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COMITÉ FRANÇAIS DE MÉCANIQUE DES SOLS ET DE GÉOTECHNIQUE

ACADÉMIE DES SCIENCES
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POUR UN PROGRÈS RAISONNÉ, CHOISI ET PARTAGÉ

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Charles-Augustin COULOMB - A geotechnical tribute

Paris, september 25 & 26, 2023

Charles Augustin de Coulomb

The artisan of modern geotechnical engineering

L'artisan de la géotechnique d'aujourd'hui

S. Lacasse, NGI

BUREAU VERITAS Shaping a World of Trust
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Contents

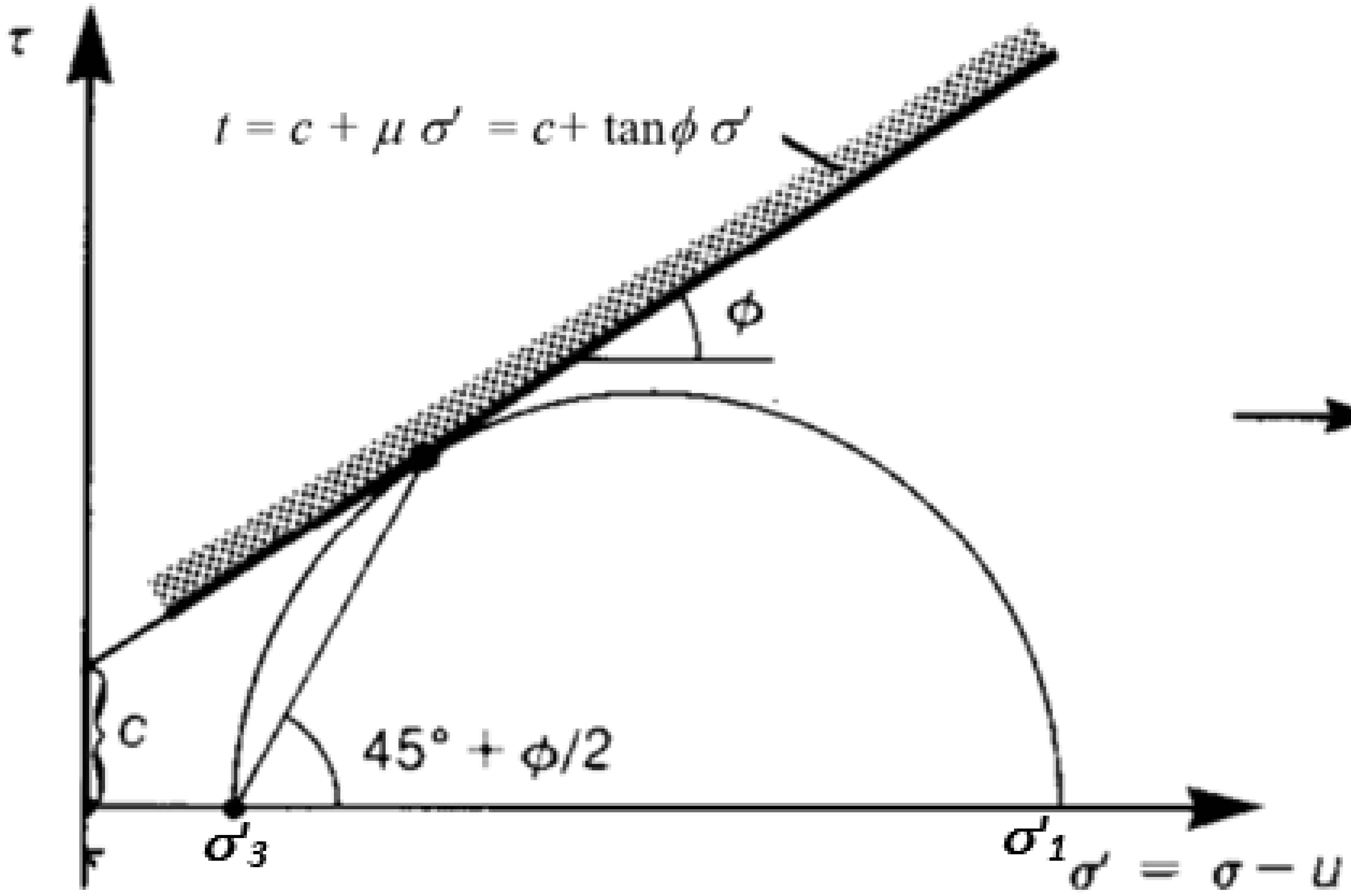
Coulomb’s fundamental equation

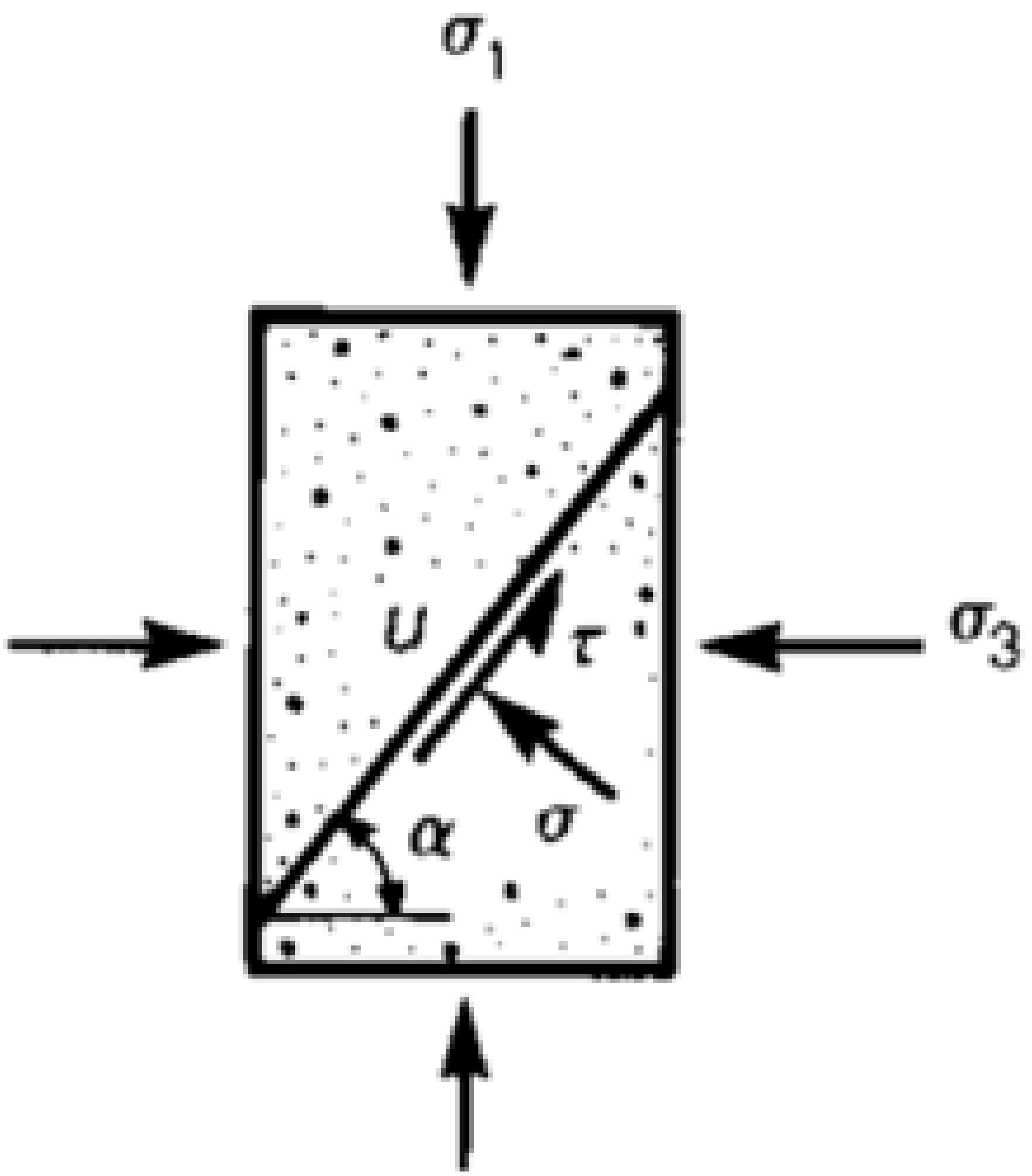
This last paper attempts to put a last perspective on the central role Coulomb's works are still playing in present day research and practice of geotechnical engineering. The clear propositions by Coulomb of friction and cohesion on a sliding plane, with the fundamental equation:

$$\pm t = c + \mu\,\sigma = c + \tan\phi\,\sigma$$

found their applications in design.

Linear strength of a Coulomb's ideal material





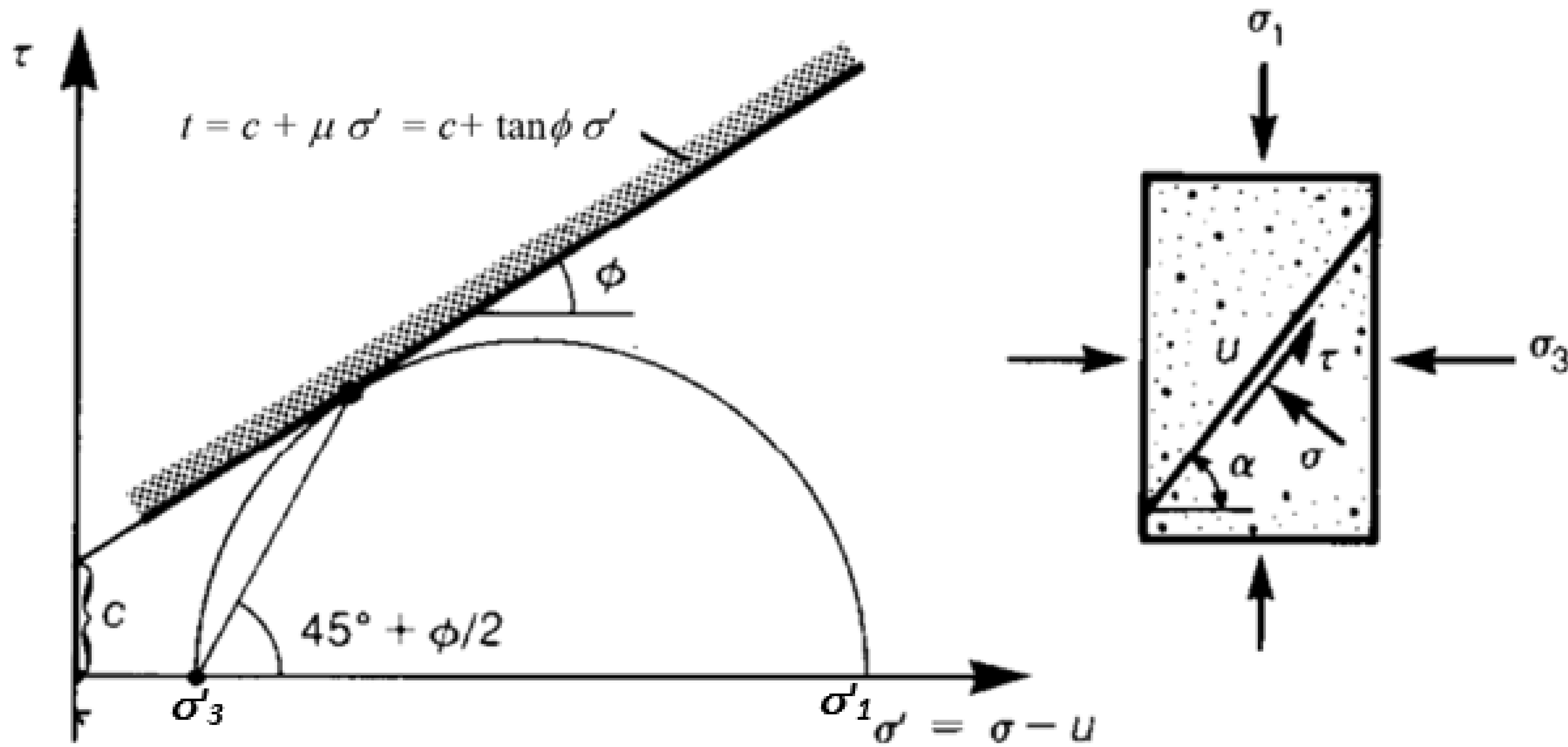
Charles Augustin de Coulomb (1736-1806)

A physicist who discovered the laws that govern magnetic attraction and repulsion and invented the torsion balance: is he still shaping the geotechnical engineering in the 2020s?

Coulomb presented his 1773 Essai on 'some statical problems'. It is a happy coincidence, on this 250th anniversary of the Coulomb's Essai, that it is also the 100th anniversary of the principle of effective stress proposed by Karl Terzaghi's in his 1923 "*Erdbaumechanik*" (Soil Mechanics).

Linear strength of a Coulomb's ideal material

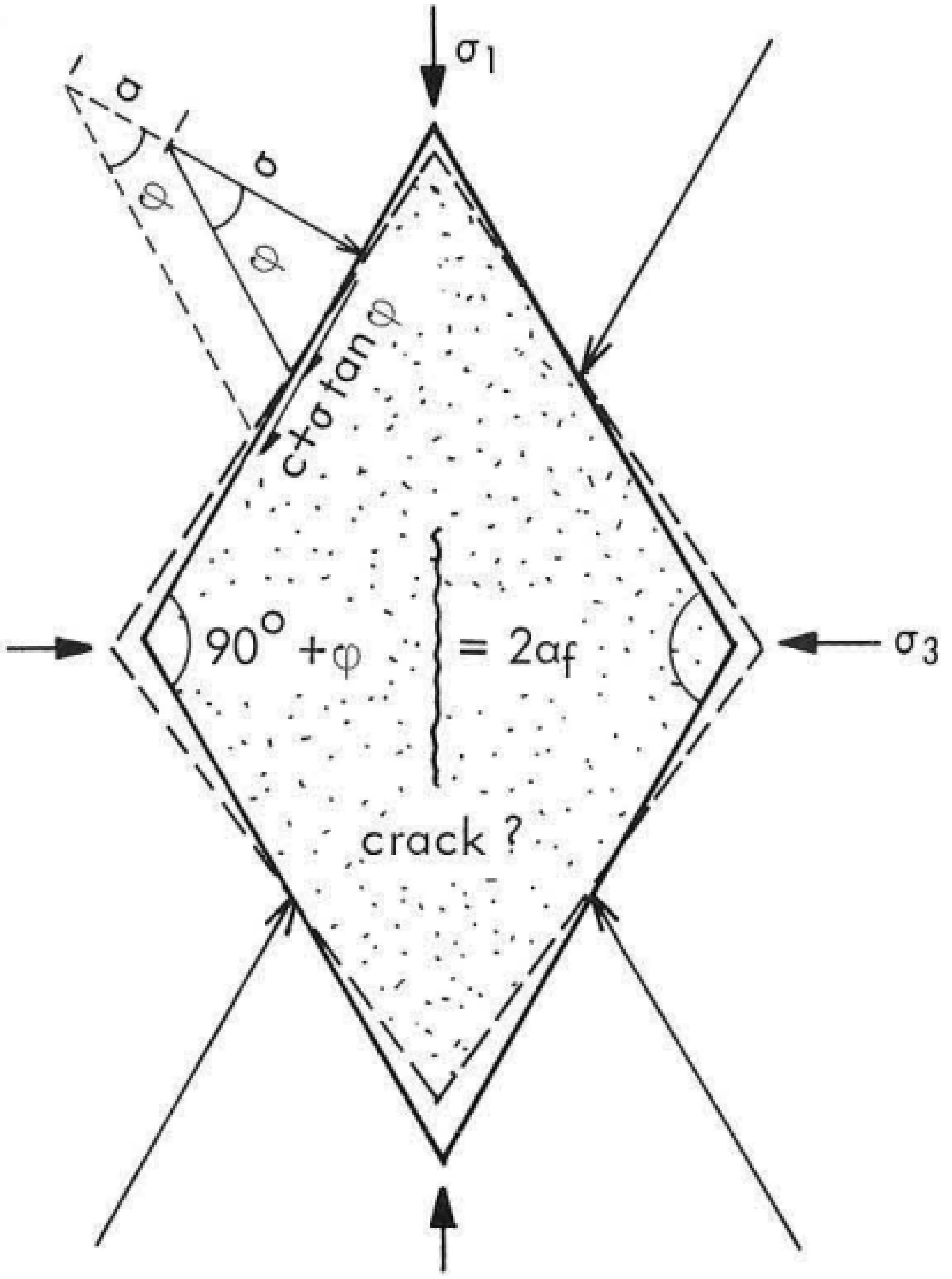
Coulomb noted that when friction and cohesion become zero, his equation gives the fluid pressure. He wrote about water pressures in 'Section X' of the Essai. Coulomb was aware that pore pressure played a role.



Plastic vs brittle behaviour

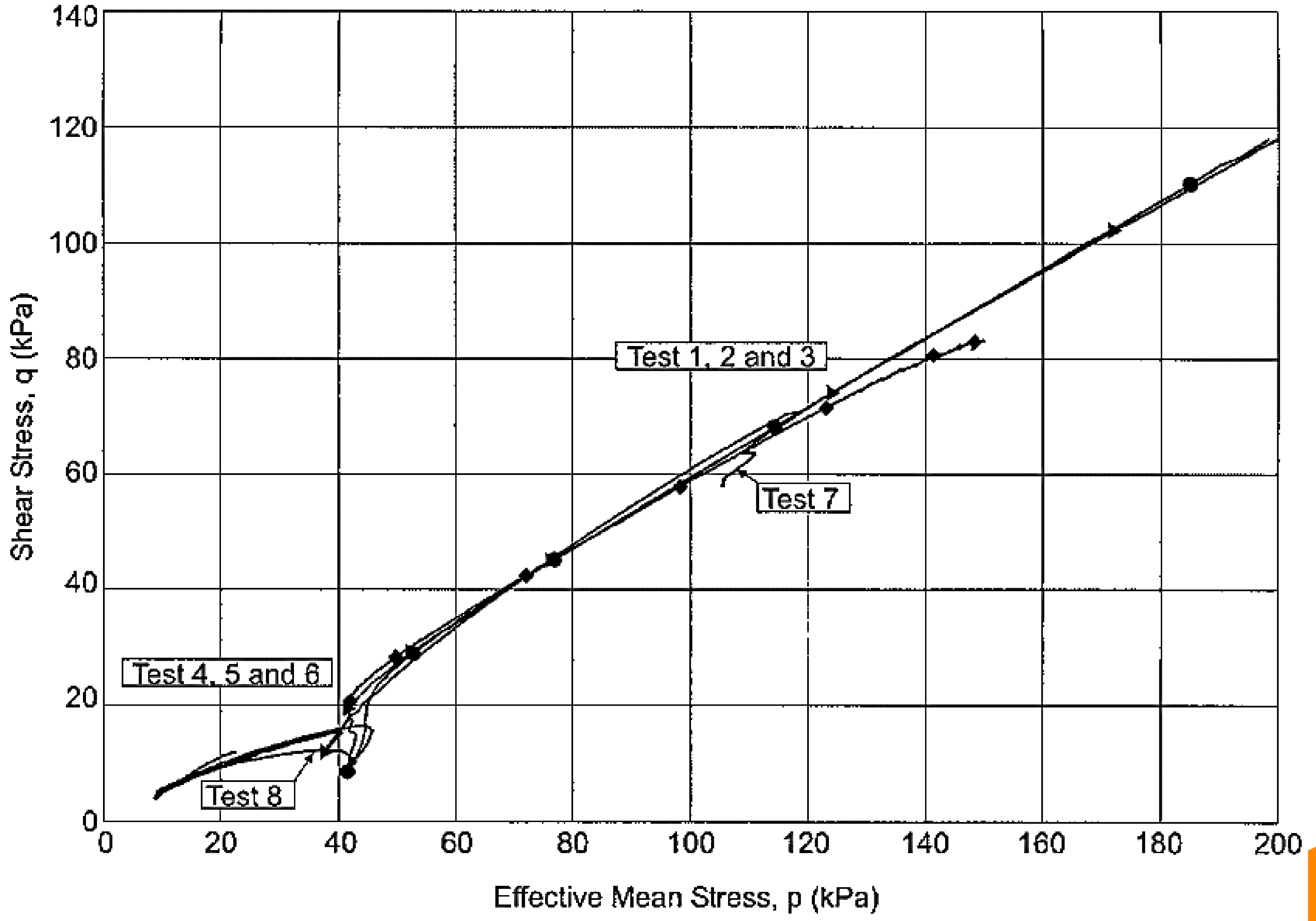
In an ideal Coulomb material, failure takes place along two sets of parallel shear planes inclined at an angle $\pm\, \alpha_f = \pm(45° + ½\phi')$. The planes form rhombic elements, with 2 modes of failure in a test specimen.

A shear failure is likely to dominate in plastic (ductile) materials, while strain-induced cracks may dominate in brittle materials. In dilatant materials, failure takes longer to develop.



Plastic vs brittle behaviour

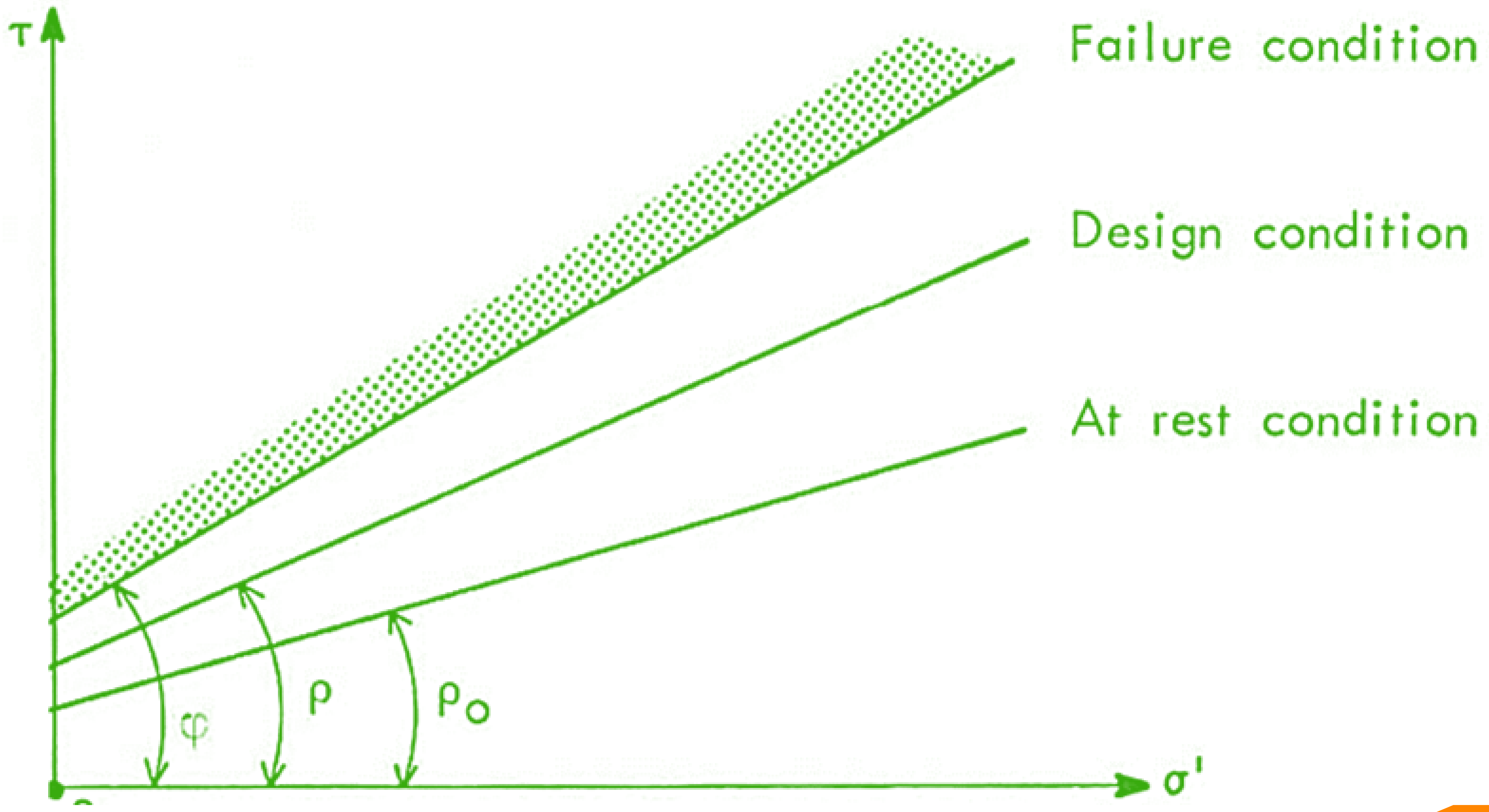
A cut slope in a dilatant material may start to crack at the crest (giving early warning) and the soil mass may gradually slump over a limited distance before reaching a new, temporary equilibrium. Contractant materials can fail abruptly, without warning and often at small strains, for example in sensitive clays.



Plastic vs br

Cohesion-friction model assists design

With its simple representation, the 3 states of design are available: at rest under K_0-conditions, the limit state (at failure), and the actual design conditions with a prescribed FS.



Terzaghi's verification of Coulomb's equation

Terzaghi (1935 - 1959) tested the suitability of the Coulomb equation to represent the shear strength of cohesive soils. While looking at the values of *c* and tan *f*, Terzaghi noted that:

"If the shear strength T of a cohesive soil is determined for a given initial water content and under equal test conditions but for different values of the vertical stress per unit area of the horizontal shear plane, the result of the tests in most cases can be expressed with sufficient accuracy by the Coulomb equation."

Terzaghi's verification of Coulomb's equation
Vienna Clay (void ratio = 0.77)

No.	Test series	Measured shear stress t (kg/cm^2)
1	Undrained	$\tau = 0.58 + \sigma \tan(0°50')$
2	"Quick"	$\tau = 0.58 + \sigma \tan(13°30')$
3	"Slow"	$\tau = 0.18 + \sigma \tan(24°0')$

Strength anisoptropy

Terzaghi observed: the most important difference in the tests was the extent to which the void ratio was allowed to adapt itself to the changes in stress on the sample. If this “adaptation” was not fully completed, part of the force on the specimen was carried by the pore water (or increased pressure in the pore water).

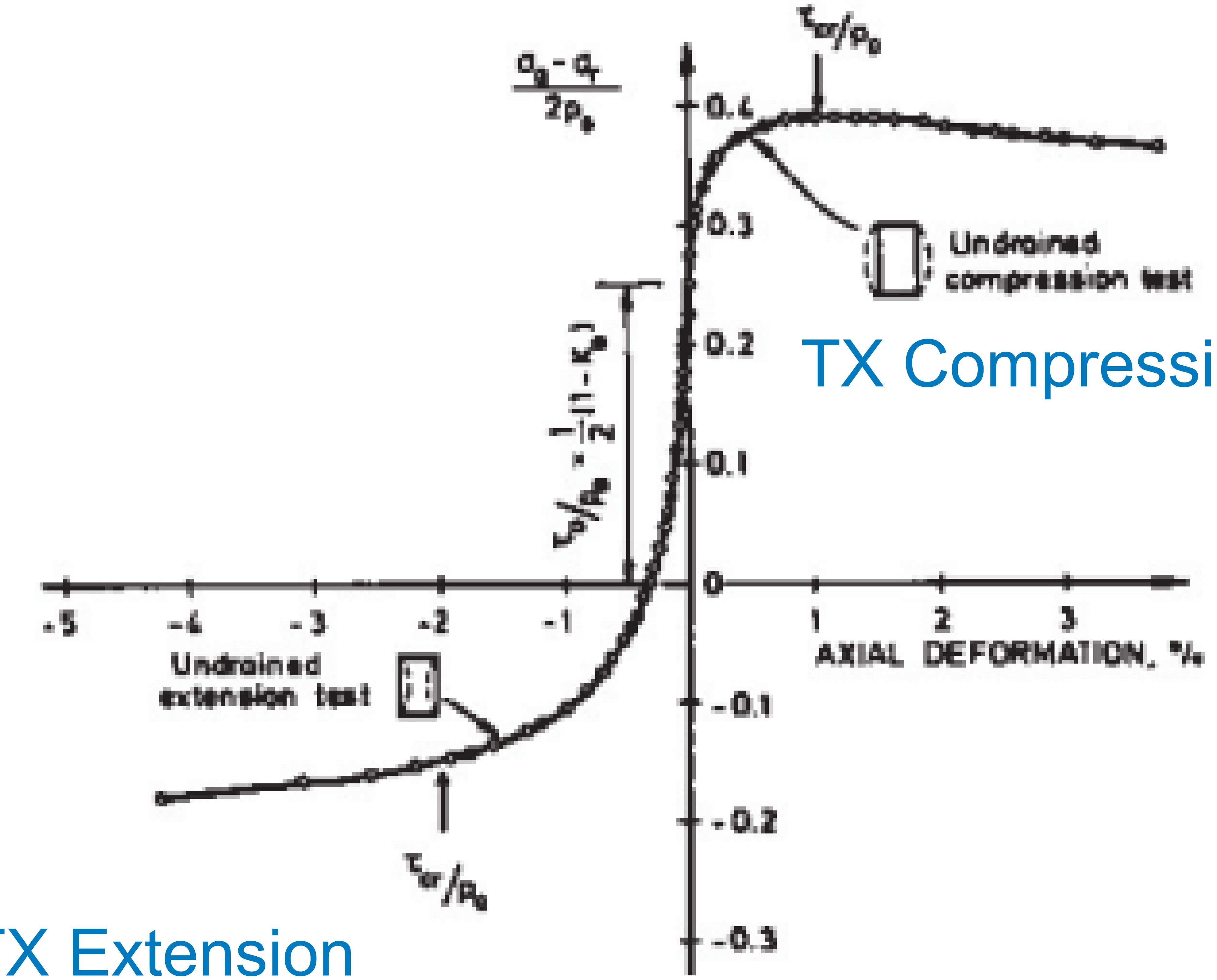
Terzaghi therefore emphasized that Coulomb’s equation gives the relationship between τ and σ for soil specimens but that one needs to consider initial void ratio, testing apparatus, drainage conditions, test procedures, etc.

Strength anisoptropy

Thereby, using Coulomb's law, Terzaghi introduced the concept of stress-induced and inherent anisotropy for clays already in the 1930s.

Coulomb provided the general relationship for shear resistance. The practitioner and theoretical modeller still have to determine which test conditions are the relevant ones. This is still true today, and one of the most common errors committed in design occurs when a designer selects shear strength values obtained from non-relevant tests, procedures, stress history or stress path.

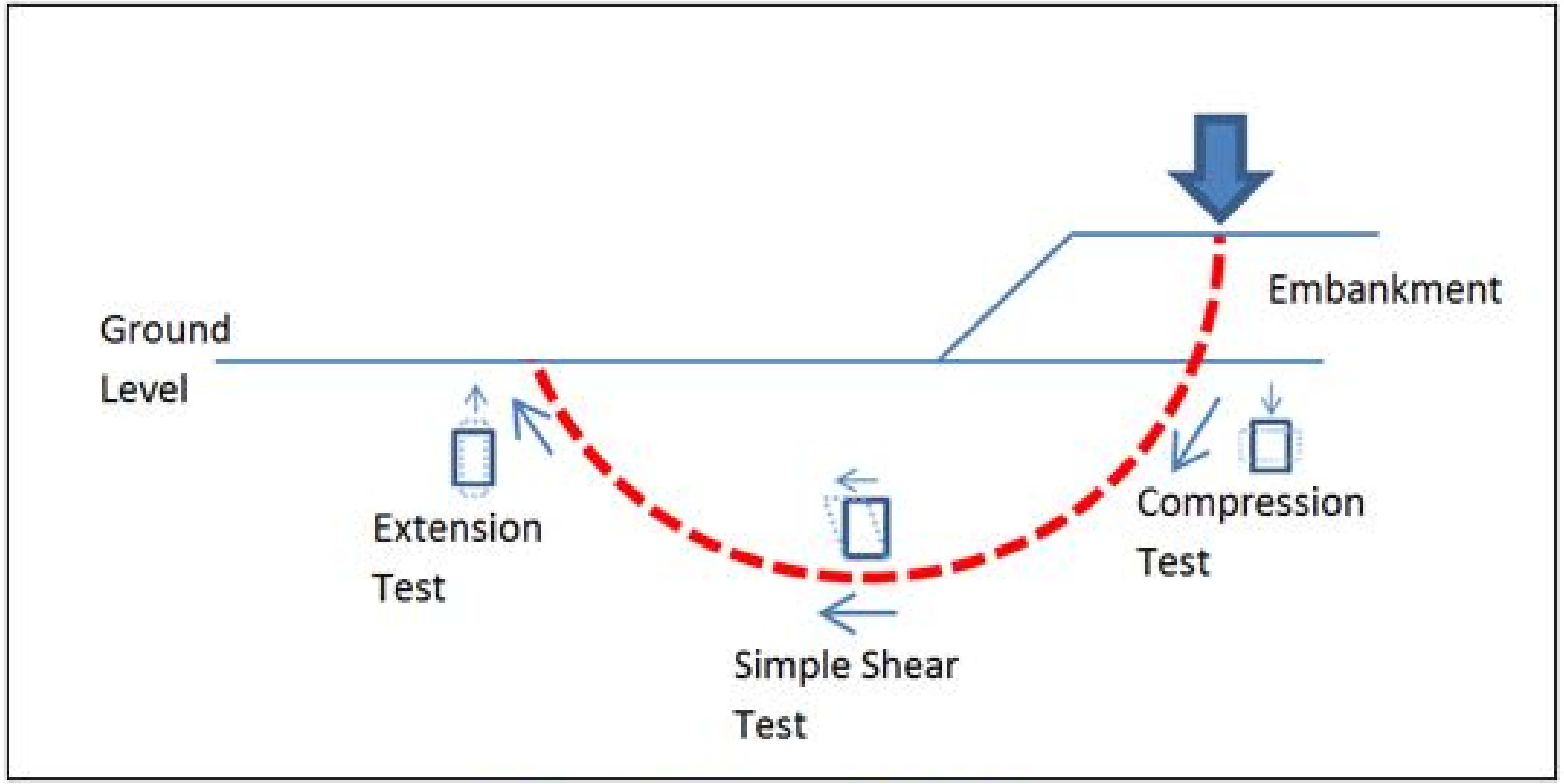
Strength anisoptropy



TX Compression

TX Extension

Drammen Clay



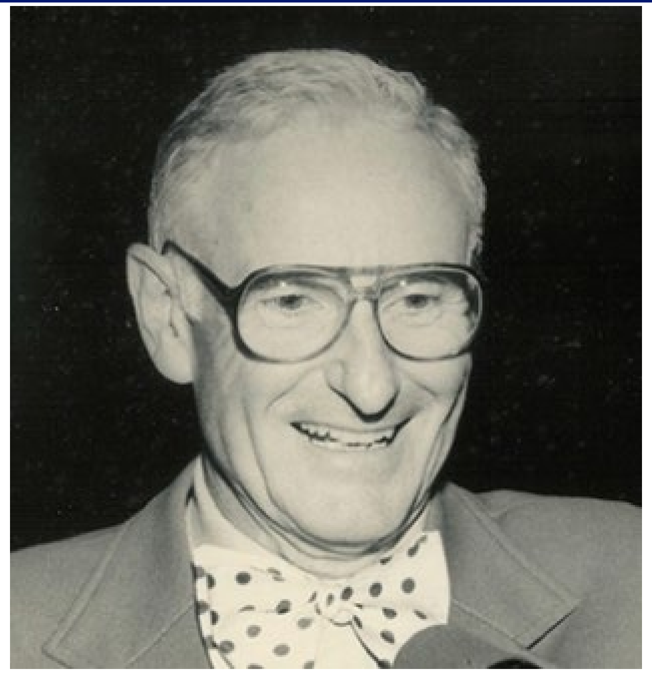
Strength anisoptropy

Terzaghi even suggested adding an index to make it very clear that the validity of the Coulomb's equation depends on the conditions analysed. Terzaghi suggested:

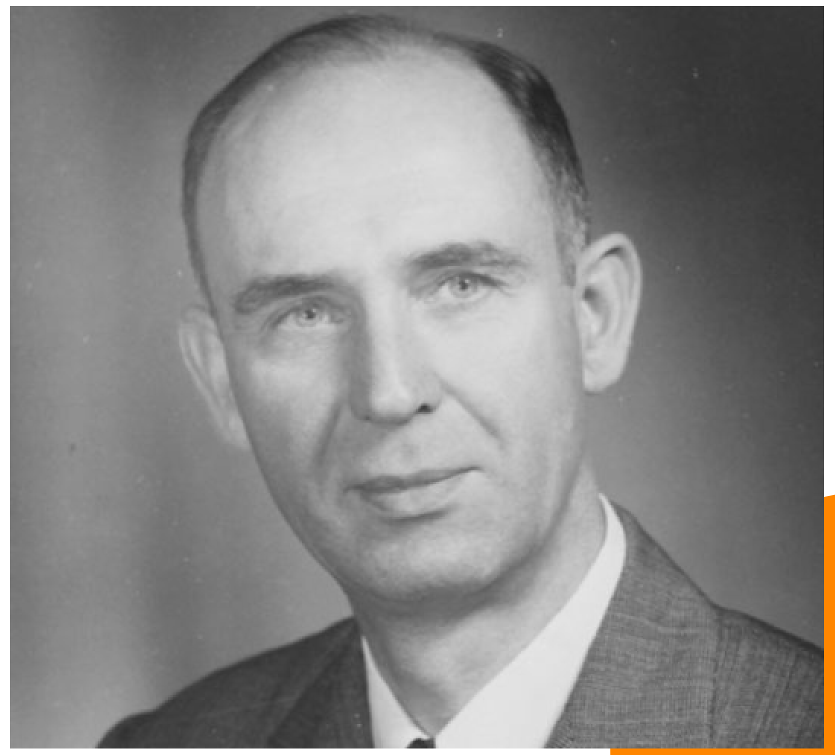
$$t = (c + \sigma \tan\phi)_d \quad \text{or} \quad t = (c + \sigma \tan\phi)_s$$

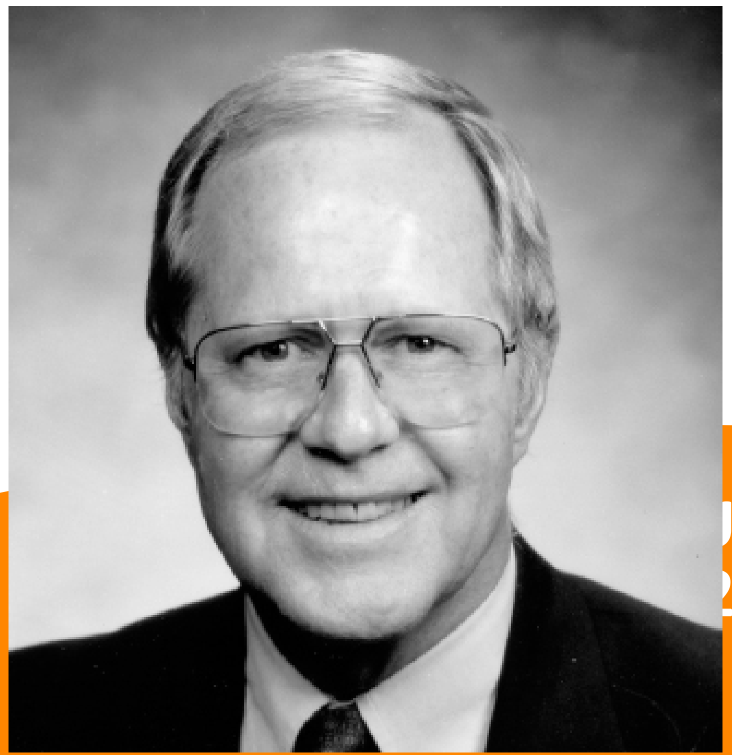
where the index "*d*" would indicate drained ("slow“) conditions, and "*s*" would indicate that the shear force is increased so rapidly that the void ratio of the soil remains unchanged. Terzaghi even suggested that a partial fulfilment of these lower and upper boundaries could be indicated by the index ”*ds*”.

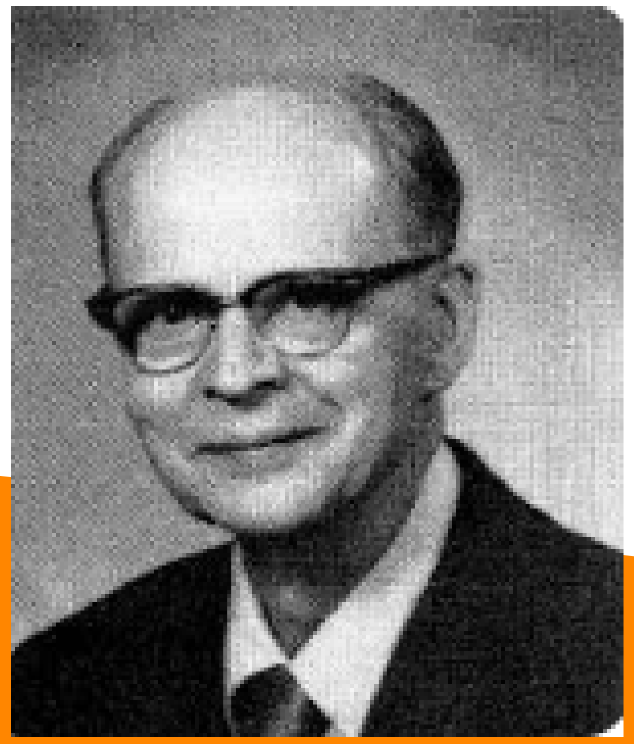
Strength anisoptropy



From then on, Terzaghi and researchers at MIT (Lambe, Taylor, Whitman, Ladd and Einstein) and Harvard (Casagrande), Imperial College (Skempton and colleagues) and NGI (Bjerrum) pursued their theoretical and experimental works, confirming the validity of Coulomb's equation under different stress and strain conditions. Each of them stressed the necessity of replicating the in situ conditions in the laboratory. Even in the UK, where Coulomb's work has been critically discussed, critical state soil mechanics agrees with this Coulomb’s law.







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A fundamental error in the Mohr-Coulomb equilibrium?

Schofield's hypothesis of an error

Schofield (1998; 2005) suggested that the error rests on the observation that Coulomb’s equation omits a component of strength, called "interlocking" (first mentioned by Taylor (1948)). Schofield wrote that the Coulomb theory, with a straight envelope, takes no account of "strain boundary conditions", that Coulomb wrongly interpreted strength data, and that Coulomb, when considering failure on an inclined plane, assumed that the slip direction was the same as the direction of the plane.

A fundamental error in the Mohr-Coulomb equilibrium?

Schofield's hypothesis of an error

Schofield used as argumentation that Taylor's tests showed that the peak strength of a sand was equal to a combination of an angle of friction plus a term for "interlocking".

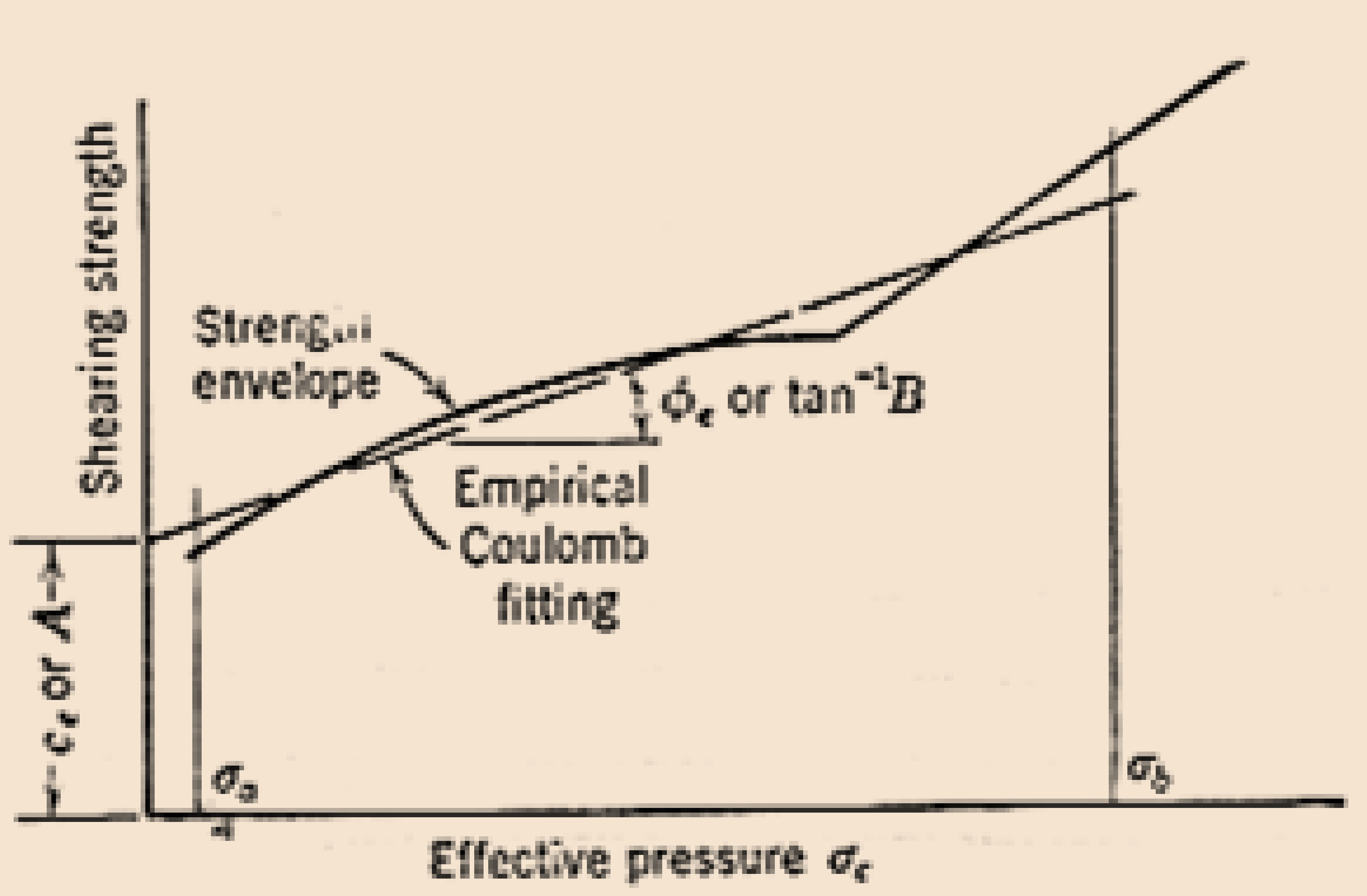
Schofield added: "… mistaking interlocking for cohesion, Terzaghi and Hvorslev attributed peak strengths of their stiff reconstituted clay soil to surface chemistry among soil grains". That last statement is not a Coulomb error.

A fundamental error in the Mohr-Coulomb equilibrium?

What did Taylor (1948) say?

Taylor's original figure illustrates the Coulomb and Taylor interpretation.

Very little difference in the two interpretations can be seen.



Taylor's original figure of Coulomb's strength envelope and his own strength envelope. Taylor's caption: "Plot illustrating the rational interpretation of Coulomb's empirical law"

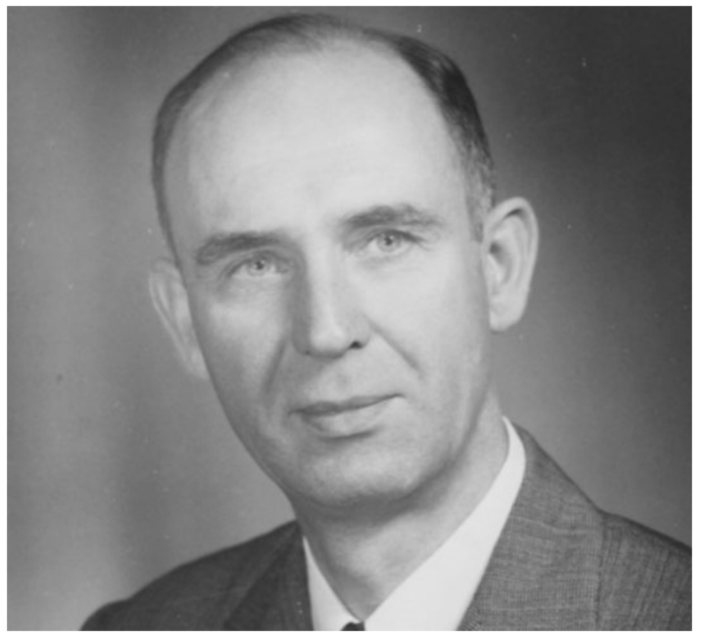
A fundamental error in the Mohr-Coulomb equilibrium?

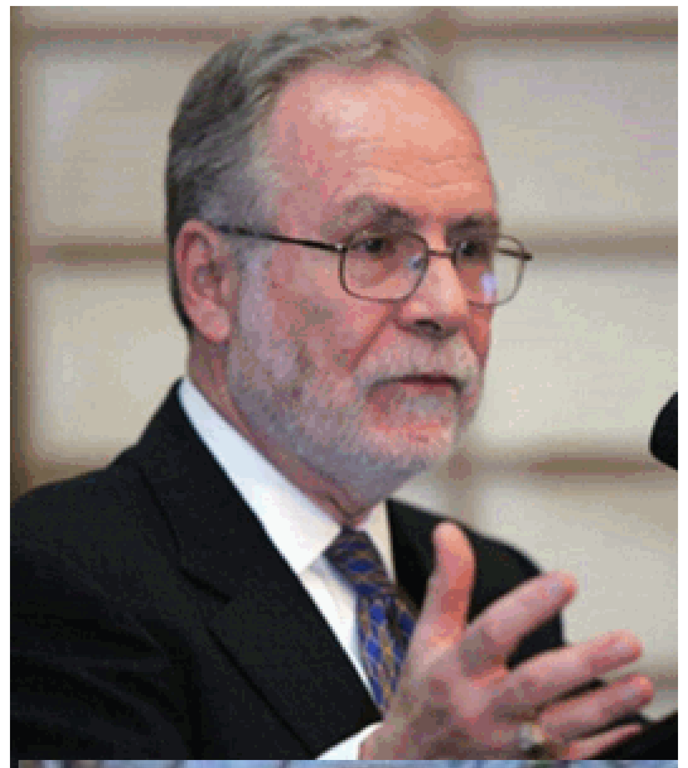
What did Taylor (1948) say?

Taylor (1948) did not describe Coulomb's theory as an error. On the contrary, Taylor called the Coulomb's parameters as empirical coefficients, and mentioned that different conditions should be accounted for by the engineers, using the Coulomb equation. He added:

> *"Curves of the Coulomb type are representative of the strength of a given soil when it has been subjected to a given magnitude of precompression. At different depths in a deposit, there usually are different amounts of precompression; thus, a different envelope may hold for each different depth. "*

A fundamental error in the Mohr-Coulomb equilibrium?





What did Taylor (1948) say?

Taylor's paragraph validates Coulomb's strength theory. Christian & Baecher (2015) and Burland (2008) reiterated that Taylor's concept of interlocking is not inconsistent with Coulomb's theory, but complementary, bringing in new results from Taylor's research. For Coulomb, shear strength was the sum of friction and cohesion, whereas for Taylor, after reflection and new testing of sands, shear strength was the sum of friction and interlocking.

A fundamental error in the Mohr-Coulomb equilibrium?

Conclusion on the so-called Coulomb error

That the vocabulary has changed or expanded over 250 years is **expected and normal**. That one refines the components and physical understanding of soil and rock resistance is **expected and normal**. That one finds, though testing, observations and/or reasoning, additional factors and sub-classes of conditions that affect soil and rock resistance is also **expected and normal**.

Conclusion on the so-called Coulomb error

To suggest that the Mohr-Coulomb's equation wrongly interpreted strength data or developed a faulty relationship is questionable. To declare a 250-year old theory to be in error because part of a component of the resistance is not fully explained with today's terms and understanding, given the experimental facilities available in the 18th century compared to today's, is shortsighted and simply bids for unfair comparisons.

A fundamental error in the Mohr-Coulomb equilibrium?

Conclusion on the so-called Coulomb error

Coulomb's principle was the precursor for developing the tools to meet the engineering profession's responsibility of ensuring the safety of people and preserving material property and the environment.

Through Coulomb's theory, the engineer can become intimate with the foundation soil and continue to rely on experience, peer review, and simple (hand-calculation) checks.

Coulomb, without doubt, laid the groundwork for the understanding of soil and rock resistance.

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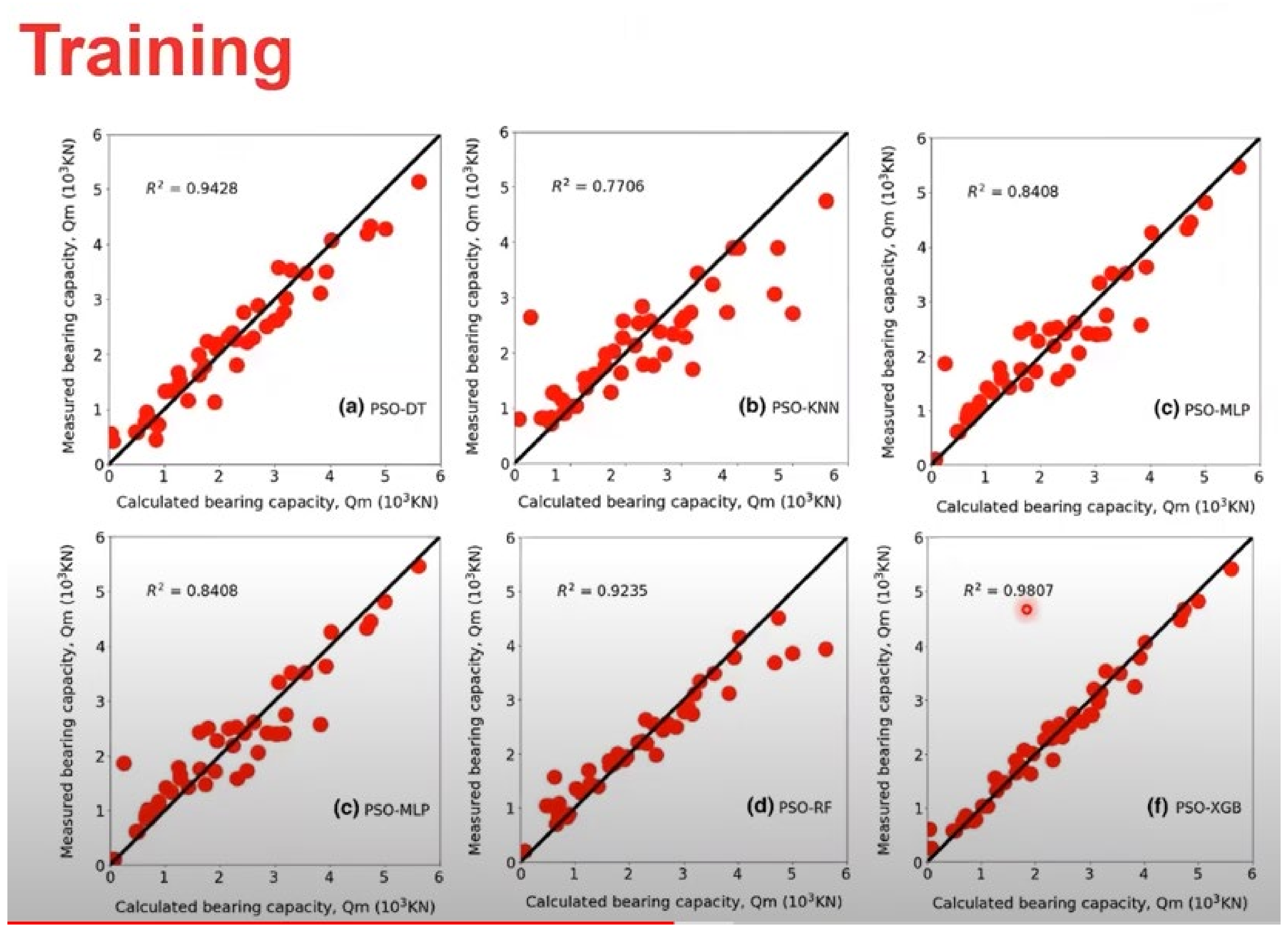
Coulomb, the Artisan of modern geotechnical engineering

The profession has greatly evolved, 100 years after the statement of the effective stress principle by Terzaghi (1923) (reported in English in Terzaghi, 1925; Skempton, 1960). The principle of effective stress, 150 years after Coulomb's equation, changed soil mechanics.



With the computer/IT revolution, the profession is today endeavouring to develop machine learning applications, massaging multi-data and considering digital twinning to generate massive "laboratory" datasets. There is much promise in the new techniques.

Coulomb, the Artisan of modern geotechnical engineering

But … the data will still have to observe Coulomb's law. The profession is at a paradigm shift from dedicated verifiable data toward vast numbers of "measurements" in anonymous distributed systems. These new developments will be helpful, but fundamentals must not be forgotten.

Observing the Coulomb's equilibrium principles, remains an absolute requirement for the teaching and practice of engineering. Fundamental understanding should not take a back seat to numerical developments and AI interpretations.

Coulomb's principles have found applications over the entire whelm of today's geotechnical practice.

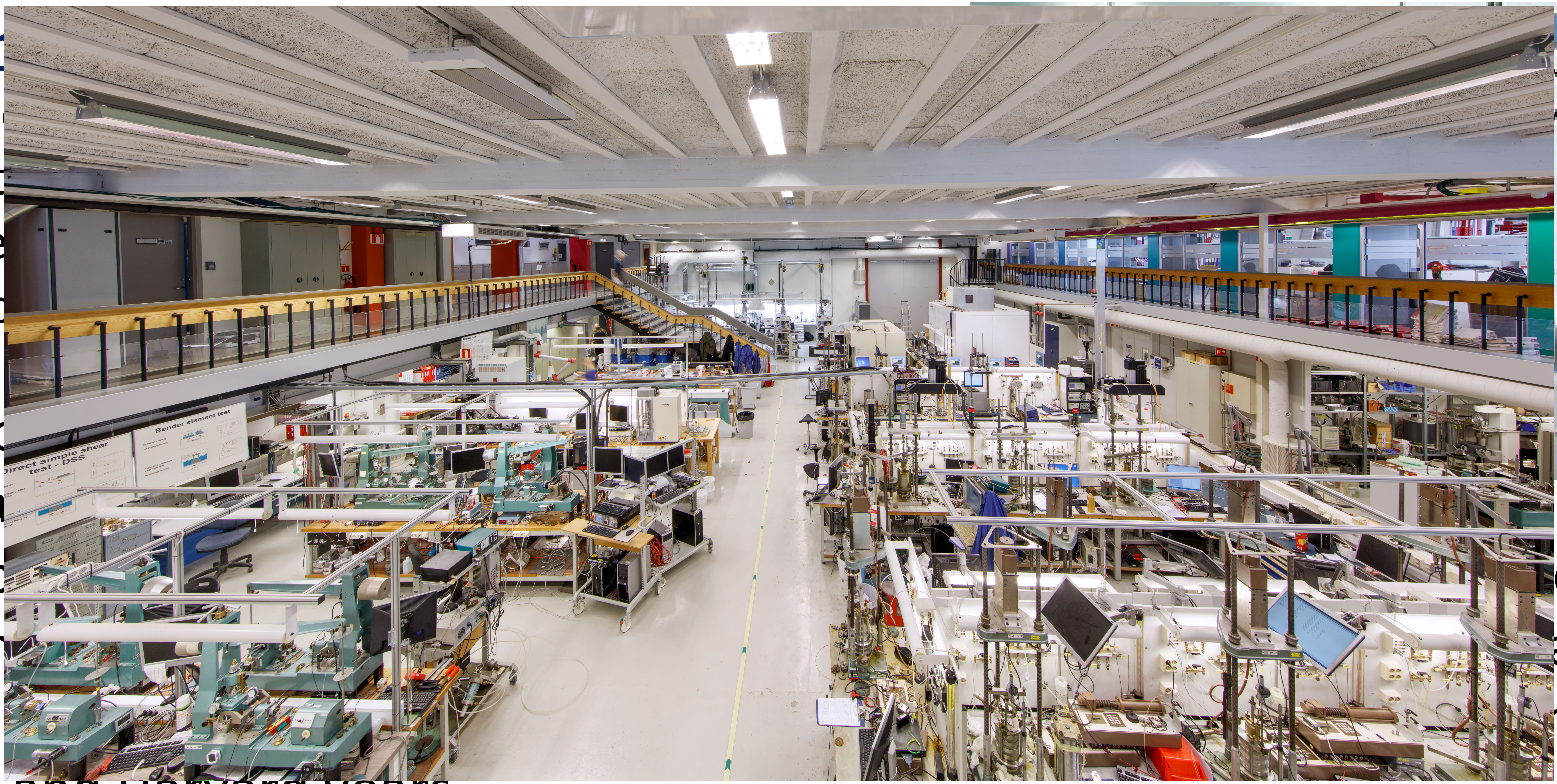
Coulomb, the Artisan of modern geotechnical engineering

In praise of testing

Coulomb was an experimental scientist. He wrote about friction and cohesion: “Only experiment can help us to decide the reality of the different causes" [of shear resistance].

Following Coulomb's philosophy, Karl Terzag measure soil and interface properties everywhere he worked (Royal Ottoman College of Engineering in Istanbul, then MIT, and so Harvard). Many of the remarkable advancements in geotechnical engineering are due to Terzaghi's, Casagrande's and Hvorslev's experimental works in the MIT and Harvard years.

Coulomb, the Artisan of modern geotechnical engineering



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Coulomb, the Artisan of modern geotechnical engineering

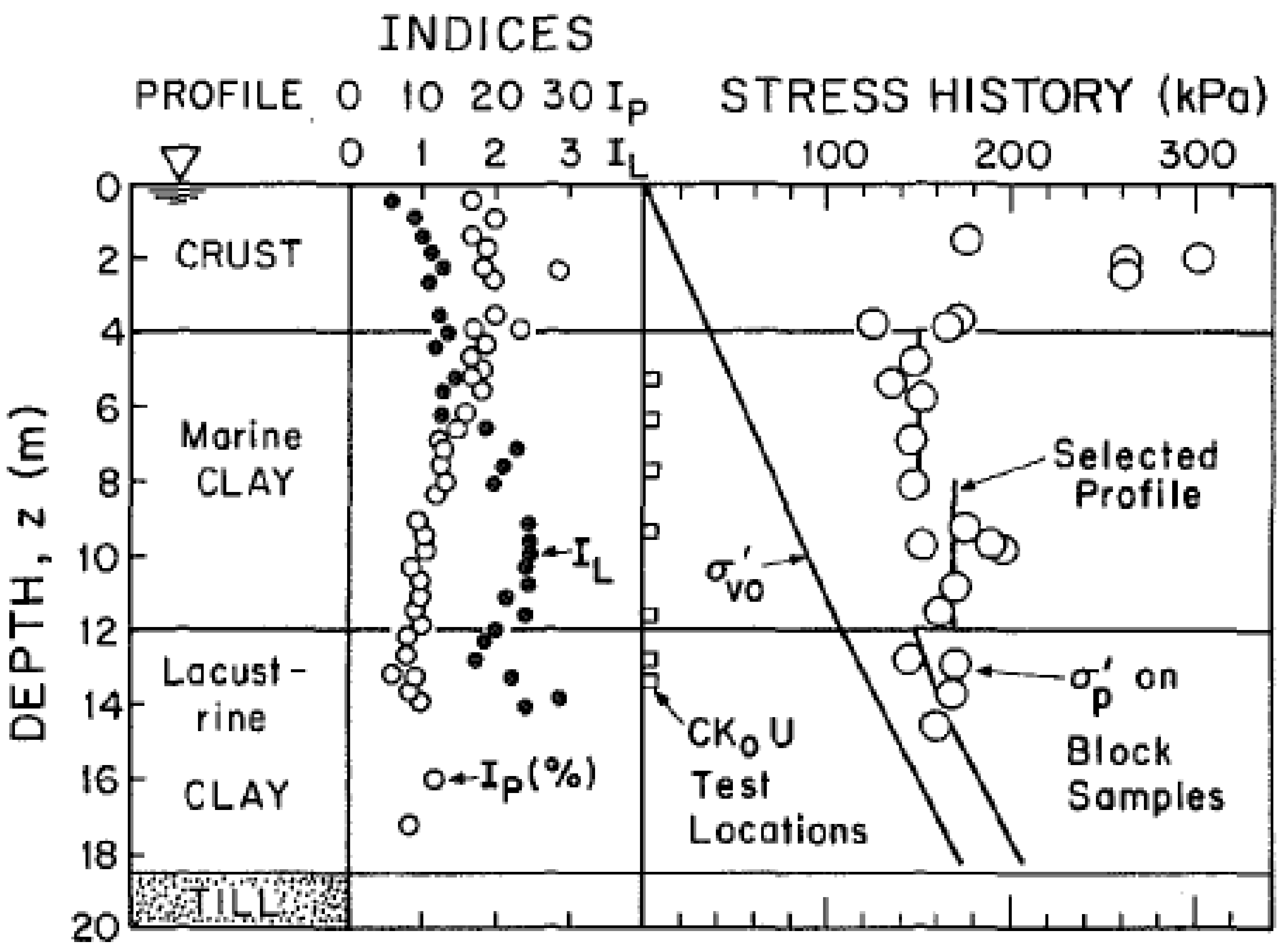
In praise of testing

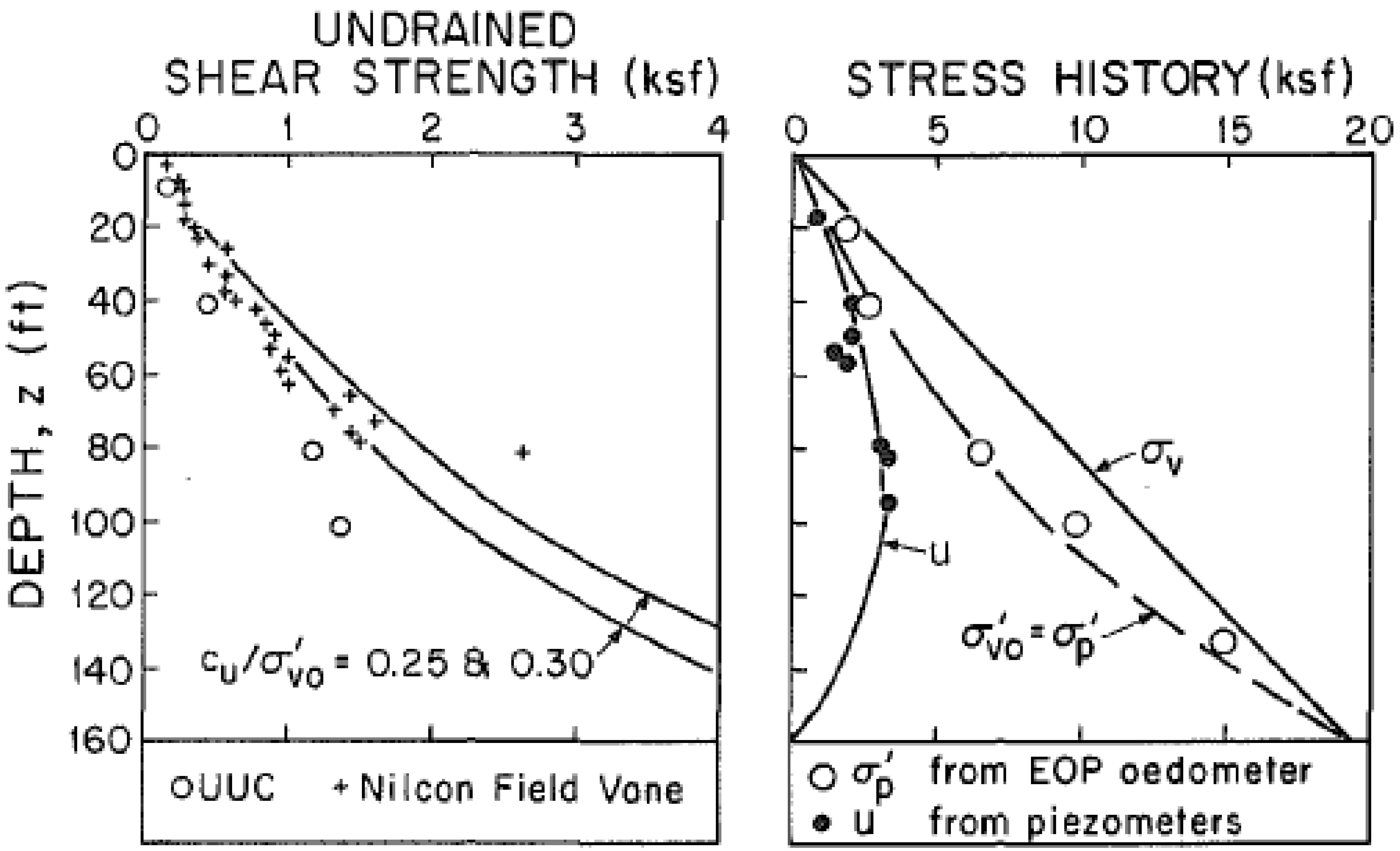
If only clay deposits could talk, like the Atchafalaya clay and New Orleans flood-protection levee sites or the sensitive clays in Norway, they would tell how important it is to thoroughly understand the geology and the stress history of a deposit, and then apply Coulomb's equation.

Geotechnical engineers need to have a sense of how soil behaves by “touching and feeling a sample,” and understanding which in-situ and laboratory tests are needed to determine representative soil properties, the stress history and corresponding shear strength profile. These "in situ" conditions are then used to apply Coulomb’s equation.

Coulomb, the Artisan of modern geotechnical engineering

In praise of testing



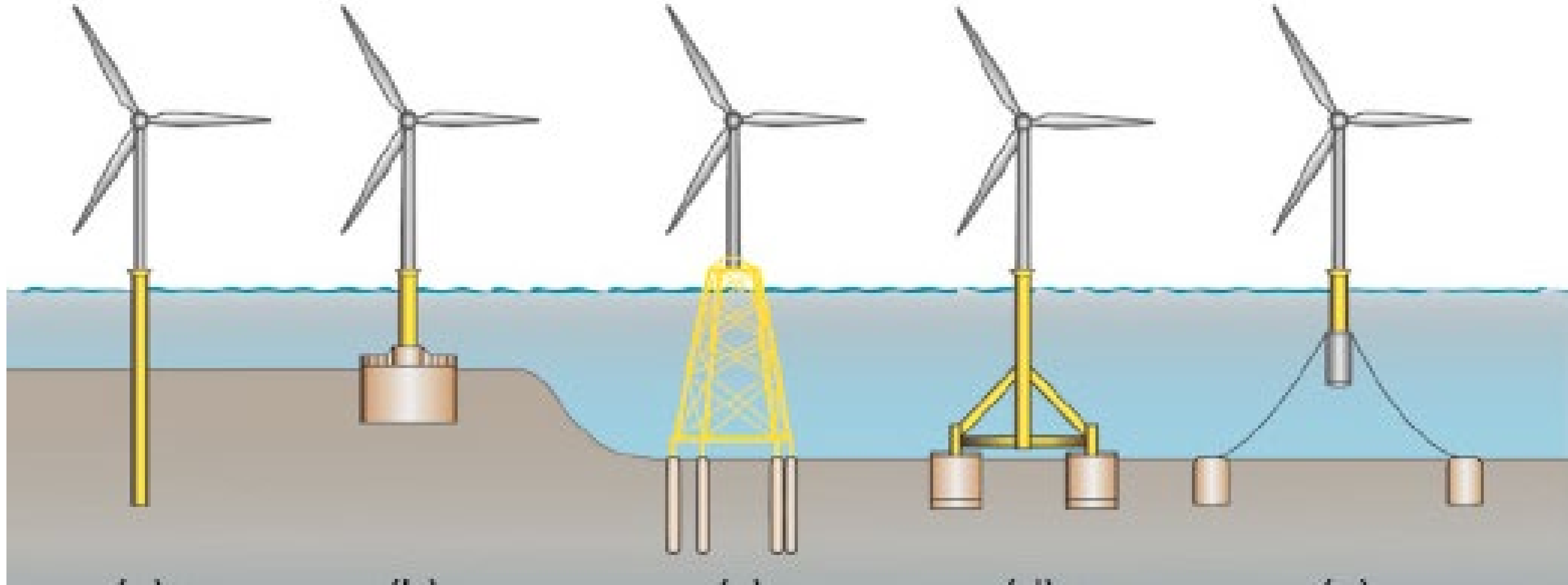


Ladd (190), Terzaghi Lecture

Coulomb, the Artisan of modern geotechnical engineering

Design of offshore installations

The yield stress design of Coulomb is also part of the solution for newer foundations, although many new factors and conditions have been found to affect the Coulomb shear strength parameters. Likewise, the behaviour of sands, silts and clays under cyclic loading (not a topic in 1773) is still represented today with stress paths, using Coulomb's law. The behaviour in the working stress ranges (serviceability limit state) is key to the solution for offshore design. The soil resistance depends on the mean normal stress level and the degree of shear mobilization, or Coulomb's principle.



Coulomb, the Artisan of modern geotechnical engineering

Reliability-based design and effect of uncertainties on computed safety

These remarks have not yet taken into account the quality of the masonry, which must always be left to dry carefully before being loaded. They have not yet taken into account frost, the enemy whose effects are without doubt the most dangerous that masonry has to fear; since, as well as the increase of pressure that frost produces in wet soils by the increase of volume, and as well as the blockage of the drainage pipes, it is certain that any wall which experiences severe frosts before being dry will necessarily lose the greatest part of its cohesion, and will have no strength.

Despite all these remarks, which seem to lead to the conclusion that retaining walls should be given individual dimensions according to the nature of the backfill which exerts a thrust on them, and that there is less danger in decreasing the size of terrace walls in dry hot countries than in damp cold countries, nevertheless I think that, for all kinds of soil, retaining walls can be designed without danger with a batter of $\frac{1}{6}$, and with the ridge one seventh of the height (as in article 11).

Excerpt from the Essai concluding on retaining wall design (translation Heyman, 1972)

Coulomb, the Artisan of modern geotechnical engineering

Reliability-based design and effect of uncertainties on computed safety

In Coulomb's Essai, one senses that Coulomb was aware that many factors are important and may affect the soil resistance, and that they should be considered to ensure that the design is safe enough (or reliable enough). Coulomb refers to the effect of pore water pressure, moisture content, potential blocking of drains and presence of ice and fissures in the retaining wall. He lists aspects not accounted for, such as quality of material and freeze-thaw effects. Coulomb then makes a value judgment on what would be an acceptable design.

Coulomb, the Artisan of modern geotechnical engineering

Reliability-based design and effect of uncertainties on computed safety

Coulomb acceptable design for a r
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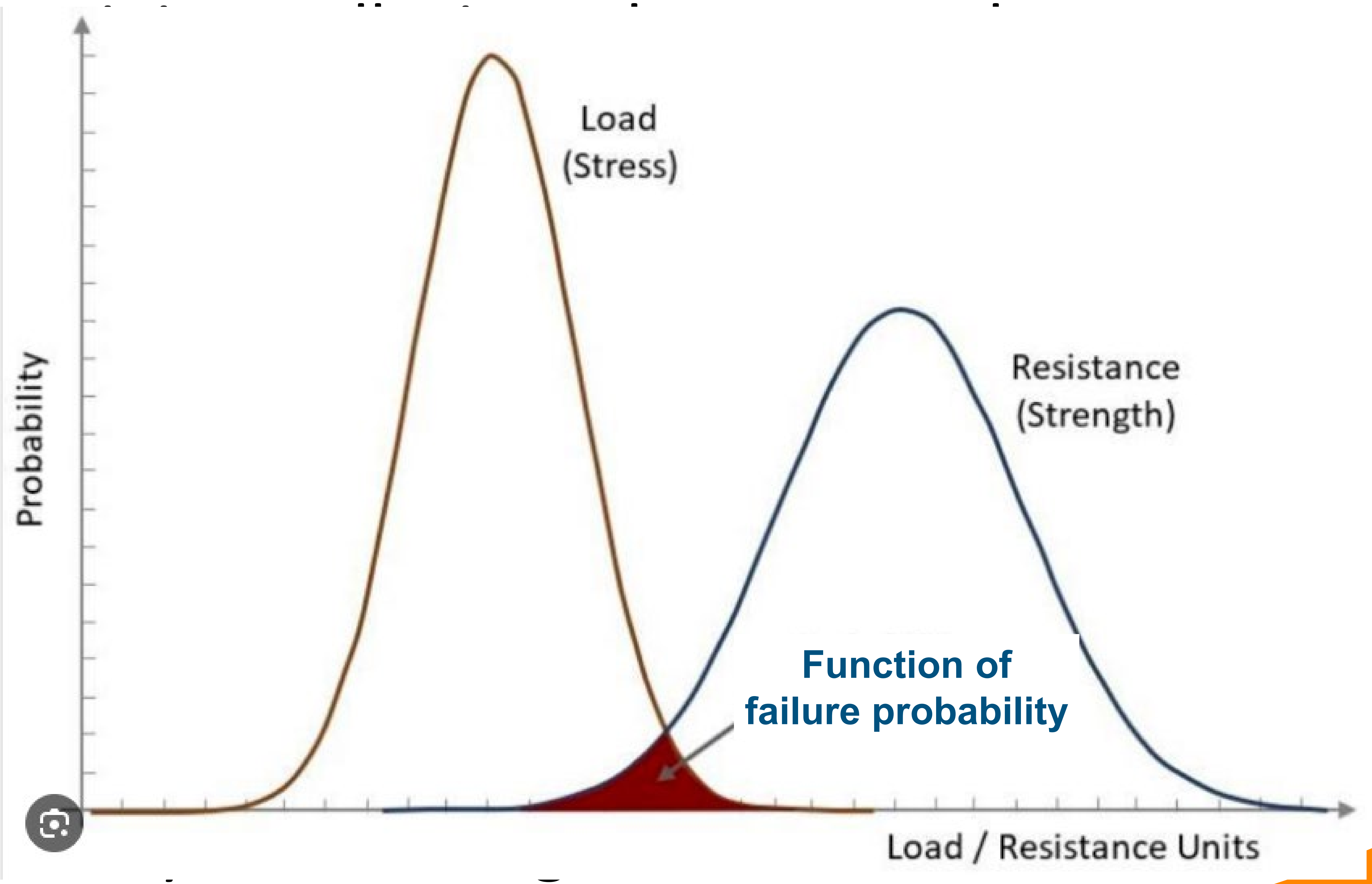


Guidelines, standards and code

Design guidelines and standards are written in terms of cohesion and friction and Coulomb's fundamental slip plane model still dominates today's recommendations for verifying the safety of foundation works. The parameters needed to check the serviceability limit state (SLS) prescribed in guidelines, standards and codes are based on Coulomb's equation. Likewise, the verification of the ultimate limit state (ULS) is based on the Coulomb principles. The yield design approach is still the essence of a majority of design codes.

Contents

Summary

Coulomb's objective when presenting his findings to the Academy

« … de rendre les principes dont je me suis servi assez clairs pour qu'un **Artiste un peu instruit** pût les entendre et s'en servir…. Ce Mémoire (…) n'était d'abord destiné qu'à mon usage particulier, dans les différents travaux dont je suis chargé par mon état; si j'ose le présenter à cette Académie."	"… to make the principles I have used clear enough for **an educated Artist** to understand and use. …This Memoir (…) was at first intended for my own use only in the different tasks with which I am charged by my state; if I dare present it to this Academy".

Summary

The value of Coulomb's works lies not only in the groundbreaking discovery he made in the 18th century, but mainly in the developments to which the discovery led over the next 250 years. Coulomb probably did not foresee the significance of his work for an entire branch of civil engineering and for resource and infrastructure development on a worldwide scale.
Coulomb's objective with his Essai was to share his knowledge and hope that it would come in use to someone likewise interested in the future. Charles Augustin de Coulomb certainly reached his objectives.

Summary

Coulomb's equation, together with the effective stress principle inspired to Karl Terzaghi by Coulomb's work, are the mainstay of geotechnical practice. Coulomb's work continues to inspire new generations of researchers to look into improved and novel interpretations of the shear strength of soil and rock.

Summary

Coulomb's contribution to the design of structures deserves the attention and recognition it has received. There have been new developments since, but the origin of the understanding was with Coulomb's equation. That the profession discovers limitations, additional influential factors (e.g., anisotropy, soil sensitivity, cyclic loading) or extensions does not invalidate the theory, rather it validates it and accentuates the outreach of the yield design approach initiated by Coulomb in his 1773 Essai.

Closure

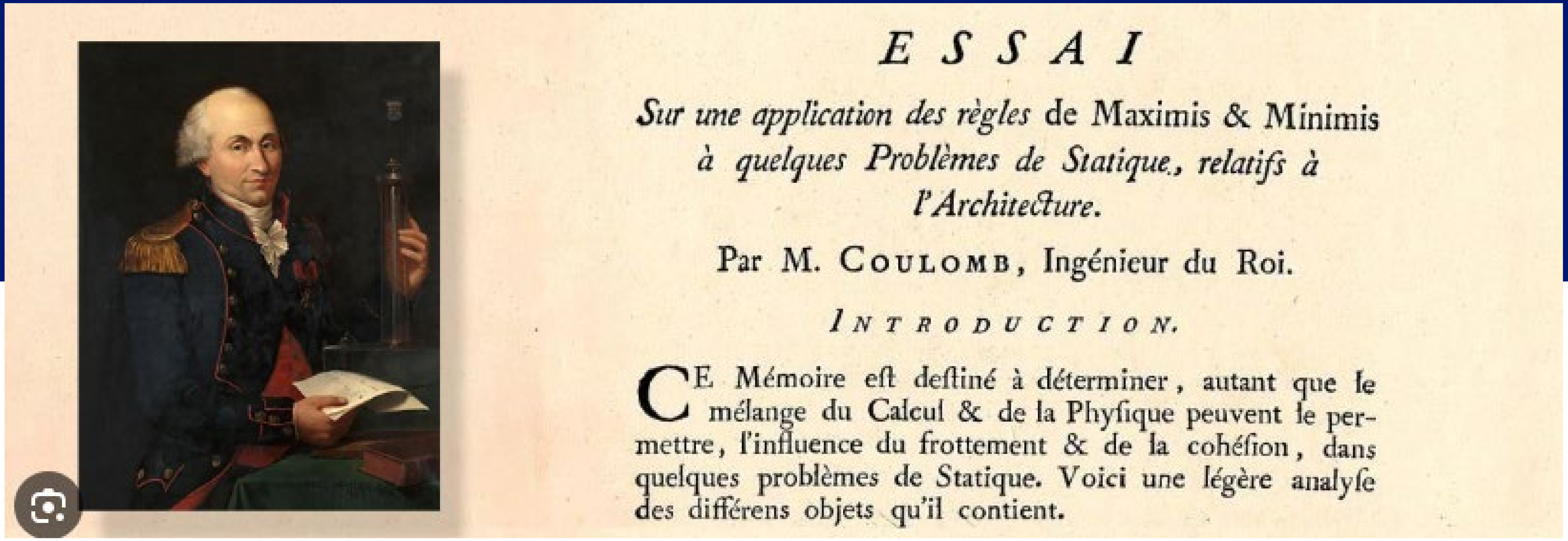
In closing, Gustave Eiffel named 72 Engineers and Scientists in the "Brotherhood of the Eiffel Tower" to outline the importance and necessity of teamwork and cooperation in engineering. The 300-m high Eiffel Tower built in 1889 is a triumph of human ingenuity and engineering collaboration. Gustave Eiffel did not think of the tower as his own. When Eiffel dedicated the Tower, he recognized the many contributors to French science and technology during the period 1789 – 1889 by inscribing 72 names on the sides of the tower (Nuttle, 2020).

Closure

Who were the scientists chosen by Gustave Eiffel to be member of the "Brotherhood of the Eiffel Tower"? Two were agronomists, six astronomers, nine chemists, four industrialists, one geographer, fifteen civil engineers, seventeen mathematicians, two physicians, two mechanical engineers, two mineralogists, one biologist, and eleven physicists. Charles Augustin de Coulomb’s name is inscribed on the Eiffel Tower, together with the likes of Ampère, Becquerel, Bréguet, Carnot, Cauchy, Coriolis, Foucault, Fourier, Lamé, Laplace, Lavoisier, Navier, Poisson, and Tresca.

Closure

Coulomb was a physicist by training but he was also an engineer.


E S S A I

Sur une application des règles de Maximis & Minimis *à quelques Problèmes de Statique, relatifs à l'Architecture.*

Par M. Coulomb, Ingénieur du Roi.

INTRODUCTION.

CE Mémoire eſt deſtiné à déterminer, autant que le mélange du Calcul & de la Phyſique peuvent le permettre, l'influence du frottement & de la cohéſion, dans quelques problèmes de Statique. Voici une légère analyſe des différens objets qu'il contient.

Being an engineer requires specialized knowledge, an insatiable interest in how things work, and a knack for solving problems. Charles Augustin de Coulomb mastered all aspects, and this gave birth to the basics for the science of today's soil mechanics and geotechnical engineering.