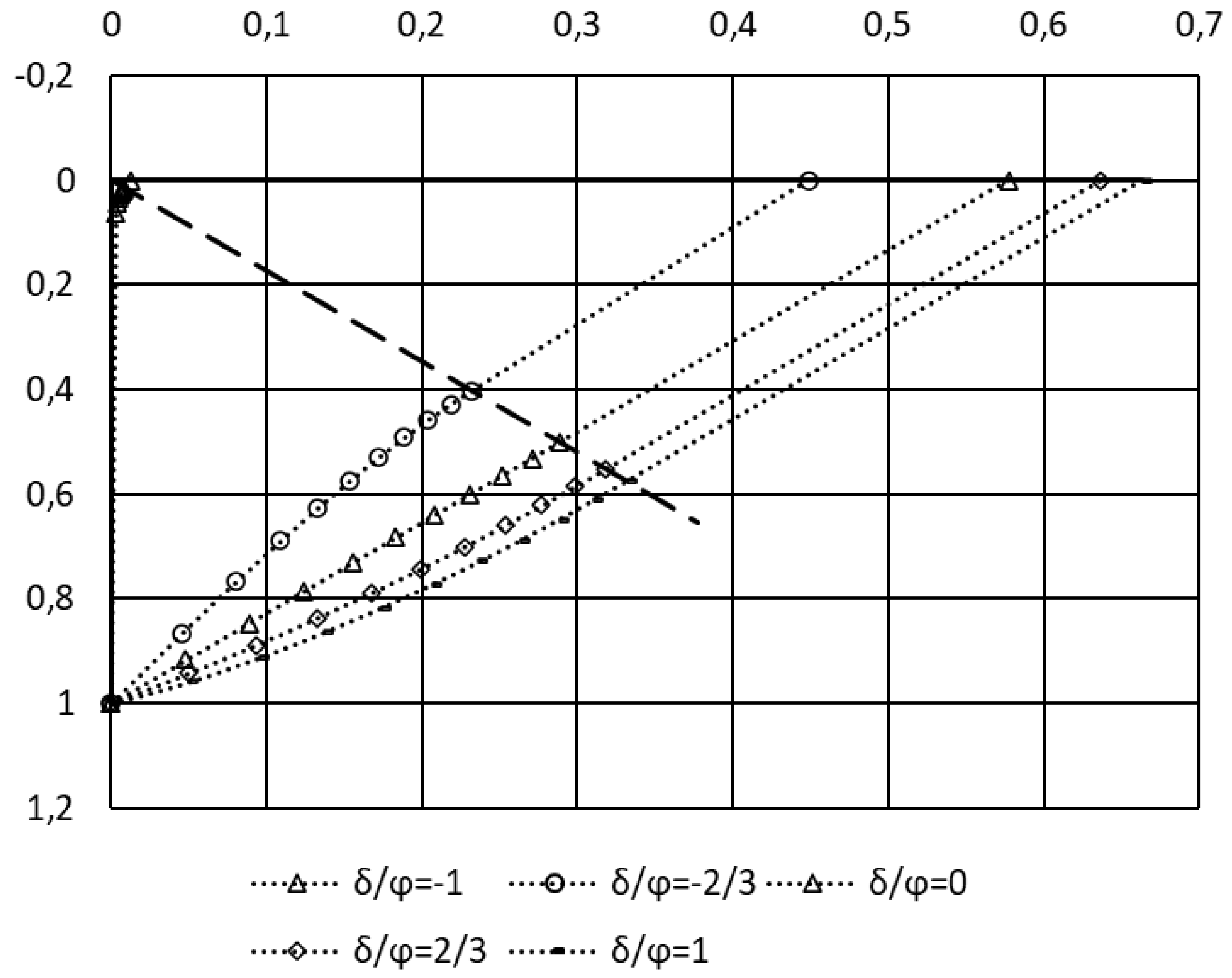
Failure mechanism – Slip lines

Active earth pressures

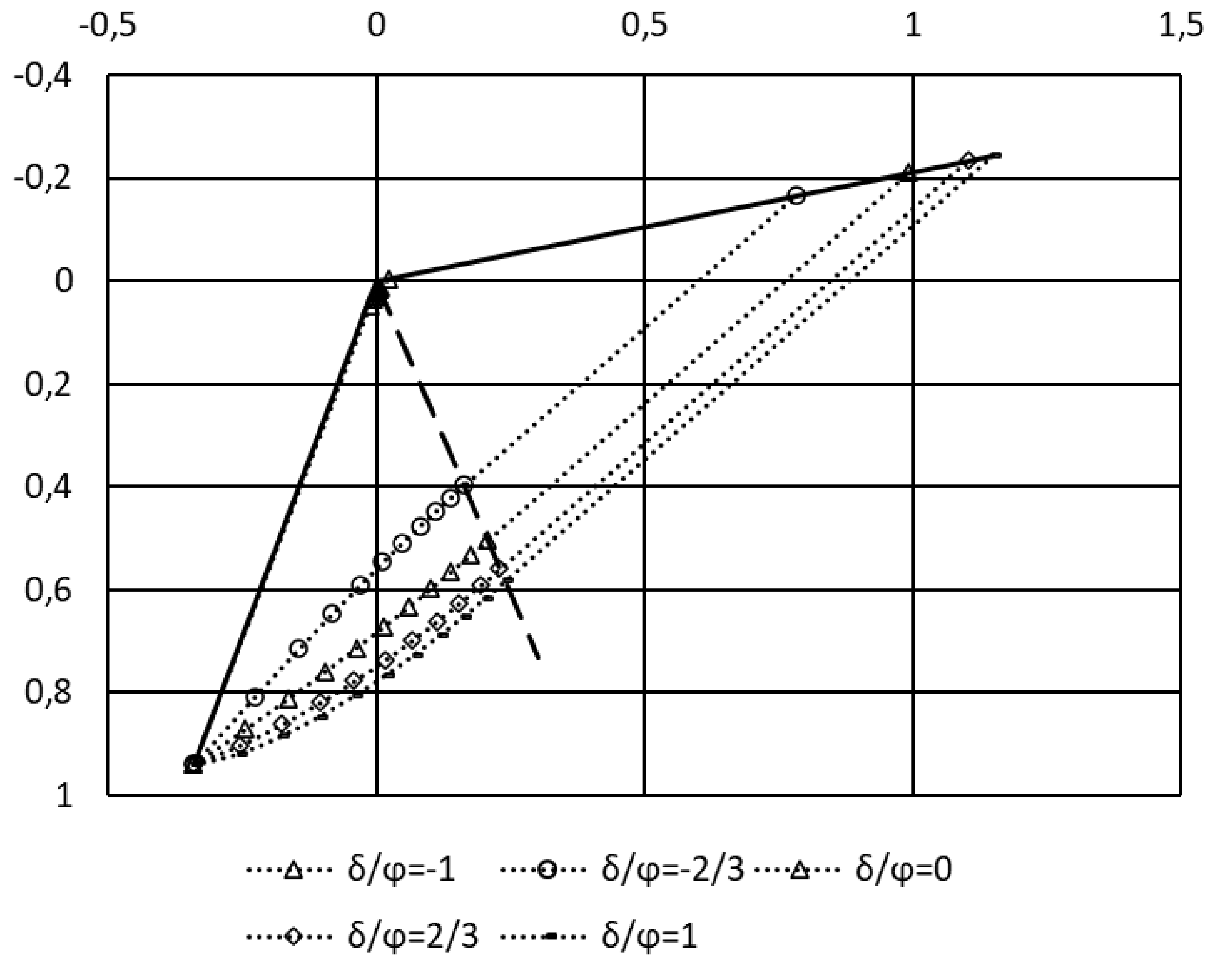
$$r = e^{-\int \tan\xi(\omega).d\omega} \text{ with:}$$

$$\xi(\omega) = \frac{\pi}{4} + \frac{\varphi}{2} - \frac{(\omega_\alpha - \alpha)}{2} \text{ and } \omega_\alpha = \arcsin\left(\frac{\sin\alpha}{\sin\varphi}\right)$$

Case 1



Case 2



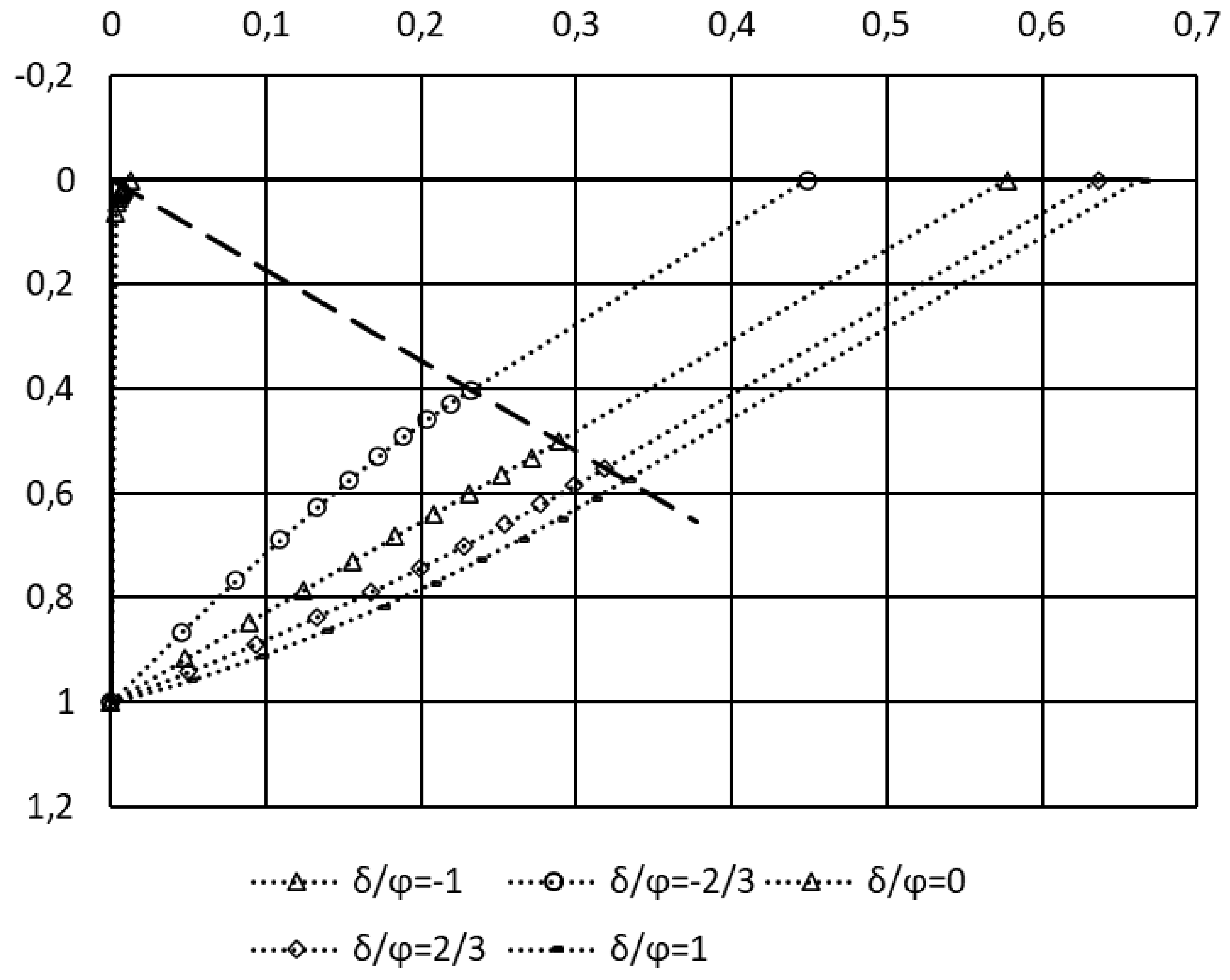
Failure mechanism – Slip lines

Active earth pressures

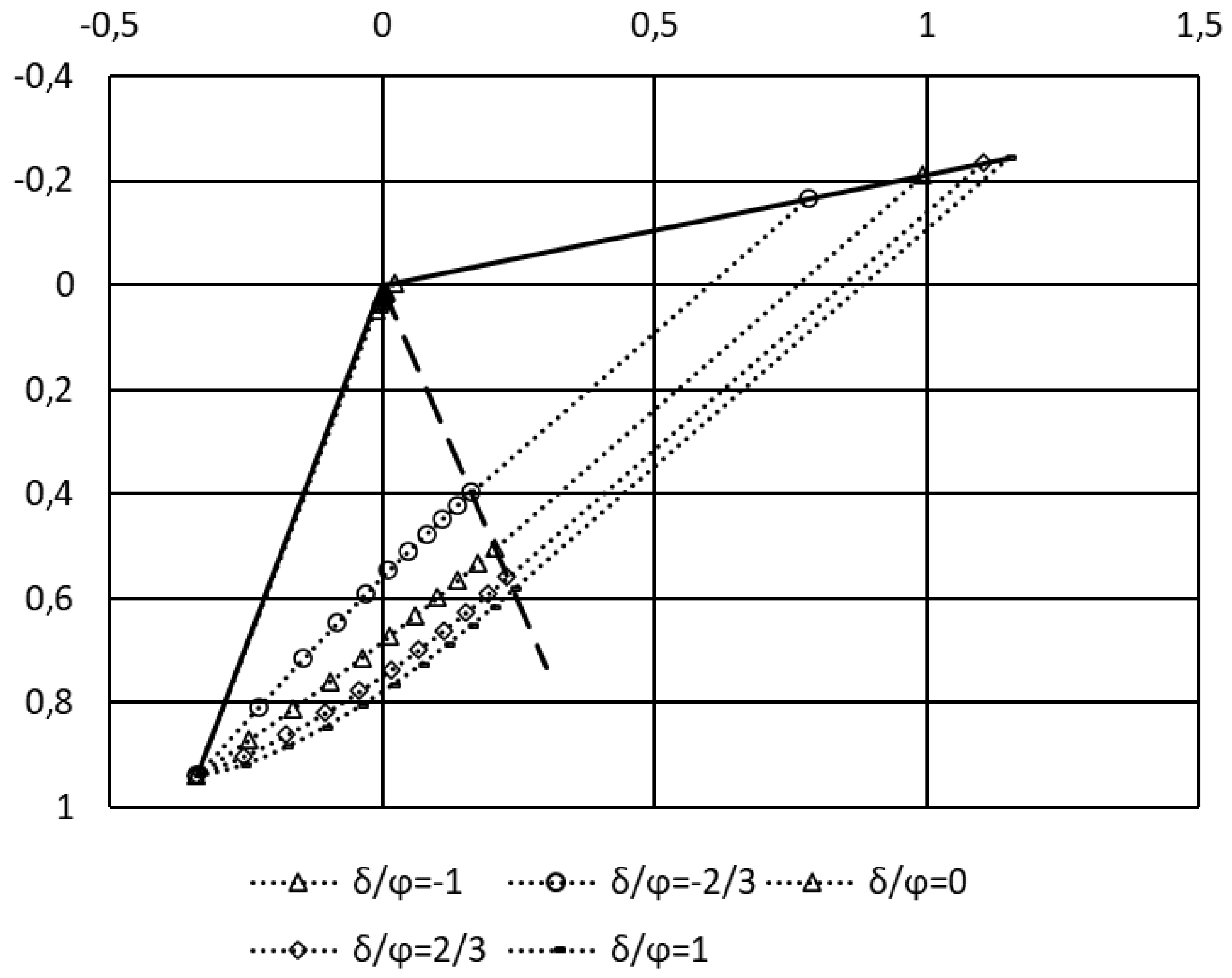
$$r = e^{-\int tan\xi(\omega).d\omega} \ with:$$

$$\xi(\omega) = \frac{\pi}{4} + \frac{\varphi}{2} - \frac{(\omega_\alpha - \alpha)}{2} \ and \ \omega_\alpha = arcsin\left(\frac{sin\alpha}{sin\varphi}\right)$$

Case 1



Case 2

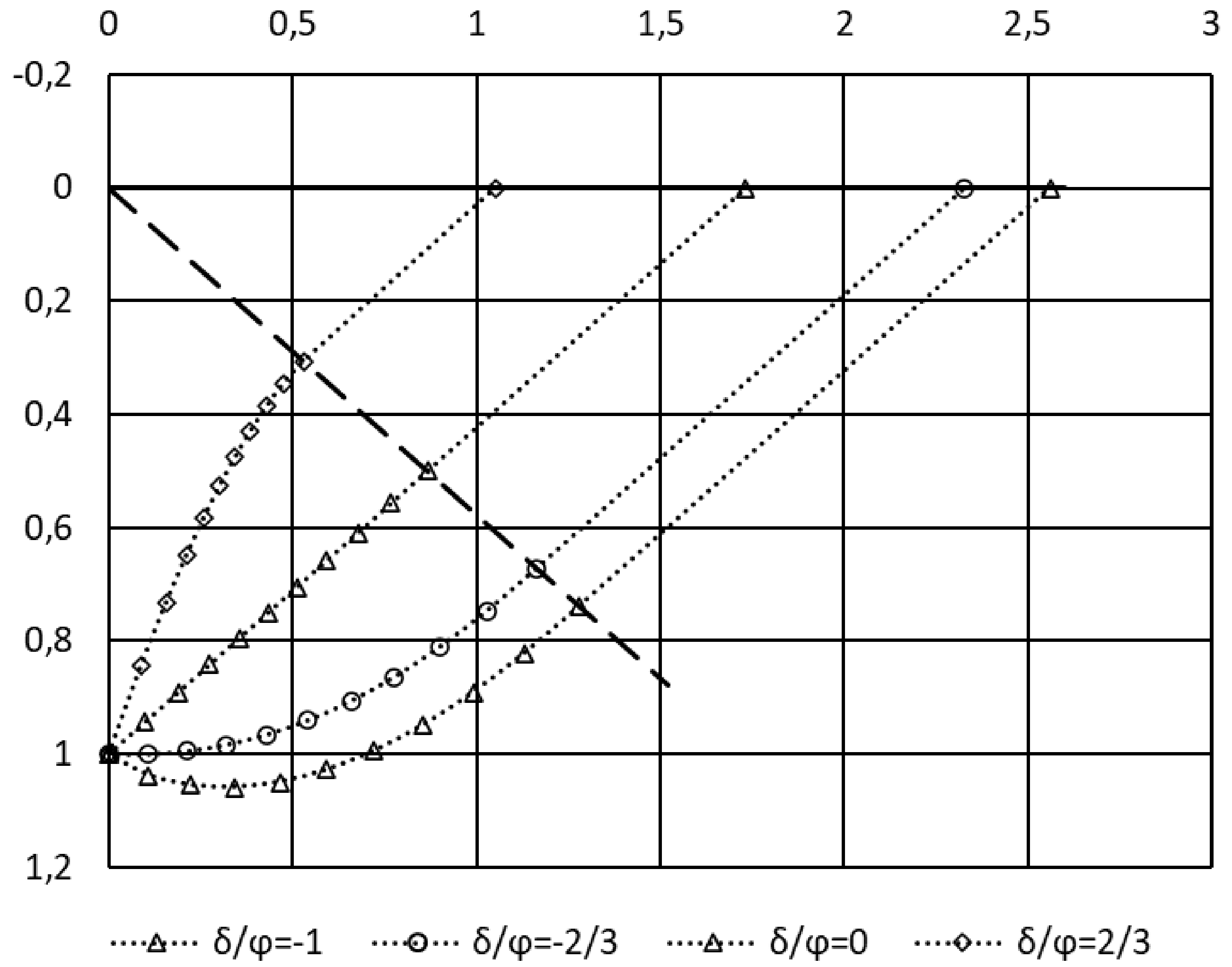


Passive earth pressures

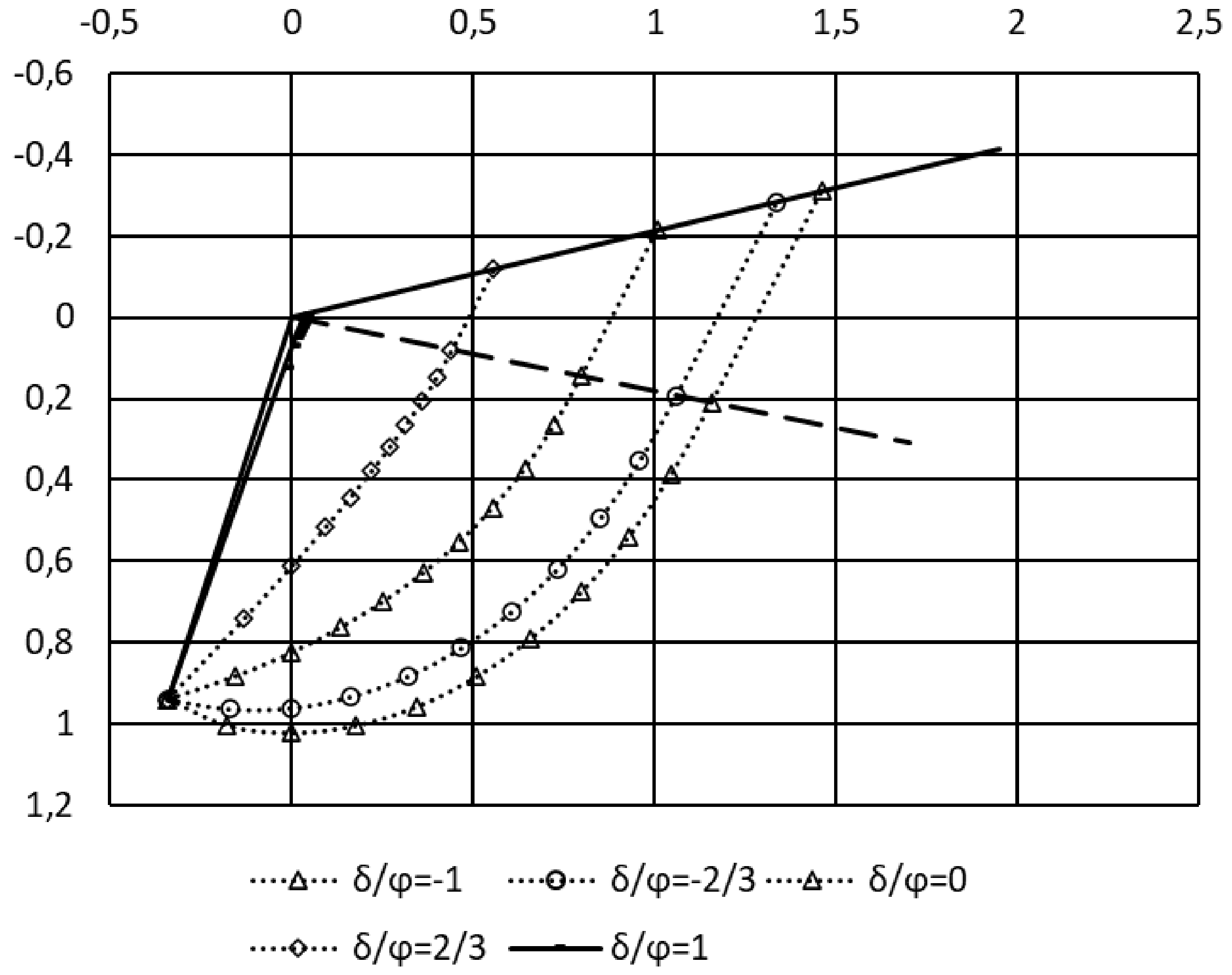
$$r = e^{-\int tan\xi(\omega).d\omega} \ with:$$

$$\xi(\omega) = \frac{\pi}{4} - \frac{\varphi}{2} + \frac{(\omega_\alpha + \alpha)}{2} \ and \ \omega_\alpha = arcsin\left(\frac{sin\alpha}{sin\varphi}\right)$$

Case 1

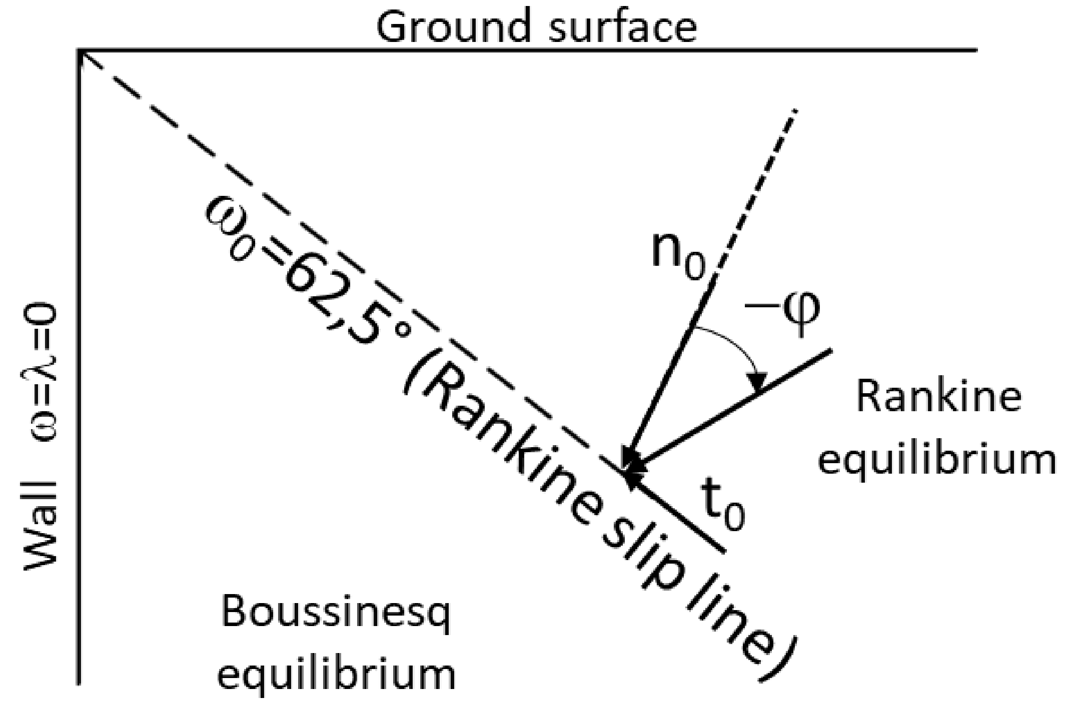


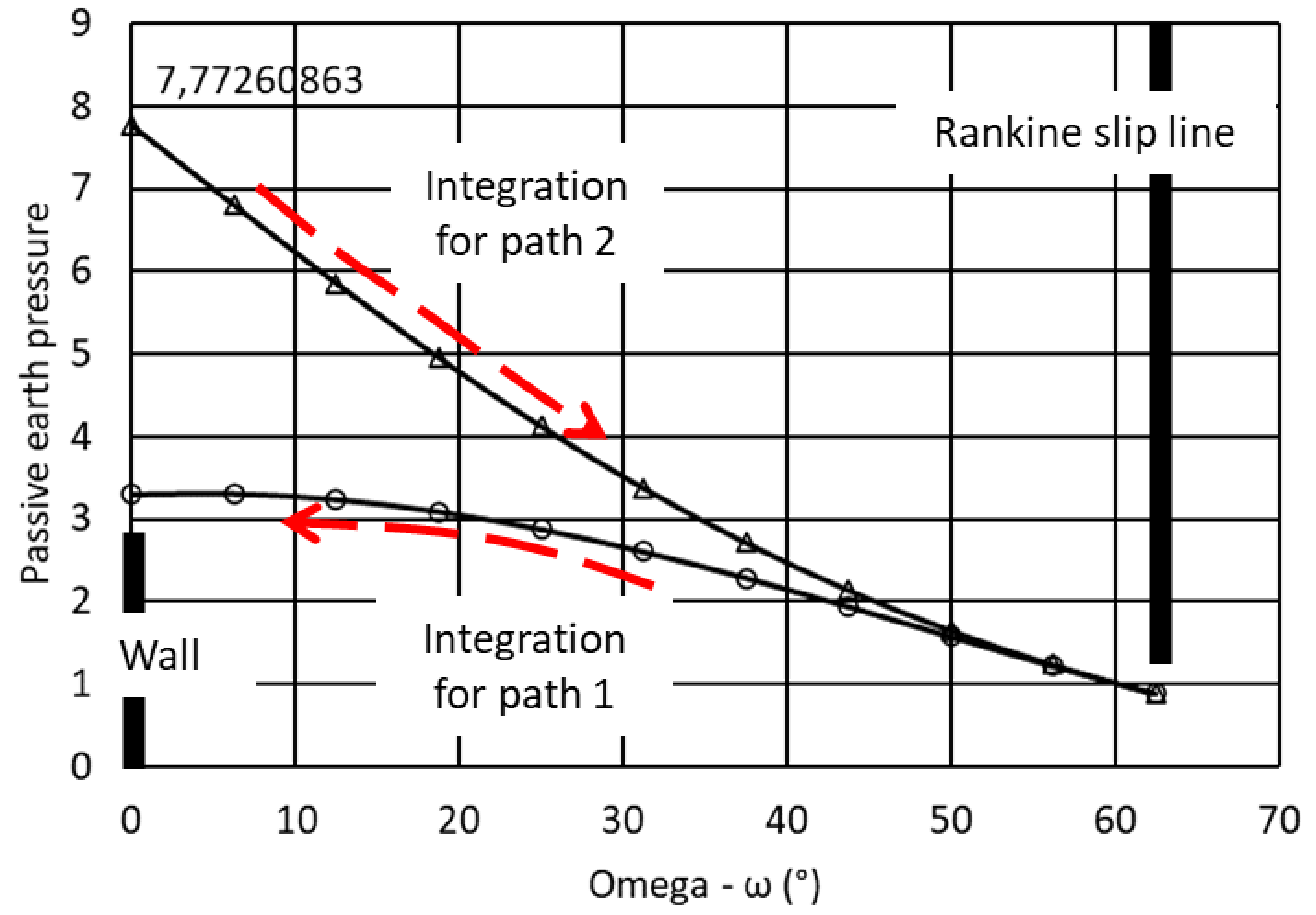
Case 2

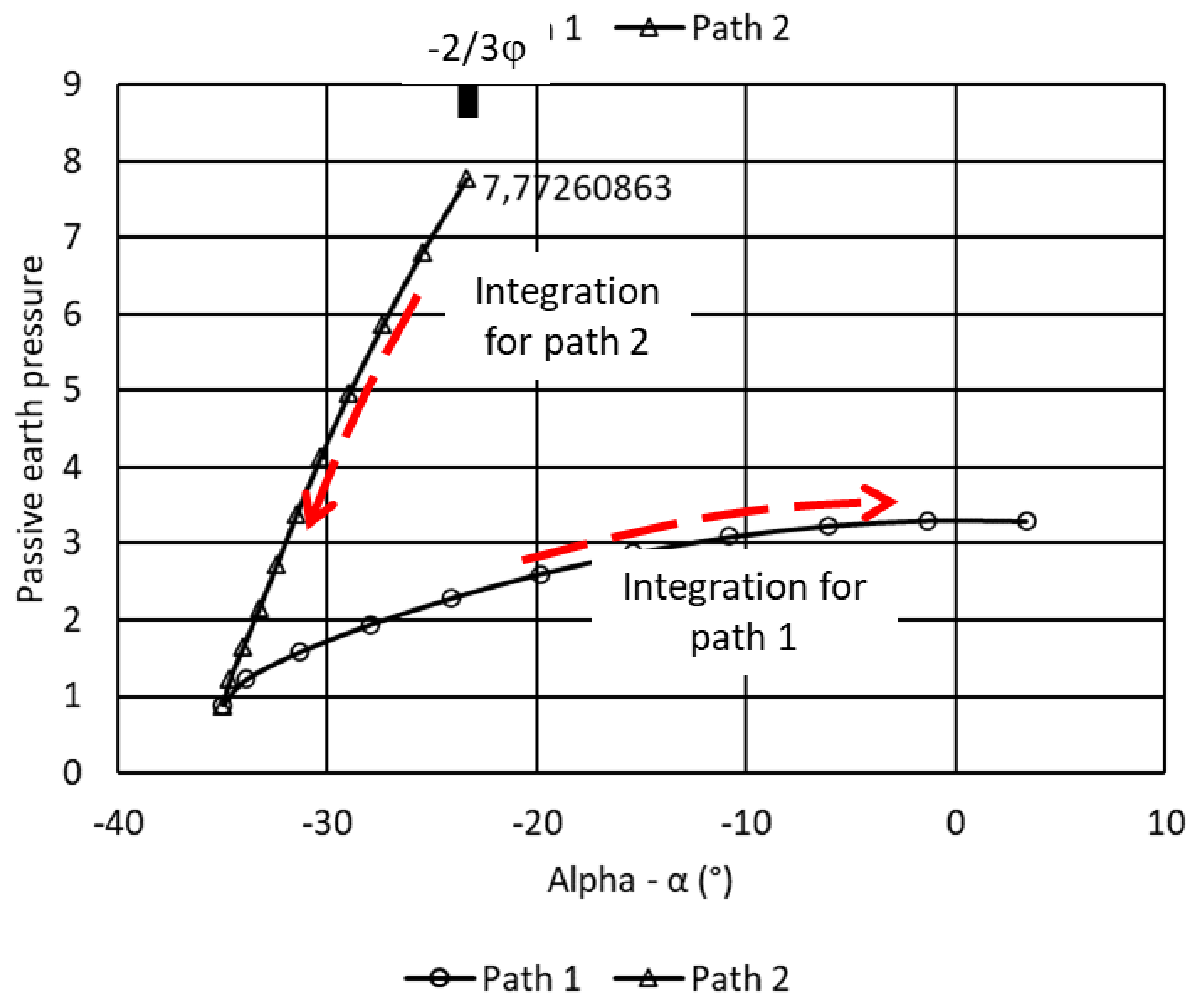


A last example – Comparison with the kinematical approach of the yield analysis

φ=35°, β=0, λ=0, δ/φ=-2/3





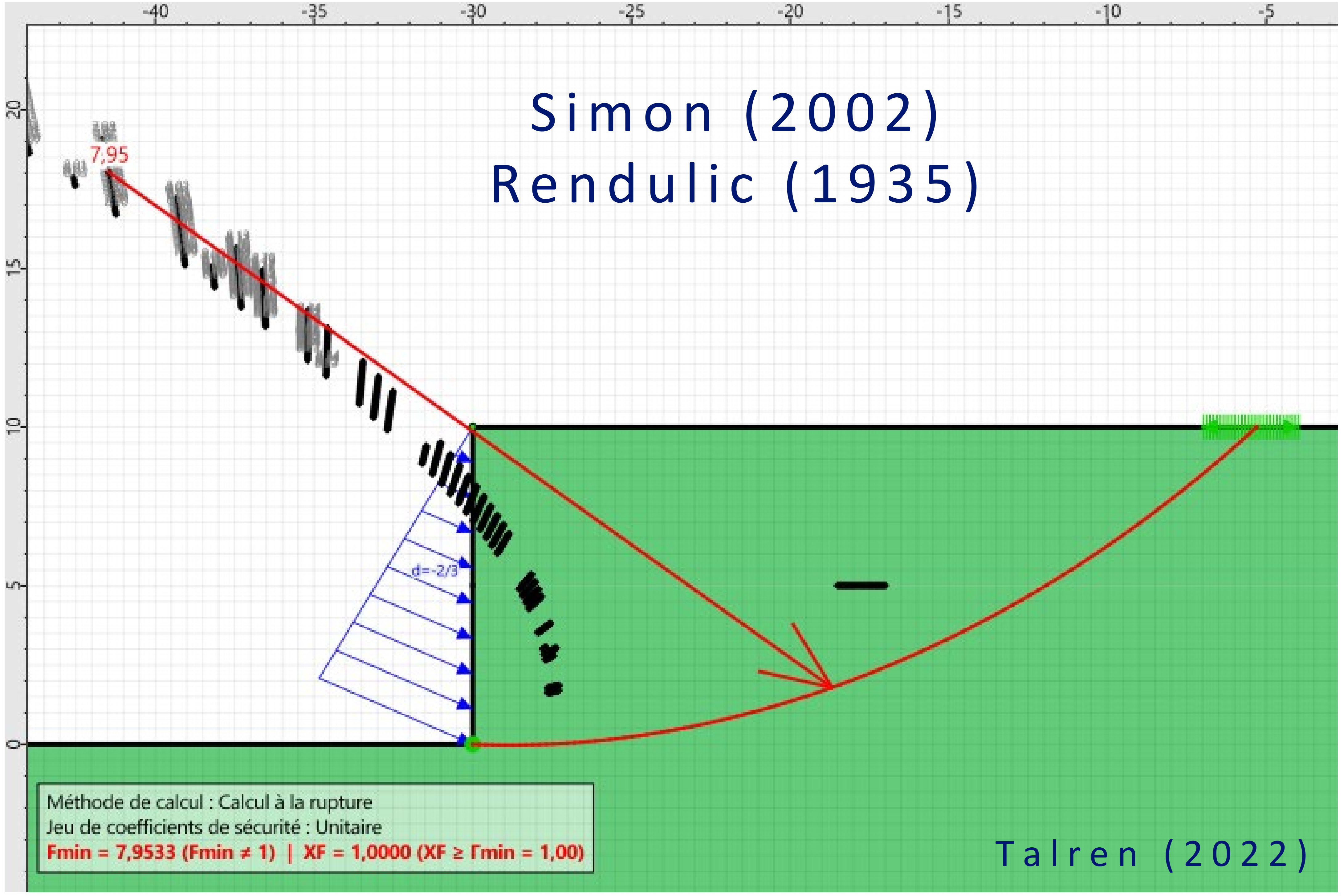


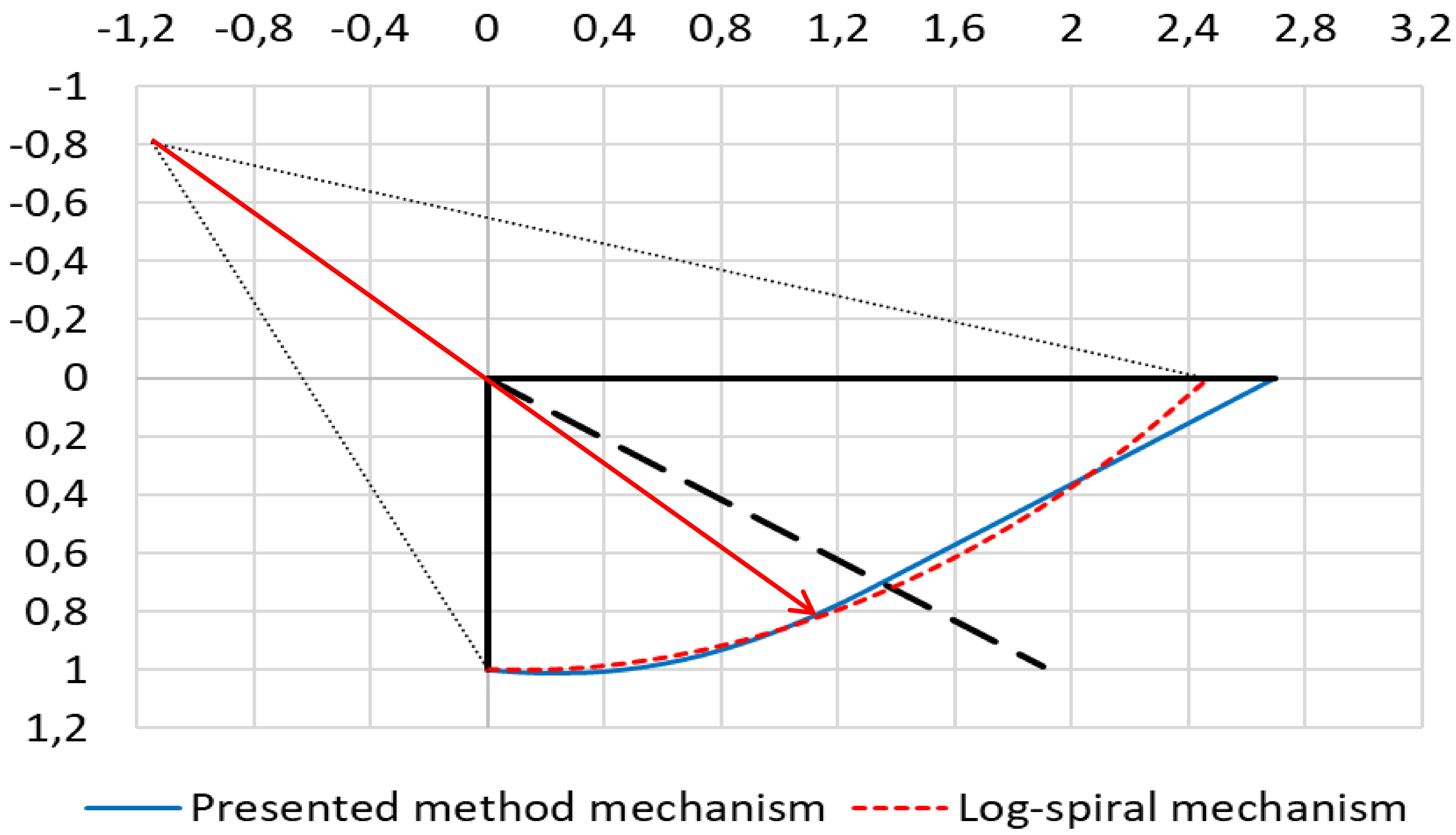
Presented method: K_p=7,77

Caquot-Kerisel: K_p=8,0

The difference is significant even if it remains very low and negligible for the practice.

A last example – Comparison with the kinematical approach of the yield analysis





A kinematical approach provides: K_p=7,95

The two mechanisms are very similar.

→ The « true » value should be between 7,77 and 7,95.

Conclusions

Around 175 years after Coulomb, Caquot and Kerisel developped another approach to assess active and passive earth pressures for weighted ground conditions from the contributions of Bousinesq, Résal and Ravizé.

Failure mechanisms can also be obtained and compared to those considered by Coulomb explaining why Coulomb approach is not on the safe side.

Today, Bousinesq, Caquot and Kerisel approaches can be compared to some other solutions obtained by the kinematical approach of the yield analysis.

New values of active and passive earth pressure coefficients might be thus obtained by comparing both Bousinesq, Caquot and Kerisel approaches and the kinematical approach of the yield analysis.

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Thank you for your attention