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COMITÉ FRANÇAIS DE MÉCANIQUE DES SOLS ET DE GÉOTECHNIQUE

INSTITUT DE FRANCE
Académie des sciences

ACADÉMIE DES TECHNOLOGIES
POUR UN PROGRÈS RAISONNÉ, CHOISI ET PARTAGÉ

SIMSG ISSMGE

LES TRAVAUX PUBLICS FÉDÉRATION NATIONALE

Charles-Augustin COULOMB - A geotechnical tribute

Paris, Septembre 25 & 26, 2023

Earth pressure estimation – What existed before Coulomb

Jean-David Vernhes, UniLaSalle

BUREAU VERITAS
Shaping a World of Trust

Geos

SoilCloud

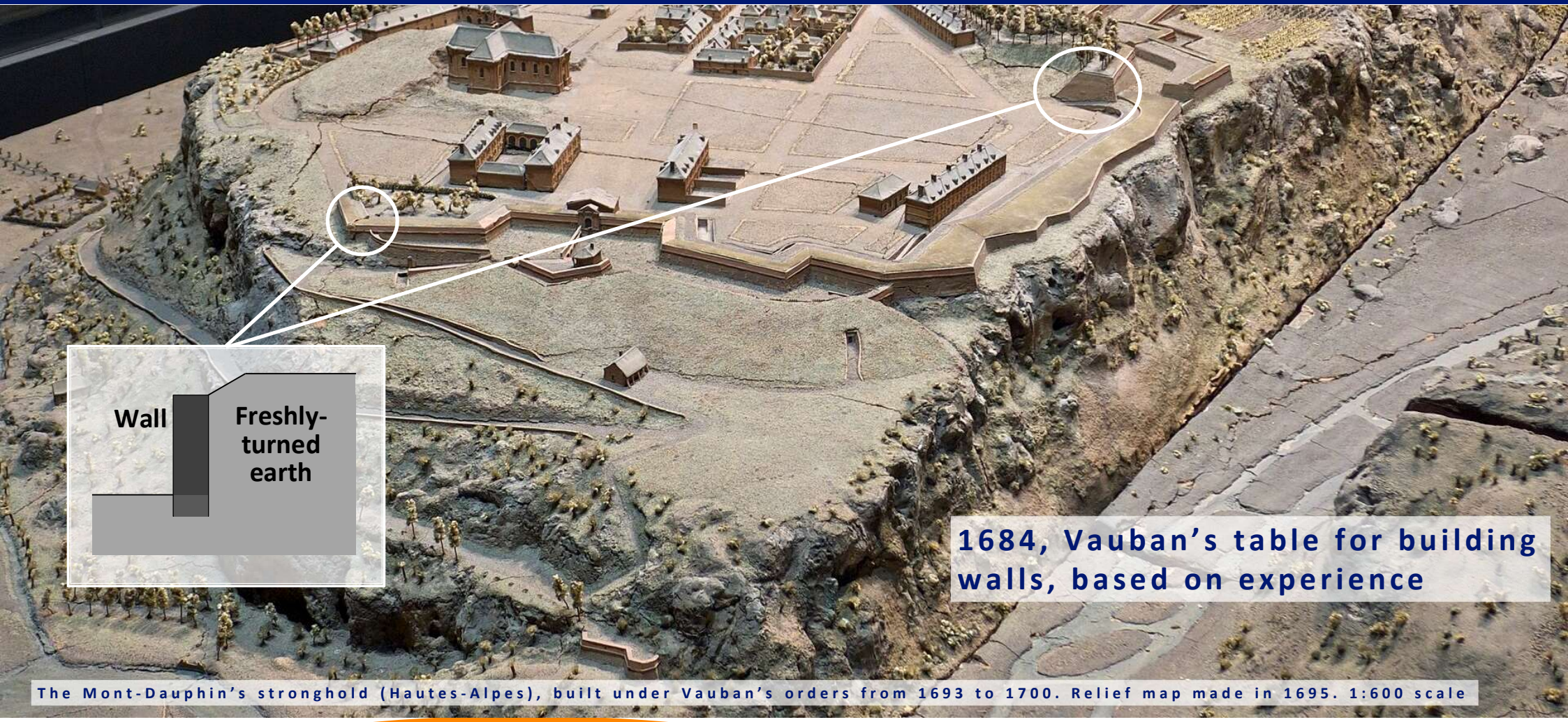
SOLETANCHE BACHY

spie batignolles
/fondations

SSI 2U

terrasol
setec

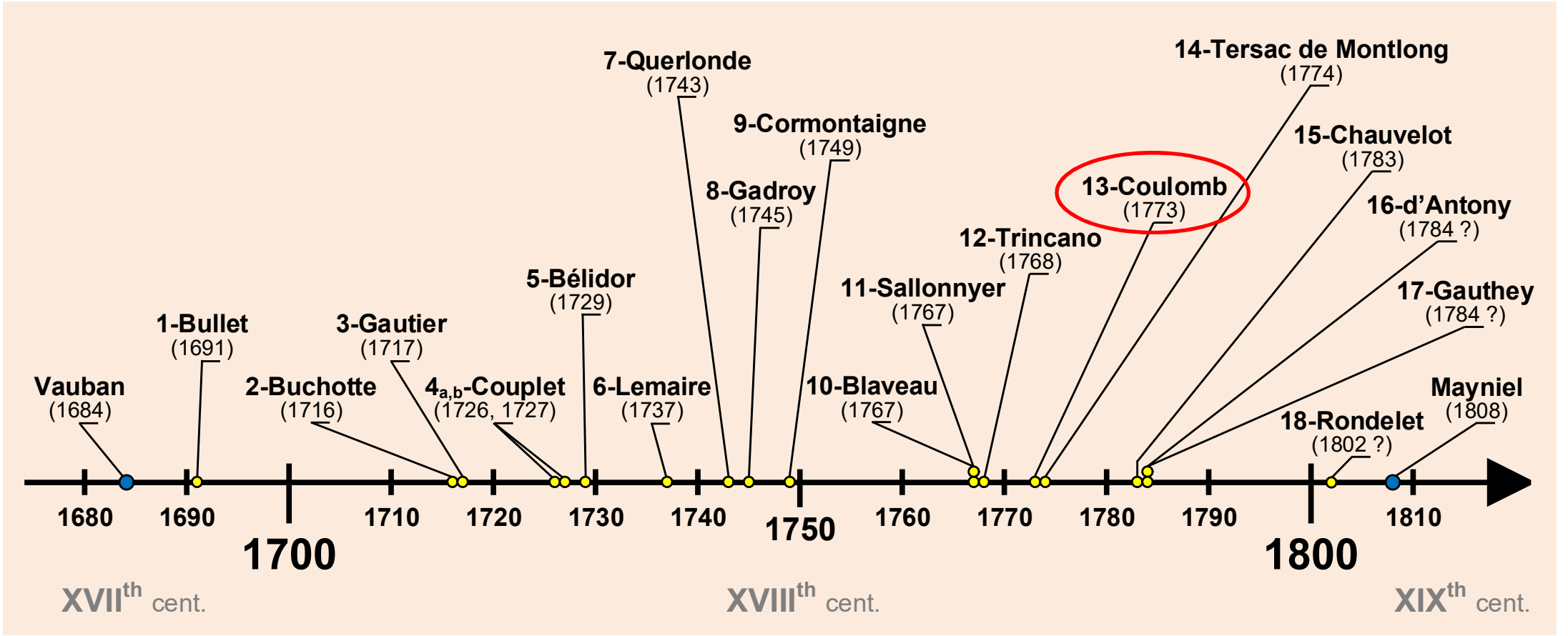
Vauban



1684, Vauban's table for building walls, based on experience

The Mont-Dauphin's stronghold (Hautes-Alpes), built under Vauban's orders from 1693 to 1700. Relief map made in 1695. 1:600 scale

Retaining walls and earth pressure : XVIIIth c. authors timeline



List after Mayniel, 1808, except for Gautier (1717)

See also : Kérisel (1956, 1973, 1993), Verdeyen et al. (1968), Skempton (1985)

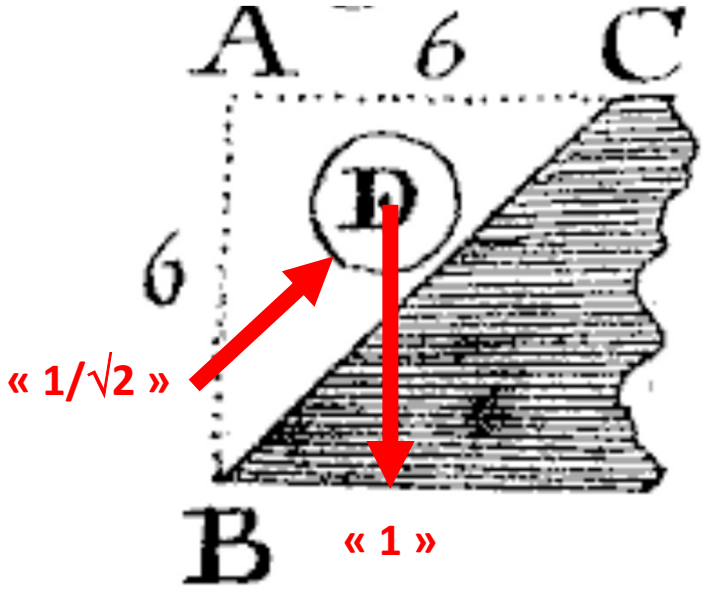
18 authors and their theories*

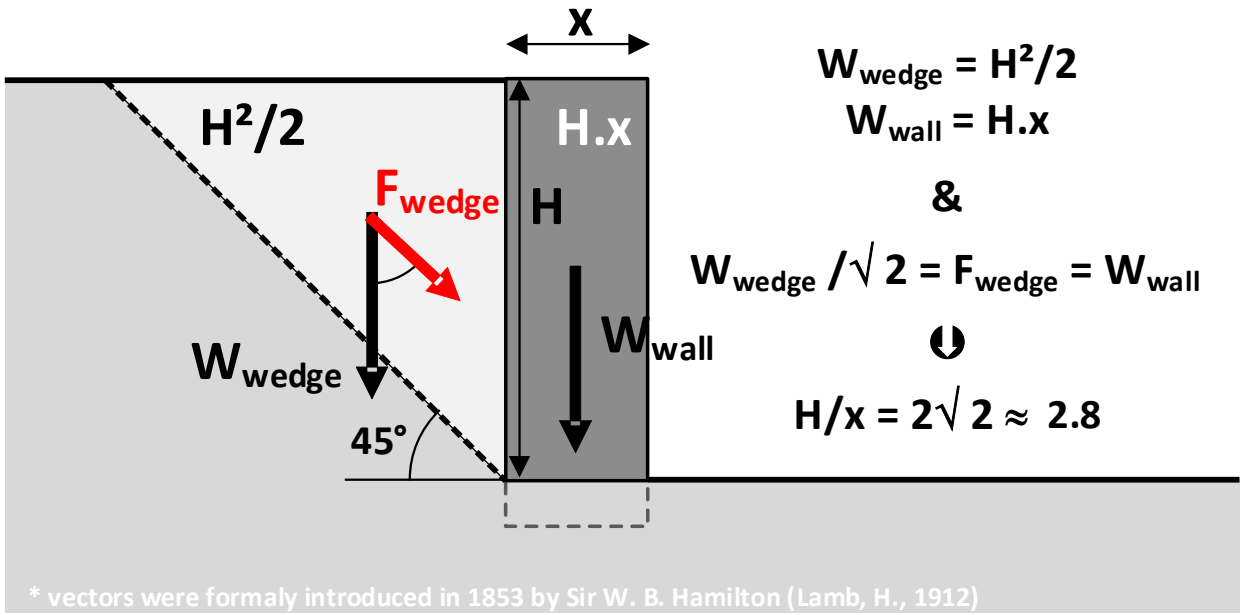
	Year	Author	Context	Elementary model	Design model	Additional theory drawn from Statics	Calculation base	Inclined plane slope angle °	Inclined plane slope angle	Surface of the wedge of height H	Densities ratio W/Earth	Internal resistance	Wall-Earth Friction	Explicit height of the lever	Theoretical height of the lever arm	Net force direction	Reduction to one componant	Weight reduction coefficient (on $H^2/2$)	Safety factor	Slenderness ratio H/x : literal expression	Slenderness ratio H/x : value
	1684	Vauban	Military eng	-	-	-	Return of experience	-	-	-	-	-			-	-	-	-	-	-	2
1	1691	Bullet	King's archit	Sphere on inclined plane stop	Solid wedge	Theory of the inclined plane	Equilibrium of factored loads	45°	1/1	$H^2/2$	1/1	Not considered	Not considered	N/A	N/A	Of slope	Not considered	1/√2 (0,71)	-	$2\sqrt{2}$	2,8
2	1716	Buchotte	Military eng	None	None: practical method	None	Equilibrium of factored loads	45° ?	1/1 ?	-	3/2	-	-	N/A	N/A	-	-	-	-	-	4,3
3	1717	Gautier	Civil engine	Stablity of a load of earth	Variable size solid prisms	None	Equilibrium of factored loads	45°	1/1	$H^2/2$	3/2	Not considered	Not considered	N/A	N/A	Of slope	Not considered	1/2 (0,5)	-	4	4
4	1726	Couplet-a	Civil engine	Tetrahedron of spheres	Solid wedge	Confused projections ?	Equilibrium of moments around the dow	71°	$1/\sqrt{8}$	$(H^2/2)/\sqrt{8}$	m/t	Not considered	Not considered	2/3	1/2	Of slope	1/√2	1/4 (0,25)	-	√(6m/t)	3
"	1727	Couplet-b	Civil engine	Tetrahedron of spheres	Parametrised Solid wedges	Confused projections ?	Equilibrium of moments around the dow	b/a	b/a	b/a.H^2/2 reduced by	m/t	Not considered	Not considered	2/3	1/2	Of slope	?	Non explicit	-	Third degree equation	
5	1729	Bélidor	Military eng	Sphere on inclined plane stop	Fixed height solid prisms	None	Equilibrium of moments around the dow	45°	1/1	$H^2/2$	3/2	1/2	Not considered	1	1/3	Horizontal	Useless	1/3 (0,33)	1,25	$\sqrt{(3m/t/(K0.\gamma_{bf}))}$	3,6
6	1737	Lemaire	Military eng	None	None: practical method	None	Return of experience	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	3
7	1743	Querlonde	Military eng	None	Parametrised Solid wedges	Modified theory of the inclined	Equilibrium of moments around the dow	27°, 45°, 56°	2/1, 1/1, 2/3	H^2, $H^2/2$, $H^2/3$	3/2	Not considered	Not considered	2/3	1/2	Horizontal	1/2	3, 1/4, 1/5 (0,33; 0,25; 0,2	-	$3\sqrt{3}/2$, 3, $3\sqrt{5}/2$	3
8	1745	Gadroy	Military eng	Same as Bélidor (Mayniel)	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	3,6
9	1749	Cormontaigne	Military eng	None	Solid wedge	None	Equilibrium of factored loads	45°	1/1	$H^2/2$	1/1	1/2	Not considered	N/A	N/A	-	-	17/60 (0,28)	-	110/17	6,5
10	1767	Blaveau	Military eng	None ?	Solid wedge	Theory of the inclined plane + P	Equilibrium of moments around the dow	b/a (45° suggested)	b/a	b/a.H^2/2	m/t	Not considered	Not considered	1/3	1/2	Of slope	cosine(slope)	1/2 (0,5)****	-	150/(11.(2√7-3))	6,0
11	1767	Sallonnyer	Military eng	Sphere on inclined plane stop	Solid wedge	Theory of the inclined plane + P	Equilibrium of moments around the dow	b/a (45° suggested)	b/a	b/a.H^2/2	m/t	Not considered	Equal to inclined plan slope	1/3	1/2	Of slope	cosine(slope)	1/2 (0,5)****	-	6/(√5-1)	4,9
12	1768	Trincano	Military eng	None	Solid wedge	None	Equilibrium of moments around the dow	45°	1/1	$H^2/2$	3/2	1/4	Not considered	2/3	1/2	Horizontal	Not considered	1/3 (0,33)	-	3,0	3
13	1773	Coulomb	Military eng	None	Solid wedge	Theory of the inclined plane + P	Equilibrium of moments around the dow	b/a (45° suggested)	b/a	b/a.H^2/2	m/t	Includes resistance the	Taken into account	0 -> 1/3+effet d	1/3+effet de c	Horizontal	cosine(slope)	(1-1/2)/(1+1/2 (0,33)	1,25	100/24	4,2
14	1774	Tersac de Montlong	Military eng	None	Solid wedge	Theory of the inclined plane + re	Equilibrium of moments around the dow	45°	1/1	$(H-y)^2/2$	m/t	Not considered	Not considered	1/3	"1/2 + y/2"	Of slope	nsidered in his first	$1/\sqrt{2}$. (1-x/H)²	-	6/(√5-1)	4,9
15	1783	d'Antony	Italian engi	Sphere on inclined plane stop	Fixed height solid prisms	None	Equilibrium of moments around the dow	b/a (45° suggested)	b/a	b/a.H^2/2	m/t	1/4	Not, but wall-foundation c	2/3	1/2	Horizontal	Not considered	1/4 (0,25)	-	$3\sqrt{(7/10)}$	1,5
16	1784	Chauvelot	Civil engine	None	Solid wedge	None	Equilibrium of moments around the dow	45°	1/1	$H^2/2$	m/t	"m/n" (K_0)	Not considered	2/3	1/2	Horizontal	Not considered	2/3.(1-0,5) (0,33)	-	$3/2\sqrt{2}$	1,1
17	1784	Gauthey	Civil engine	Sphere on inclined plane stop	Solid wedge	None	Equilibrium of moments around the dow	45°	1/1	$H^2/2$	m/t	2/3	Not considered	1/3	1/2	Horizontal	1/√2	1/9 (0,11)	-	$3\sqrt{(3/2)}$	3,7
18	1802	Rondelet	Civil archite	None	Solid wedge	Théorie du Pl. Inc.*	Equilibrium of moments around the dow	a/h	a/h	a/h.H^2/2	m/t	Not considered	Egale au plan incliné sans p	1/3	1/2	Of slope	Not considered	1/2 (0,5)****	-	6/(√5-1)	4,9

List after Mayniel, 1808, except for Gautier (1717)

* Couplet (1726, 1727) introduced two different theories ; Lemaire (1737) proposed wall design rules rather than a theory

Bullet, 1691 : the first attempt





Bullet's « Elementary Model »

Drawing from Gautier (1717), modified

Based on the *Inclined plane theory*

"In the case of equilibrium, the weight of the body is to force parallel to the inclined slope as the lenght of the plane to its height" ⇔ F = W sin i

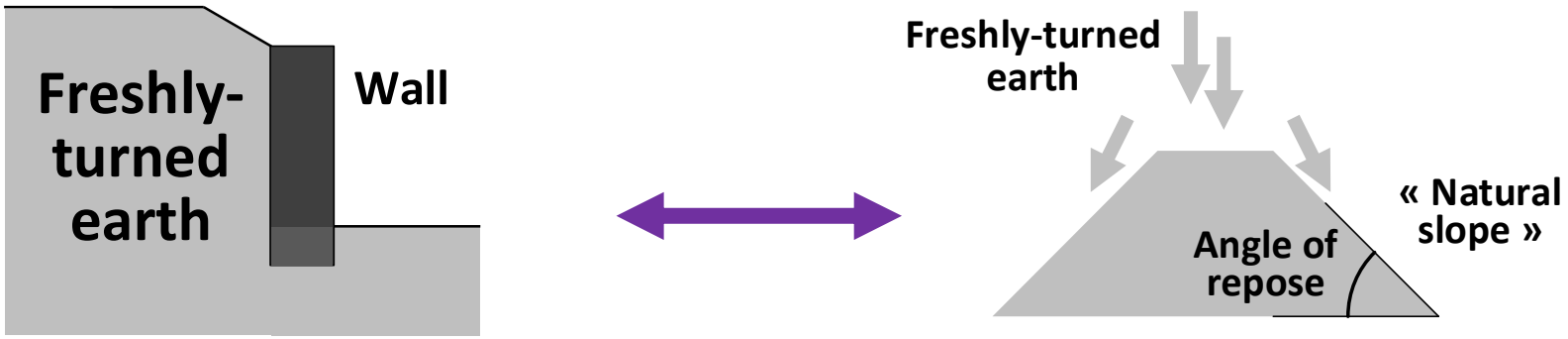
Bullet's « Design Model »

An early fact:

« *x has to be proportional to H* »

→ The slenderness ratio should be constant

Mechanical interpretation of field experience

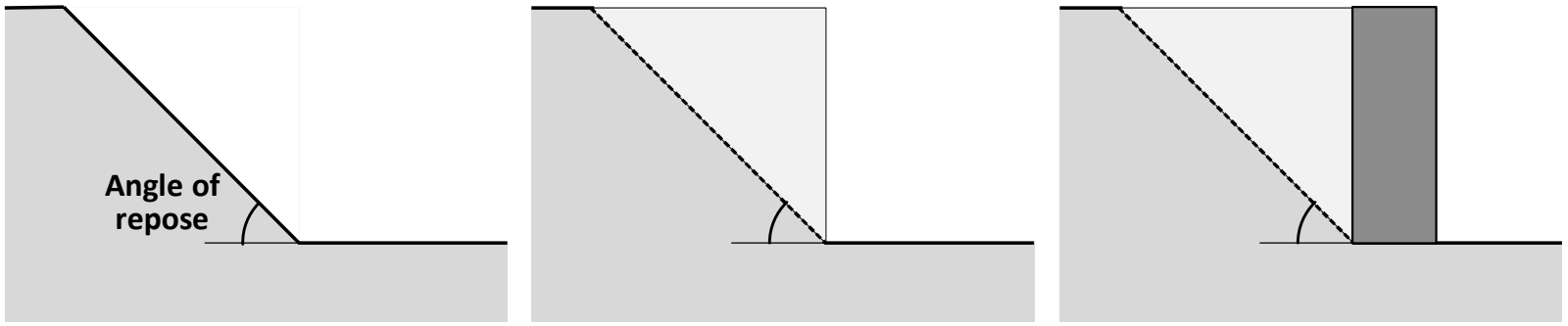


The *solid wedge* principle (and its backward reasoning)

Stable natural slope

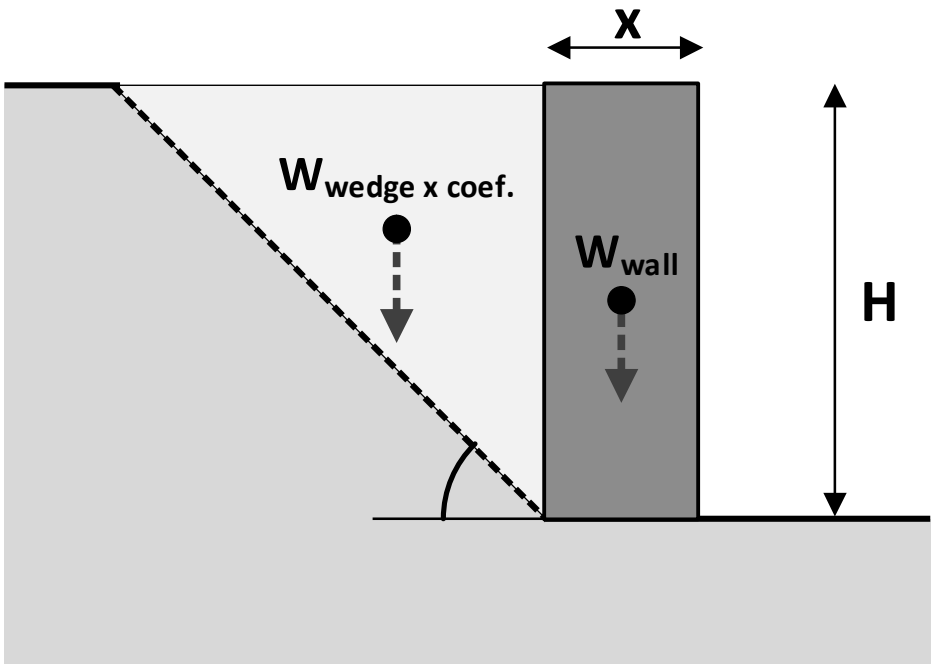
Virtual sliding wedge of earth

→ The thrust is due to the sliding wedge



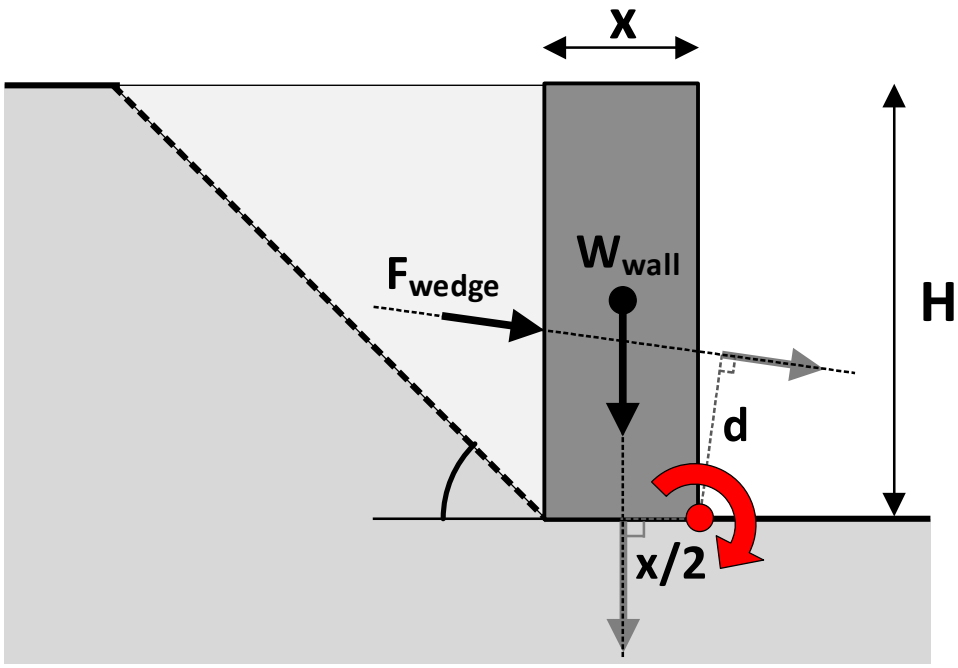
Basic wall design principle: equilibrium

Equilibrium of adjusted forces



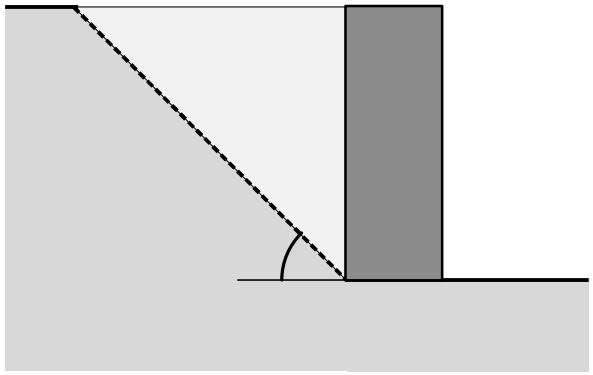
Bullet 1691, Buchotte 1716, Gautier 1717, Cormontaigne 1749

Equilibrium of adjusted moments



All the other authors, starting from Couplet (1726-27) and Bélidor (1729), to Coulomb (1773) and after

Adjusted moments: how?



A - Solid wedge surface (volume of « active earth »)

B - Relative densities

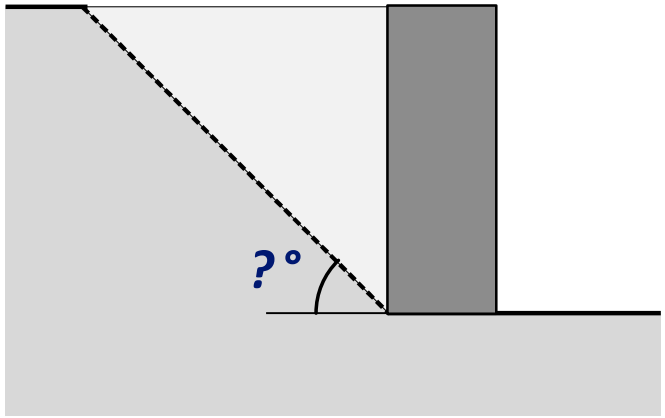
C - Converting wedge weight to wedge thrust

D - Earth's internal resistance, if any

E - Thrust loading point

Adjusted moments: A - Solid wedge surface

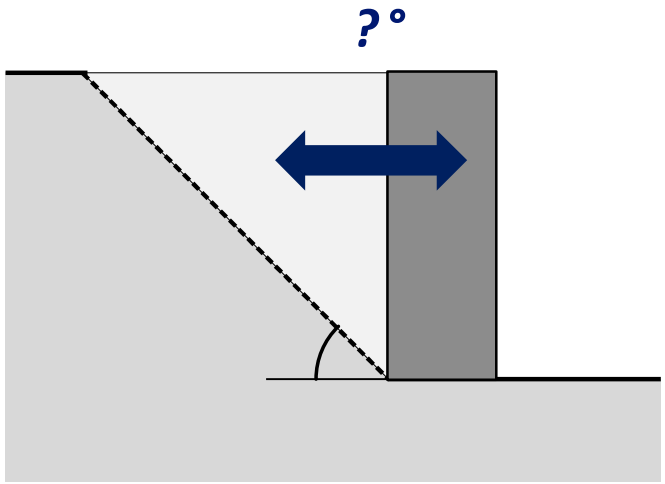
Year	Author	Inclined plane slope angle
1684	Vauban	-
1691	Bullet	**45°**
1716	Buchotte	45° ?
1717	Gautier	45°
1726	Couplet-a	**71°**
1727	Couplet-b	**b/a**
1729	Bélidor	45°
1737	Lemaire	-
1743	Querlonde	**27°**, 45°, **56°**
1745	Gadroy	45°
1749	Cormontaigne	45°
1767	Blaveau	**b/a (45° suggested)**
1767	Sallonnyer	b/a (45° suggested)
1768	Trincano	45°
1773	Coulomb	b/a (45° suggested)
1774	Tersac de Montlong	45°
1783	d'Antony	b/a (45° suggested)
1784	Chauvelot	45°
1784	Gauthey	45°
1802	Rondelet	b/a (45° suggested)



The 45° slope is dominant throughout the entire century

Adjusted moments: B - Relative densities

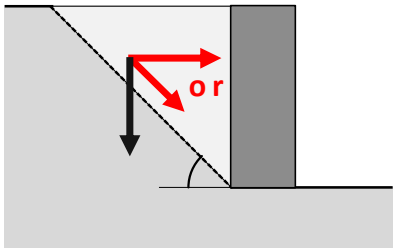
Year	Author	Wall/Earth densities ratio
1684	Vauban	-
1691	Bullet	1/1
1716	Buchotte	3/2
1717	Gautier	3/2
1726	Couplet-a	m/t
1727	Couplet-b	m/t
1729	Bélidor	3/2
1737	Lemaire	-
1743	Querlonde	3/2
1745	Gadroy	3/2
1749	Cormontaigne	1/1
1767	Blaveau	m/t
1767	Sallonnyer	m/t
1768	Trincano	3/2
1773	Coulomb	m/t
1774	Tersac de Montlong	m/t
1783	d'Antony	m/t
1784	Chauvelot	m/t
1784	Gauthey	m/t
1802	Rondelet	m/t



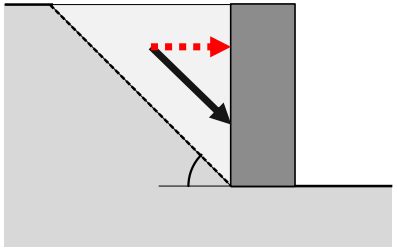
The 3/2 ratio was quite popular after Buchotte (1716), but most authors leave it up to the reader…

Adjusted moments: C - From wedge weight to thrust

The main two aspects

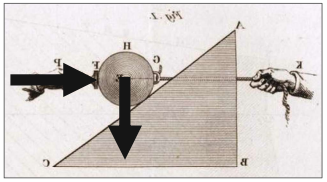


Projection of the wedge weight effect
- inclined [W/√2] (Bullet 1691 et al.)
- or horizontal [W] (Bélidor 1729 et al.)



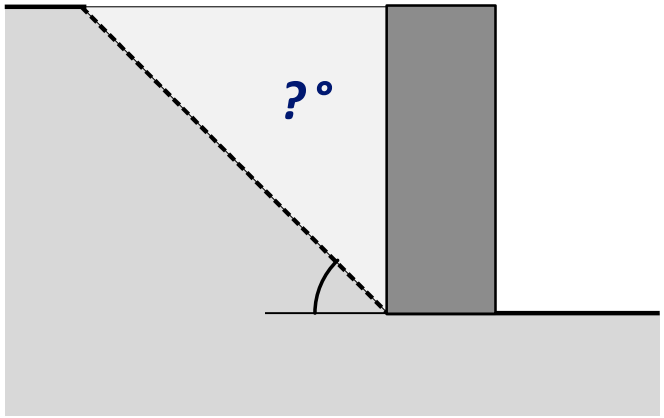
If inclined,
- kept as it is
- or projected again, horizontally

Elementary models



Adjusted moments: D - Earth's internal resistance, if any

Year	Author	Internal resistance
1684	Vauban	-
1691	Bullet	Not considered
1716	Buchotte	-
1717	Gautier	Not considered
1726	Couplet-a	Not considered
1727	Couplet-b	Not considered
1729	Bélidor	**1/2**
1737	Lemaire	-
1743	Querlonde	Not considered
1745	Gadroy	1/2
1749	Cormontaigne	1/2
1767	Blaveau	Not considered
1767	Sallonnyer	Not considered
1768	Trincano	**1/4**
1773	Coulomb	**Includes resistance theory, with c and ϕ**
1774	Tersac de Montlong	Not considered
1783	d'Antony	1/4
1784	Chauvelot	**"m/n" (K_0)**
1784	Gauthey	2/3
1802	Rondelet	Not considered

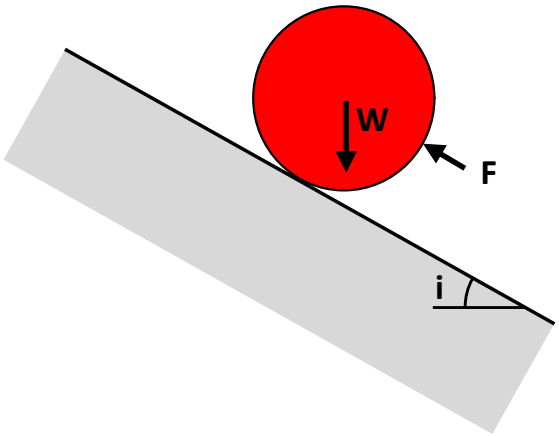


- Most authors : loss of efficiency along interfaces in the design model
- Exceptions : the global factor (1/4, 1/2, 2/3)

Adjusted moments: E - Thrust loading point

Thrust loading point: foreword

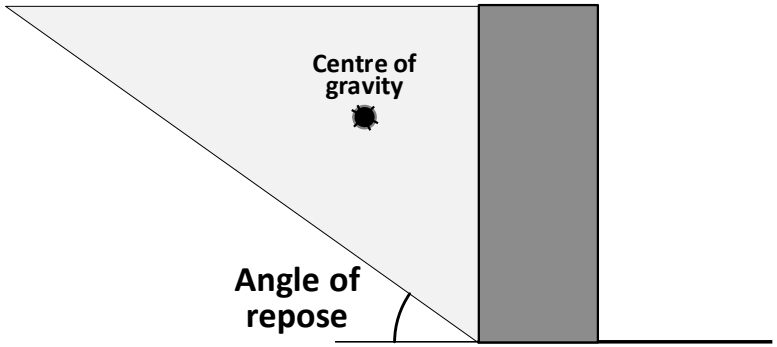
Elementary model



F = W.sin(i)

Earth homogeneity

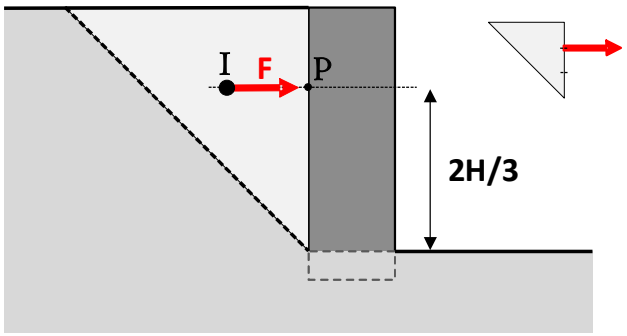
Elementary model properties concentrated on the centre of gravity of the wedge



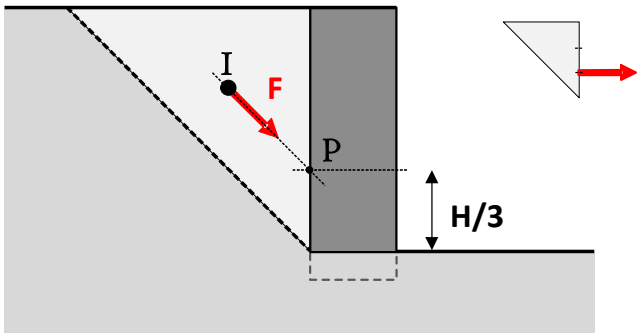
Scale of the wall

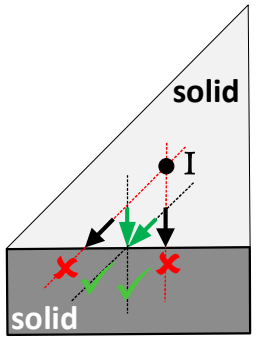
E - The thrust loading point of « *freshly turned earth* »

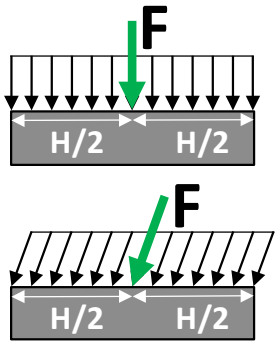
5 authors



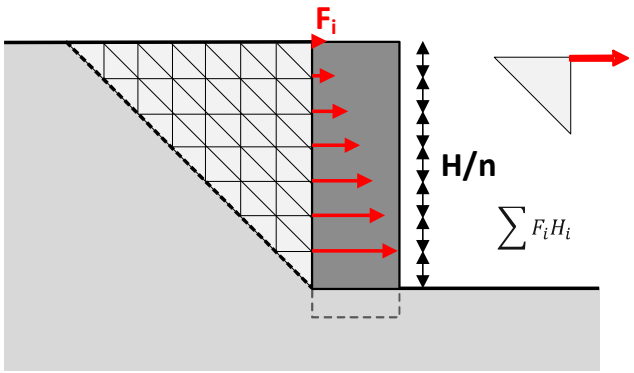
5 other authors



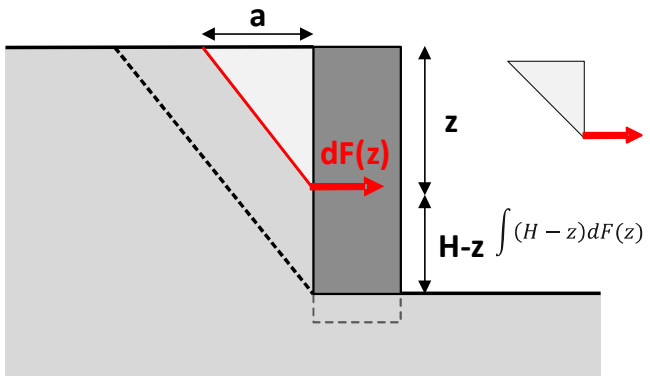




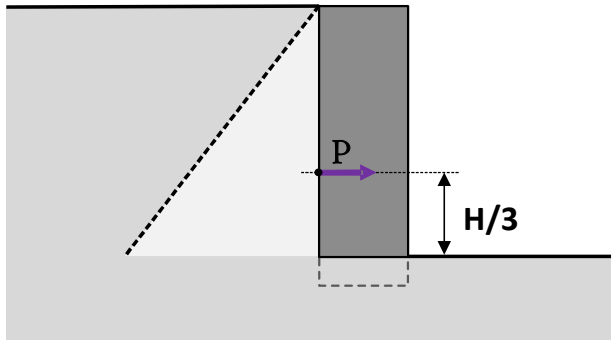
Bélidor, 1729



Coulomb, 1773

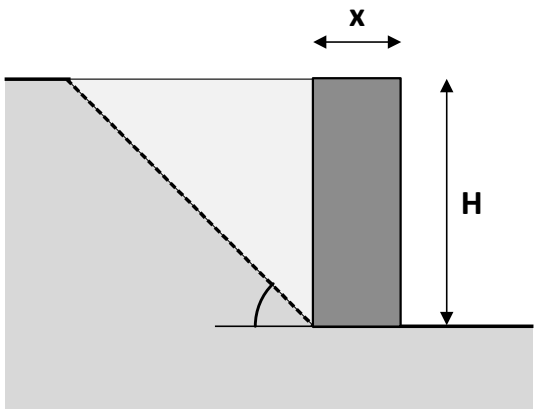


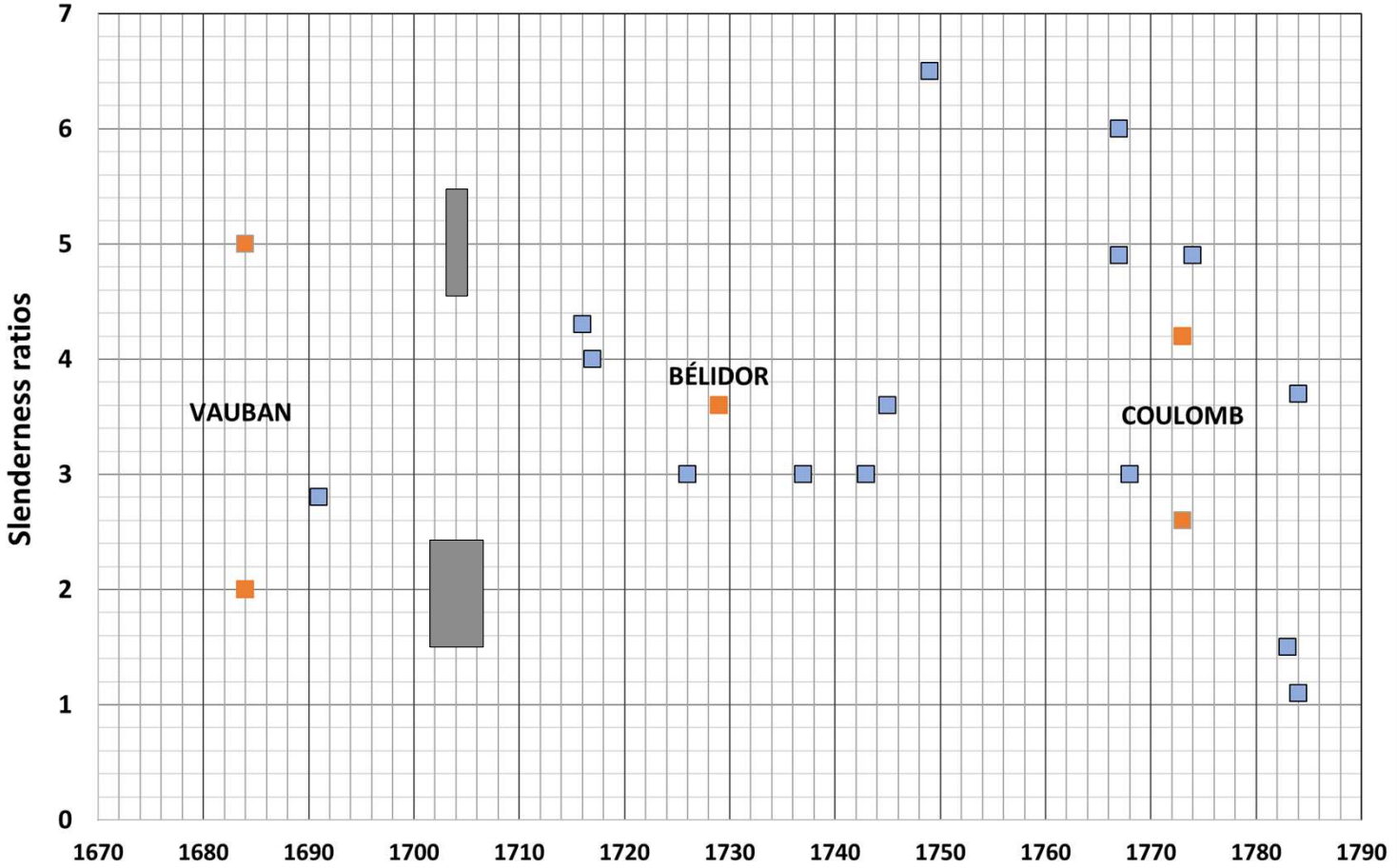
Only one theory allowed to find an *hydrostatic* pressure net force level



Compared* wall slenderness ratios (H/x)

Almost all theories refered to Vauban's 1684 table (including Coulomb)



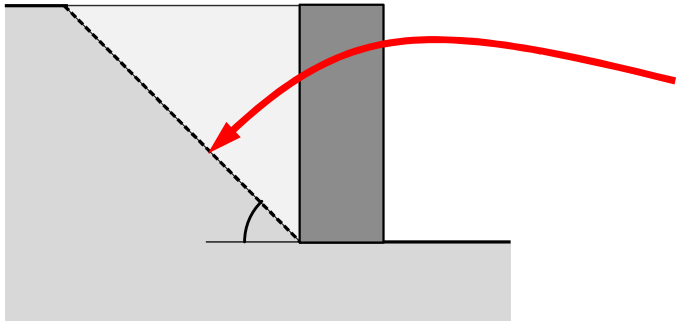


* When needed and/or possible, fixed values were : Wall/Earth density = 3/2 ; slope angle 45° ; ϕ = 30° ; c=0

Earth pressure estimation before Coulomb: conclusion

Coulomb's (1773) faithfullness to Bullet's (1691) solid wedge:

- **as an elementary model introducing δ (c) and 1/n (ϕ), is long-lasting !**
- **as a design model, may be still discussed, especially when c = 0**



Is it a cause or a consequence ?
(see Collin, 1846)

An oustanding engineer in the long period before Coulomb : Bélidor.
His 1729's elementary model and *sliced* solid wedge:

- **from earth thrust, leads to *earth pressure,* proportionnal to depth**
- **can be related to Rankine's (1862) earth pressure coefficient approach**

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