COMITÉ FRANÇAIS DE MÉCANIQUE DES SOLS ET DE GÉOTECHNIQUE

Charles-Augustin COULOMB - A geotechnical tribute

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Yield Design-based analysis of reinforced soil structures, making use of different models of reinforced soils

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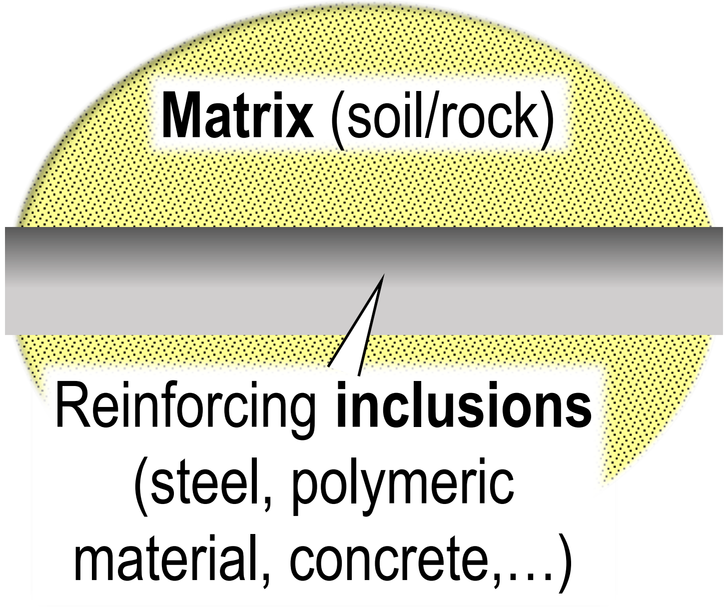
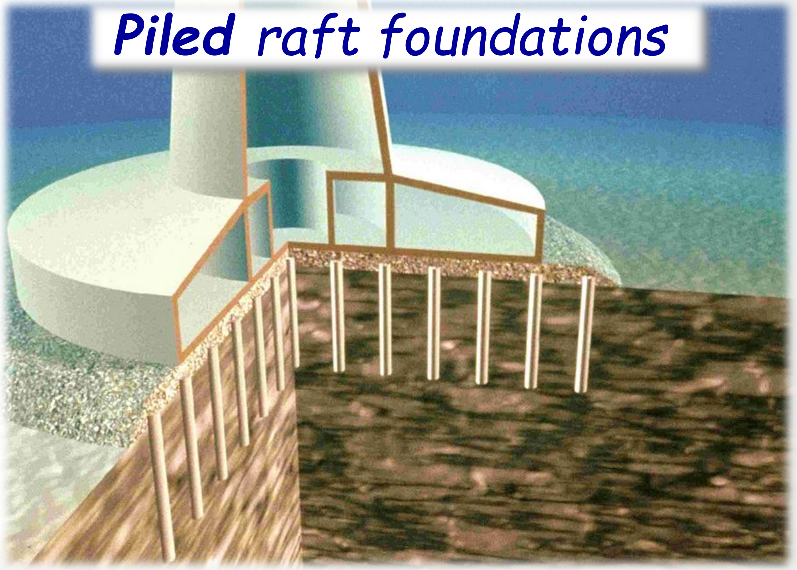
Reinforced soils/rocks regarded as ***«geocomposite»*** materials





Stability analysis of *reinforced soil* structures by means of the ***Yield Design*** approach
(Salençon, 1990, 2013, 2023)

Checking the **compatibility** between:

- **Static equilibrium**
- **Strength (failure)** conditions

…under ***prescribed loading***

Three possible ***mechanical models*** of reinforced soils:

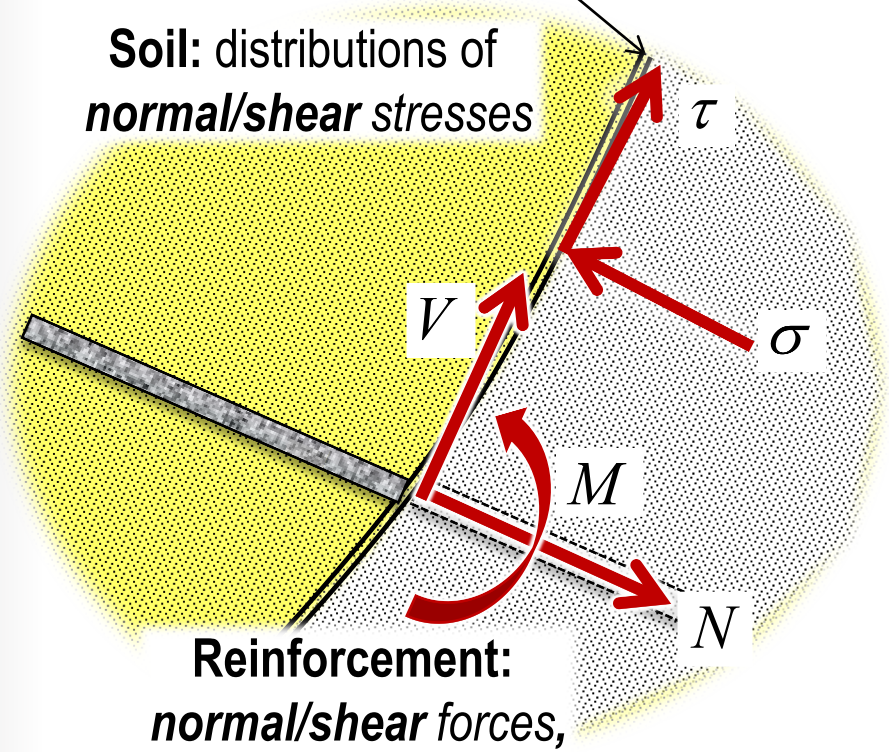
1. The ***mixed modelling*** approach : soil as a **3D** continuum vs. inclusions as **1D** structural members.
2. The ***homogenization*** method: reinforced soil as a **homogeneous anisotropic** medium.
3. The ***multiphase*** model as an **extension** of the homogenization method.

1. Mixed modelling of reinforced soils in the context of *Yield Design*

Classical stability analyses by ***«failure surfaces»***

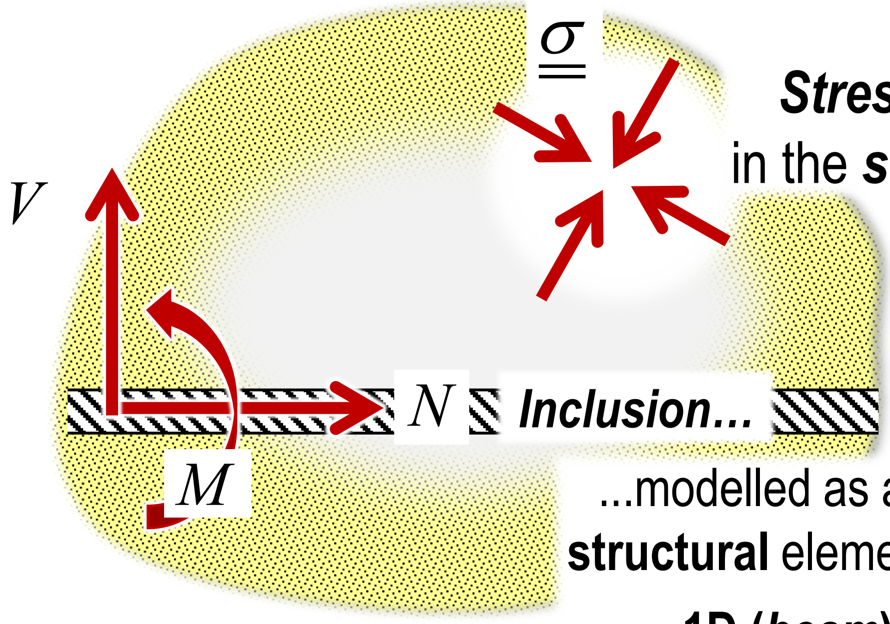
Mixed modelling of reinforced soils

Soil: distributions of ***normal/shear*** *stresses*



Reinforcement: ***normal/shear*** *forces*, ***bending*** *moment*

Stress field in the ***soil*** mass

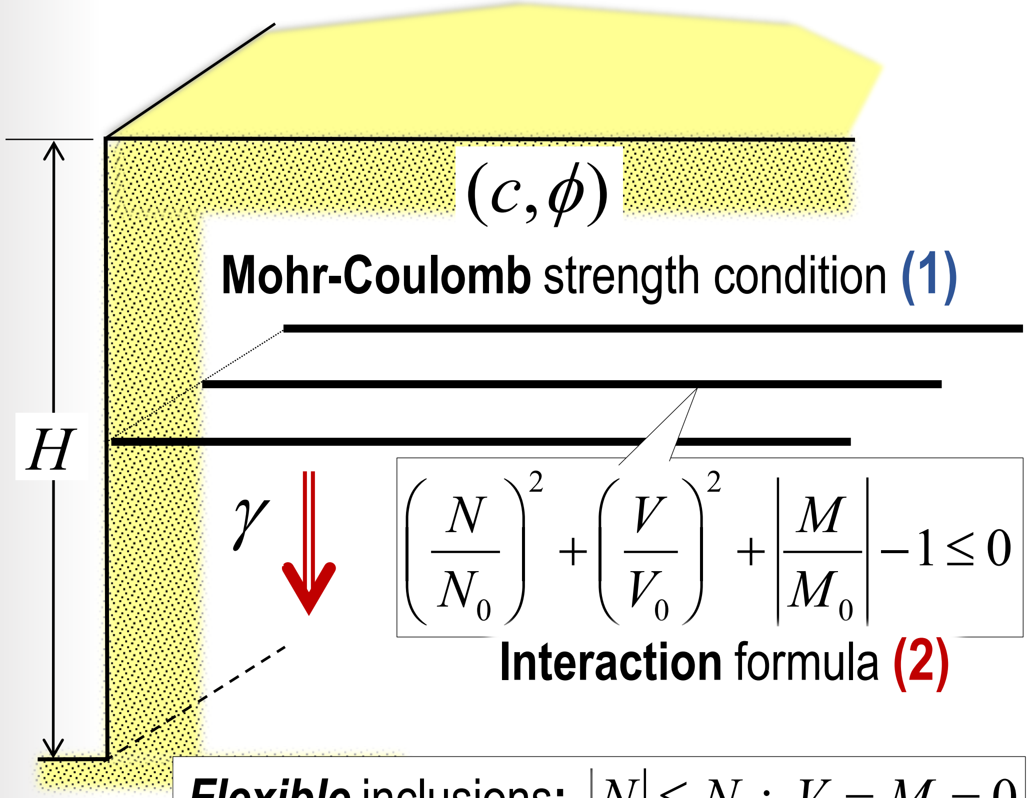


...modelled as a **structural** element

- **1D (*beam*)**
- **2D (*plate*)**

Implementation of the ***Yield Design*** reasoning: an illustrative example

Stability analysis of
a *reinforced* vertical cut



Flexible inclusions: $|N| \le N_0;\ V = M = 0$

Stability of the excavation $\Leftrightarrow$ $\Gamma = \gamma H / c \le \Gamma^+$

$\Updownarrow$

$\exists \left\{ \begin{array}{c} \underline{\underline{\sigma}} \text{ in the } \textbf{soil}; \\ (N,V,M) \text{ along the } \textbf{reinforcements} \end{array} \right\}$

....in **equilibrium** with γ

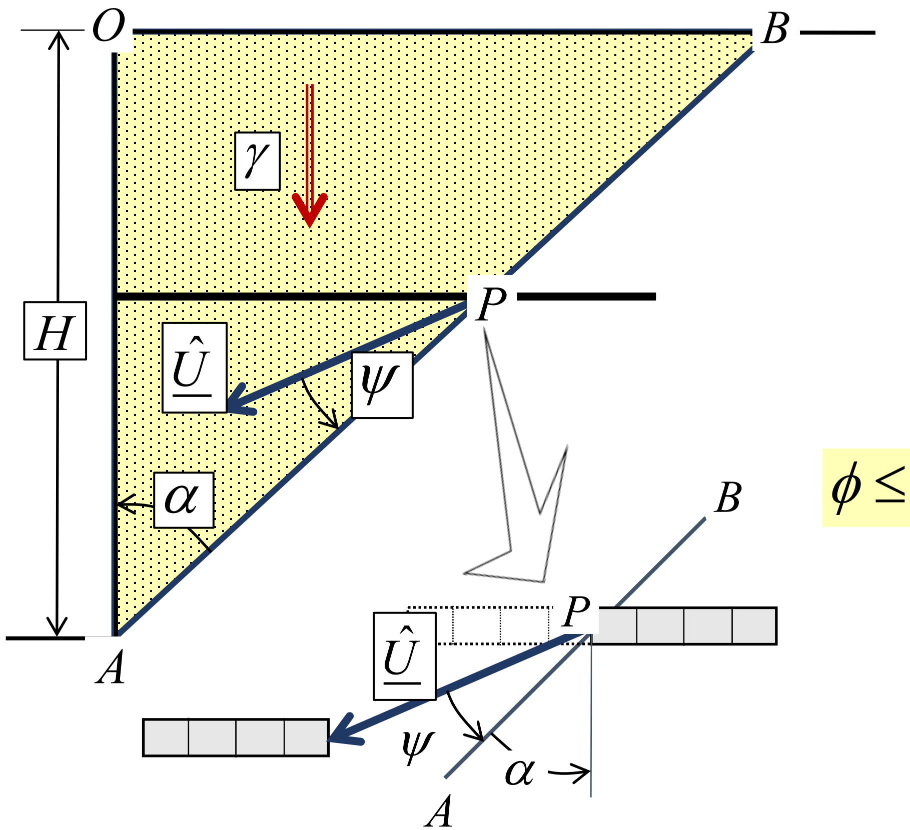
$\left. \begin{array}{c} \underline{\underline{\sigma}} \\ (N,V,M) \end{array} \right\}$ verifying **strength** conditions $\left\{ \begin{array}{c} \textbf{(1)} \\ \textbf{(2)} \end{array} \right.$

Critical stability factor : $\Gamma^+ = \Gamma^+(\phi,$
$...r = N_0 / 2cH, v = V_0 / N_0, m = M_0 / N_0 H)$

Lower bound static approach proves difficult, if not *impossible*, to implement

A simple ***analytical***
upper bound kinematic approach

Virtual translational ***failure mechanism***



$\phi \le \psi \le \pi - \phi$

Virtual **work** of ***external forces***
(per unit transverse length)

$$W_e(\underline{\hat{U}}) = \overbrace{1/2\gamma H^2 \tan\alpha}^{\text{weight } OAB} \left|\underline{\hat{U}}\right| \cos(\alpha + \psi)$$

Maximum resisting **work:**
two contributions

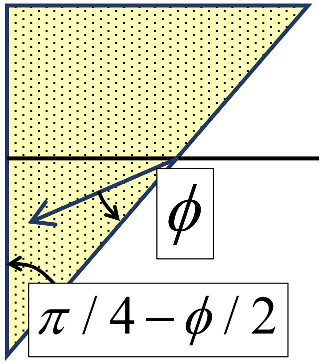
$$W_{mr}(\underline{\hat{U}}) = \begin{cases} W_{mr}^{\text{soil}} = c\left|\underline{\hat{U}}\right| H \dfrac{\tan\alpha}{\tan\phi} \sin\psi \\ + \\ W_{mr}^{\text{reinf.}} = \left|\underline{\hat{U}}\right| \left(\begin{array}{l} N_0^2 \sin^2(\alpha+\psi) \\ + V_0^2 \cos^2(\alpha+\psi) \end{array} \right)^{1/2} \end{cases}$$

Kinematic theorem* of *Yield Design

$$\text{Stability} \Leftrightarrow \gamma H / c \leq \Gamma^+ \boxed{\Rightarrow \quad \forall \hat{\underline{U}},\ W_e(\hat{\underline{U}}) \leq W_{mr}(\hat{\underline{U}})}$$

$$\Rightarrow \forall(\alpha,\psi) \in]0,\pi/2[\times]\phi,\pi-\phi[,\ \Gamma^+ \leq \Gamma^{\text{UB}}\boxed{(\alpha,\psi)} \rightarrow$$

***optimization* parameters**



***Optimal upper bound* for $\psi=\phi$ and $\alpha=\pi/4-\phi/2$**

$$\Gamma^+ \leq \Gamma^{\text{UB}}_{\text{opt.}} = 4\sqrt{K_p}\left[1+\frac{N_0}{2cH}\left(\left(\frac{V_0}{N_0}\right)^2+K_p\right)^{1/2}\right]$$

$K_p = \tan^2(\pi/4+\phi/2)$

***passive earth pressure* coefficient**

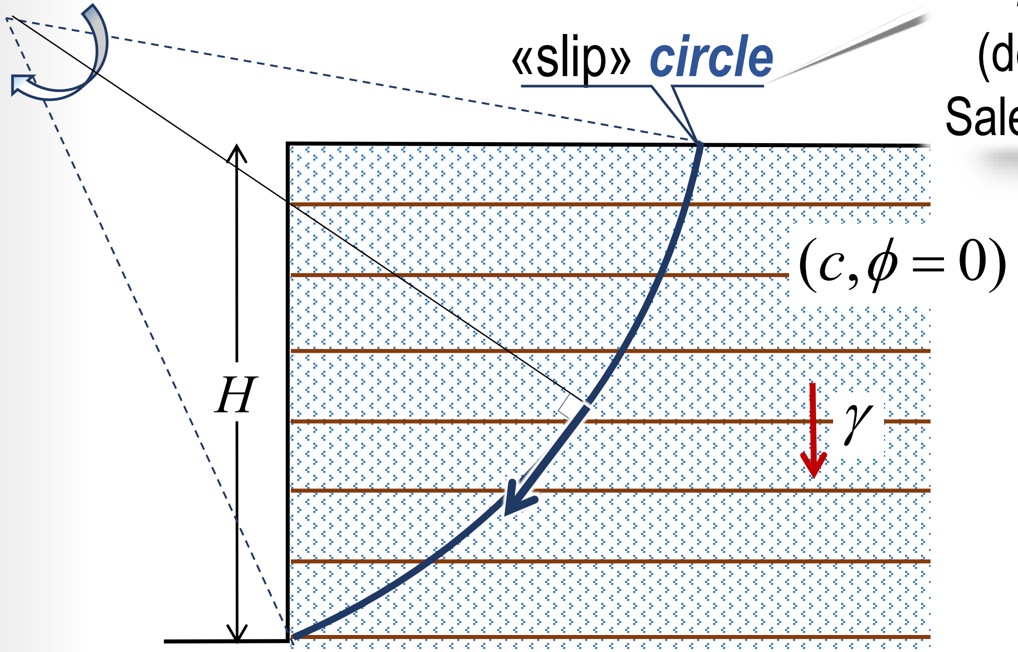
- ***Unreinforced* excavation $N_0 = V_0 = 0 \rightarrow \Gamma^+ \leq \boxed{\Gamma^{\text{UB}} = 4\sqrt{K_p}}$**
- Reinforcement with ***flexible*** inclusions

$$V_0 = 0 \rightarrow \Gamma^+ \leq \boxed{\Gamma^{\text{UB}} = 4\sqrt{K_p}\left[1+\sqrt{K_p}\frac{N_0}{2cH}\right]}$$

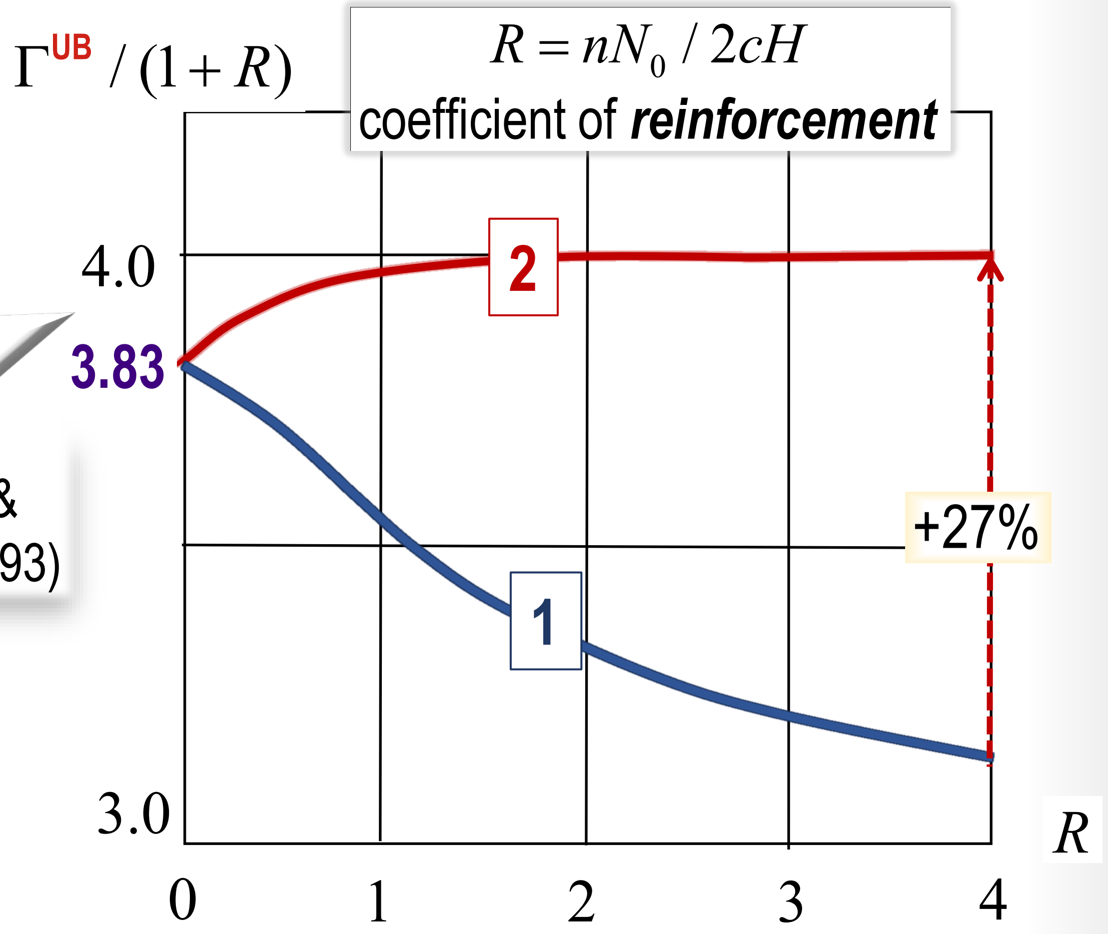
To what extent should the ***shear* (V_0)** and ***bending* (M_0) *strengths*** of the reinforcements be taken into account?

Stability of a ***uniformly reinforced*** excavation…

….using ***rotational*** *failure mechanisms*



Results
(de Buhan & Salençon, 1993)



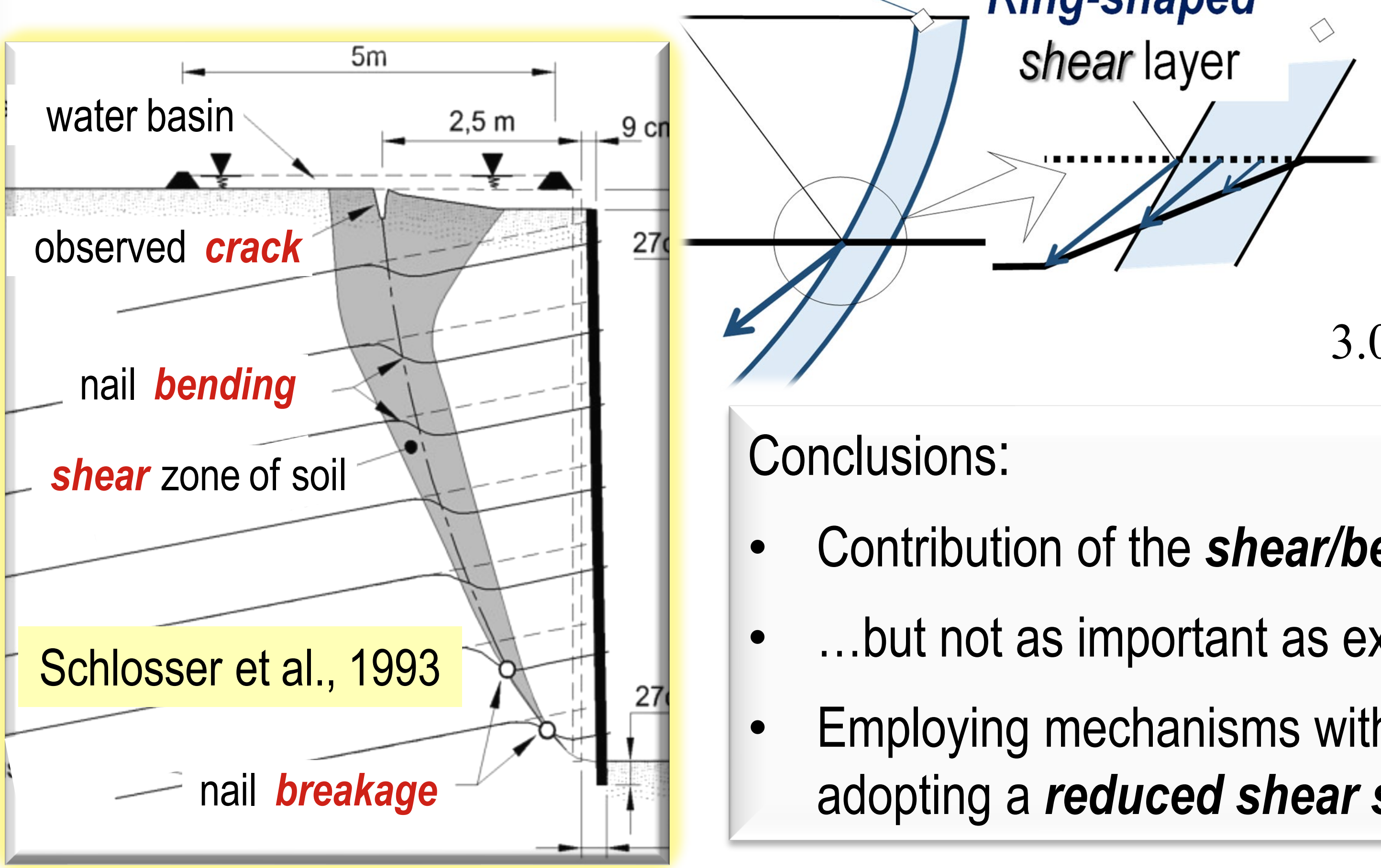
1 ***Flexible*** inclusions $V_0 = M_0 = 0$

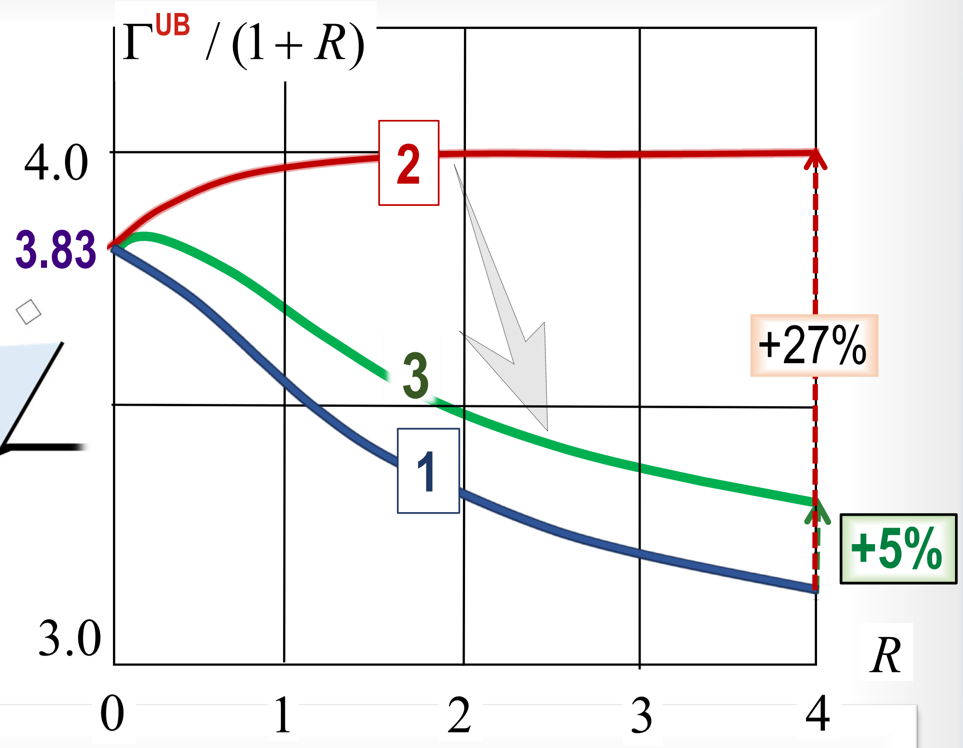
2 ***Shear-bending*** *resistant* inclusions (***soil nailing***)

$$V_0/N_0 = 0.5, \quad M_0/N_0H = 5\times10^{-4}$$

Improved *upper-bounds* derived from rotational mechanisms with ***shear zones.***

…based on **experimental observations**



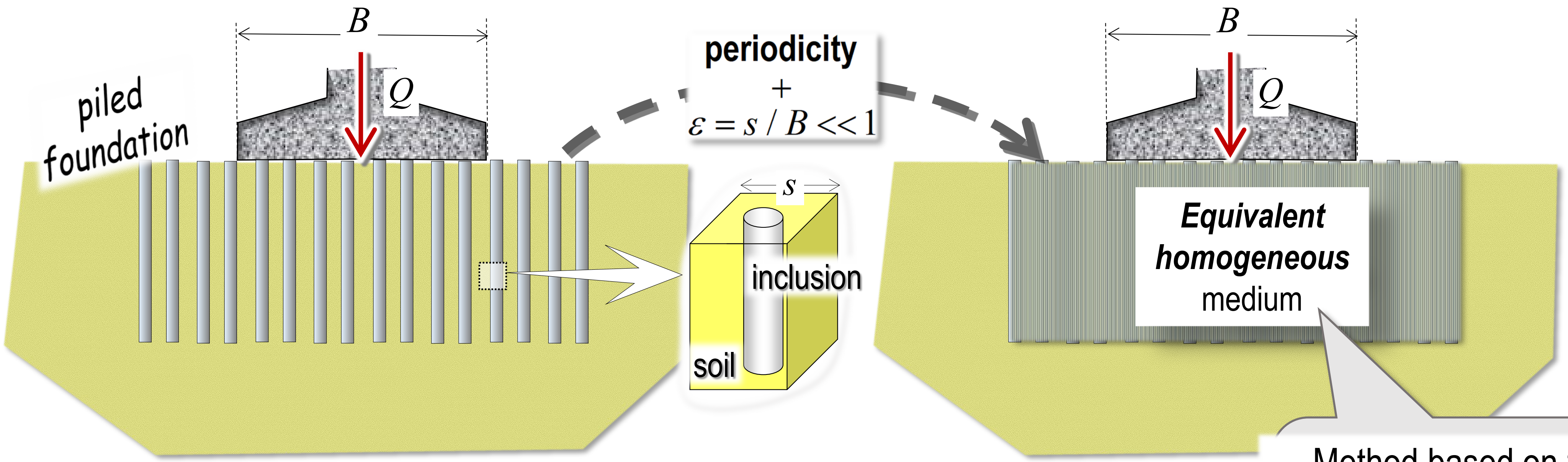


Conclusions:

- Contribution of the ***shear/bending*** strengths always **positive**…
- …but not as important as expected from a ***«slip-circle»*** analysis.
- Employing mechanisms with ***velocity discontinuity*** surfaces adopting a ***reduced shear strentgh:*** μV_0 instead of V_0

2. Yield strength of reinforced soils as ***anisotropic*** media: the **homogenization approach**

Applicable to ***densely*** and ***regularly*** reinforced soils



Method based on the prior determination of a **macrosopic strength condition**

Yield Design homogenization result
(Suquet, 1983; de Buhan, 1986)

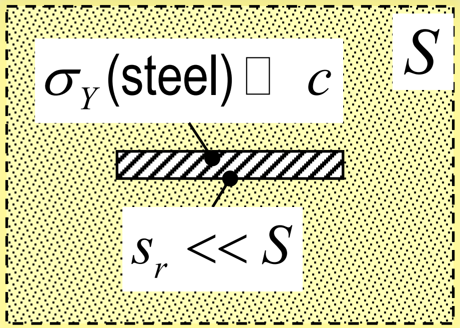
Ultimate bearing capacity of…

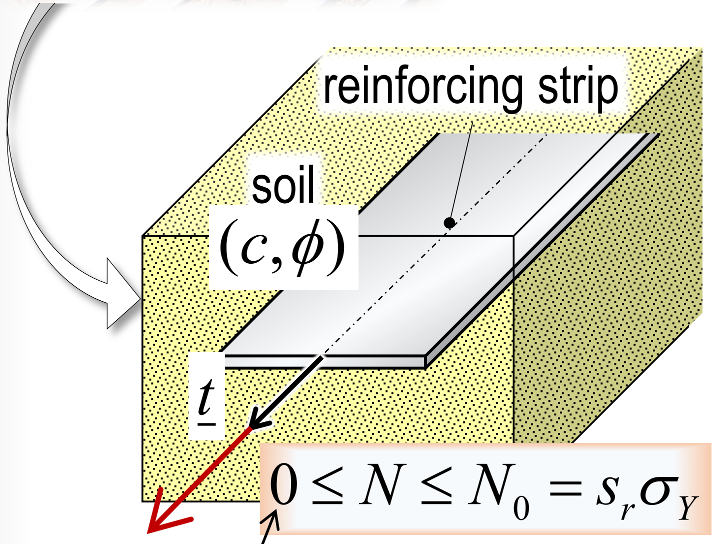
initial structure $Q^+(\varepsilon)$

scale factor $\varepsilon \to 0$

homogenized structure Q^+_{hom}

Formulation of a ***MACROSCOPIC STRENGTH CONDITION*** for ***thin highly resistant linear*** inclusions





$0 \le N \le N_0 = s_r \sigma_Y$

buckling

Tensile resistance

Derivation of a ***closed-form*** expression

(de Buhan & Salençon, 1987; de Buhan et al., 1989,…)

$$\boxed{f^{\text{hom}}\left(\underline{\underline{\Sigma}}\right) \le 0}$$ **«macroscopic»** stress

$$\Updownarrow$$

additive decomposition

$$\underline{\underline{\Sigma}} = \boxed{\underline{\underline{\sigma}}^s} + \boxed{\sigma^r \underline{t} \otimes \underline{t}}$$ **uniaxial** stress

$$\boxed{f^s\left(\underline{\underline{\sigma}}^s\right) \le 0}$$

Mohr-Coulomb ***strength*** criterion

$$\boxed{0 \le \sigma^r \le \sigma_0 = N_0 / S}$$

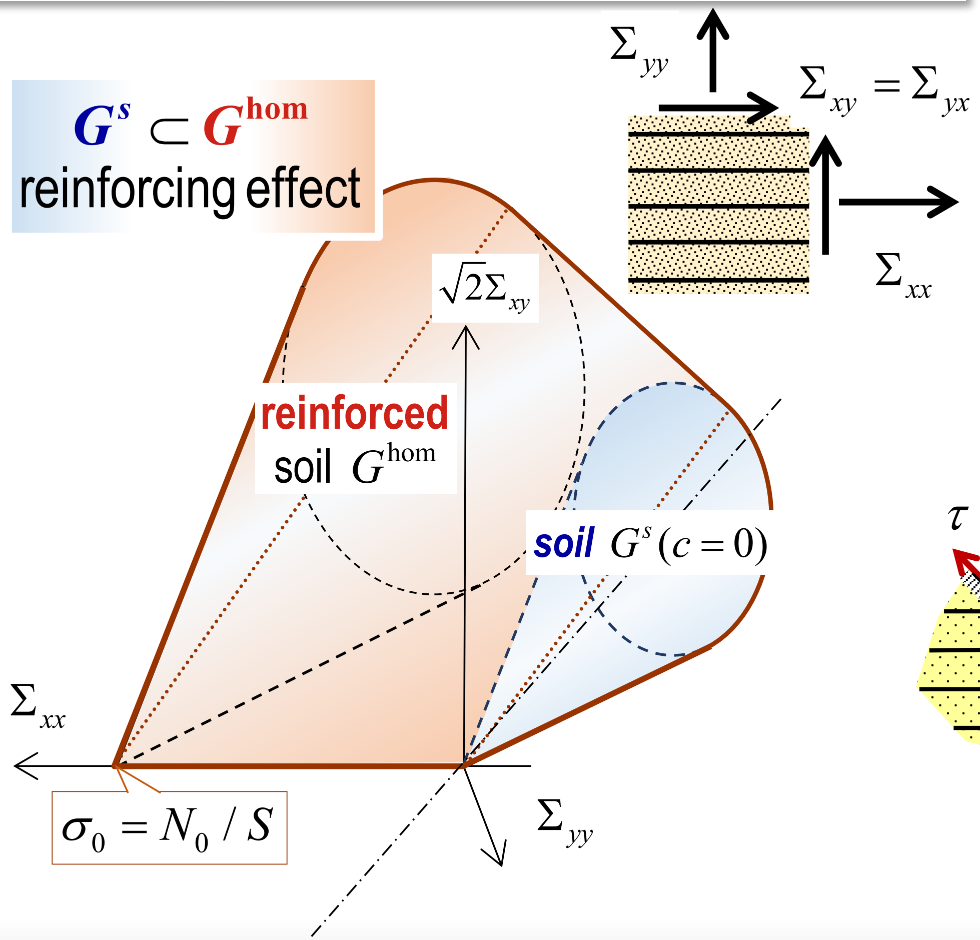
Interpretation of $\sigma_0 = \dfrac{N_0}{S} = \dfrac{s_r}{S}\sigma_Y = \boxed{\eta}\,\sigma_Y$

volume fraction <<1

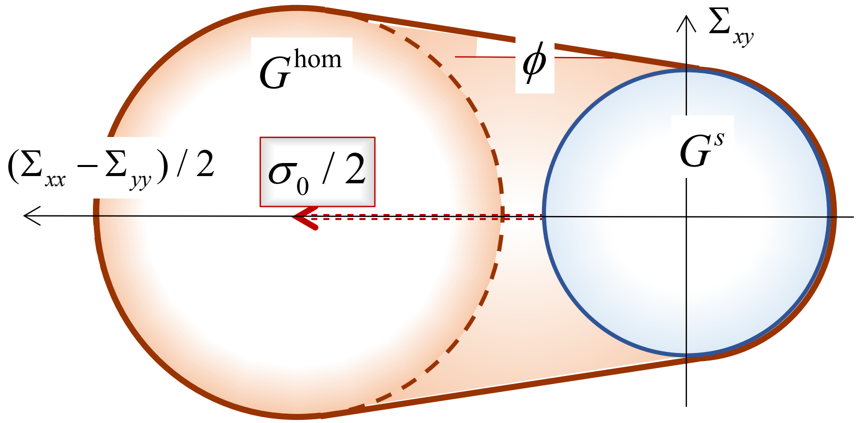
Tensile strength of the *reinforcements* per ***unit transverse area***

Three ***geometrical*** interpretations

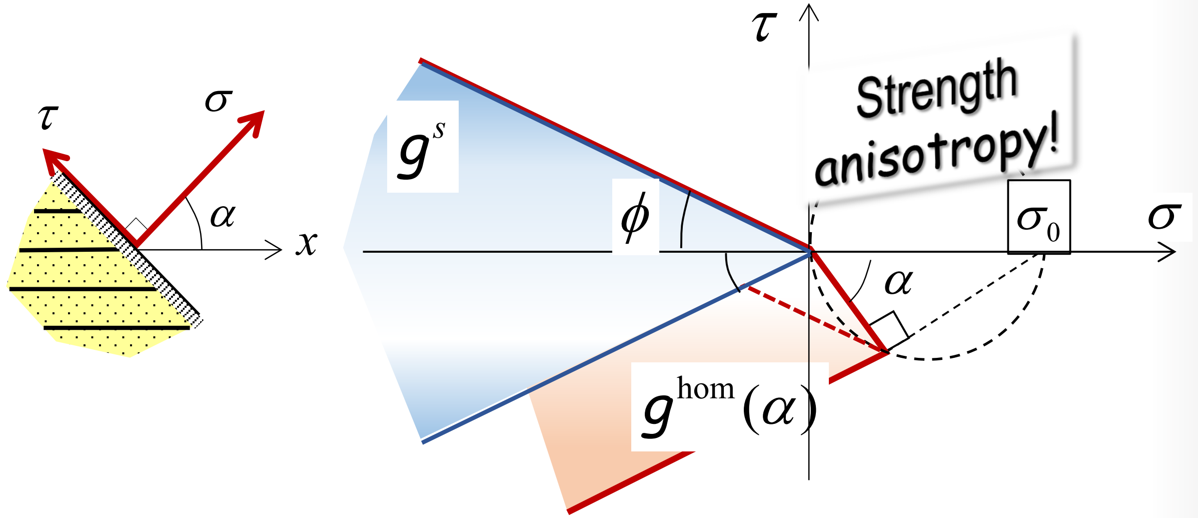
1 **Strength domains** in the space of stresses (***plane strain** conditions*)



2…in the ***deviatoric*** plane



3…in the ***Mohr's*** plane



Implementation of the **static** and **kinematic** approaches

Lower bound static approach

$$Q \le Q^{+}_{\text{hom}} \Leftrightarrow \exists\left\{\underline{\underline{\sigma}}^{s}, \sigma^{r}\right\} \text{ s.t.}$$

strength conditions

$$f^{s}(\underline{\underline{\sigma}}^{s}) \le 0 \text{ and } 0 \le \sigma^{r} \le \sigma_{0}$$

$$+$$

$$\underline{\underline{\Sigma}} = \underline{\underline{\sigma}}^{s} + \sigma^{r}\, \underline{t} \otimes \underline{t}$$

in **equilibrium** with Q

Upper bound kinematic approach

$$Q \le Q^{+}_{\text{hom}} \Rightarrow \forall \underline{\hat{U}}$$

$$W_{e}(\underline{\hat{U}}) \le W^{\text{hom}}_{mr}(\underline{\hat{U}})$$

with

$$W_{e}(\underline{\hat{U}}) = Q\dot{q}(\underline{\hat{U}})$$

and

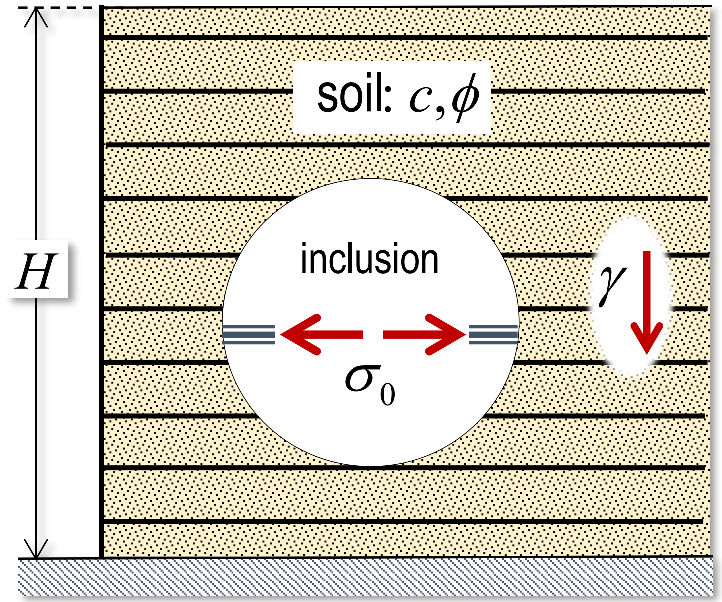
$$W^{\text{hom}}_{mr}(\underline{\hat{U}}) = W^{\text{soil}}_{mr}(\underline{\hat{U}}) + W^{\text{reinf.}}_{mr}(\underline{\hat{U}})$$

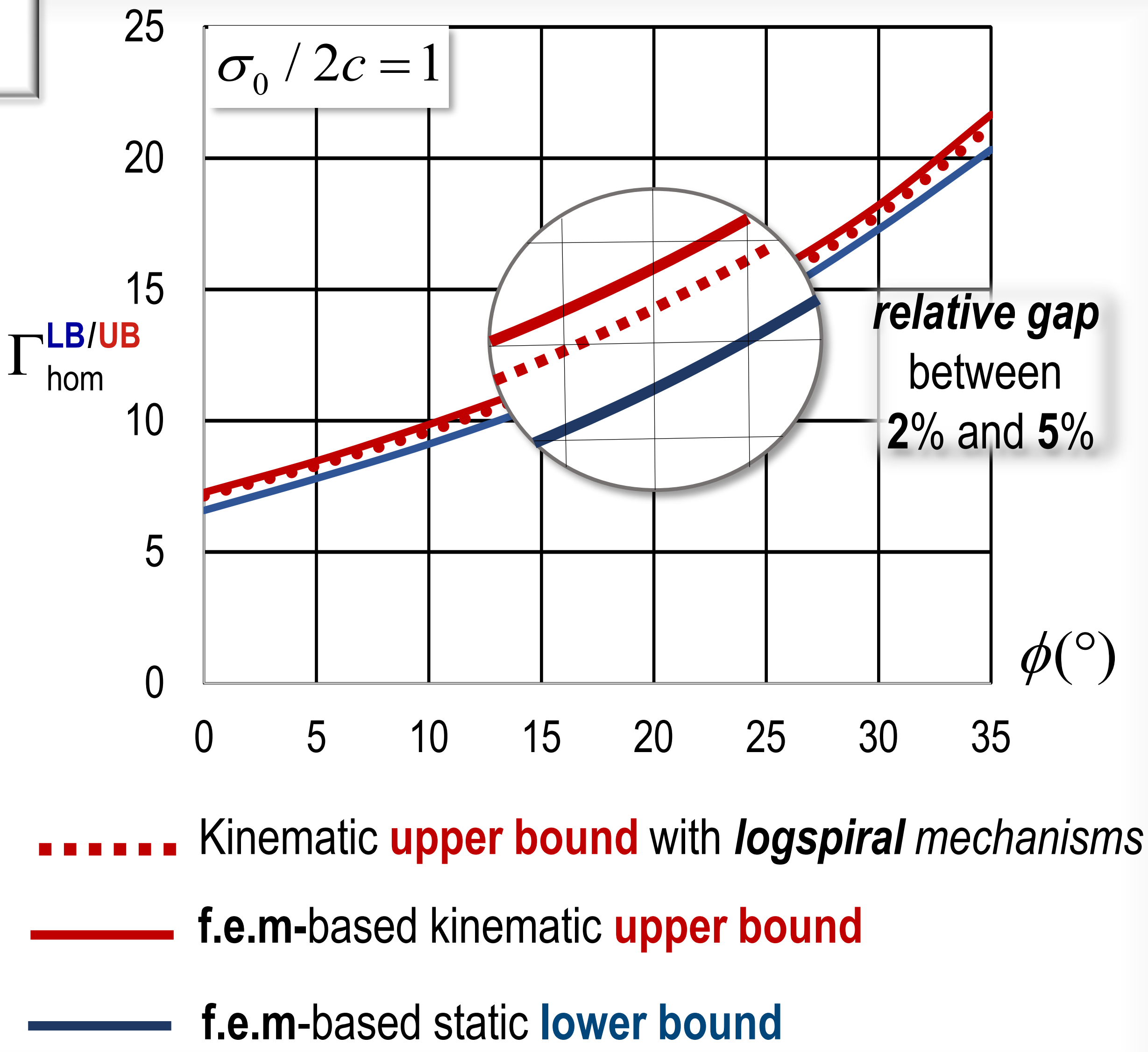
maximization → $\boxed{Q^{\text{LB}}_{\text{hom}} \le} \, Q^{+}_{\text{hom}} \, \boxed{\le Q^{\text{UB}}_{\text{hom}}}$ ← **minimization**

Stability analysis of a **reinforced retaining structure** (Abdi et al., 1994)

Evaluation of the **critical stability factor**

$$\gamma H / c \leq \Gamma^{+}_{\mathrm{hom}}(\phi)$$





Advantages of the **homogenization** approach***....***

- **Lower bound static** method of YD always feasible, unlike the *mixed modelling approach*.
- Considerable **simplification** of *Yield Design* analyzes and **sharply reduced** *computational times* by treating the reinforced soil as a **homogeneous** medium and not as a strongly **heterogeneous** material

...but ***two limitations***

- Fails to capture the **shear/flexural strength** properties of the reinforcements, implicitly considered as ***flexible.***
- Relies upon the assumption of **perfect bonding** between the *inclusions* and the *surrounding soil*.

….towards a ***multiphase description*** of reinforced soils

3. The **multiphase model** as an ***improved*** homogenization method

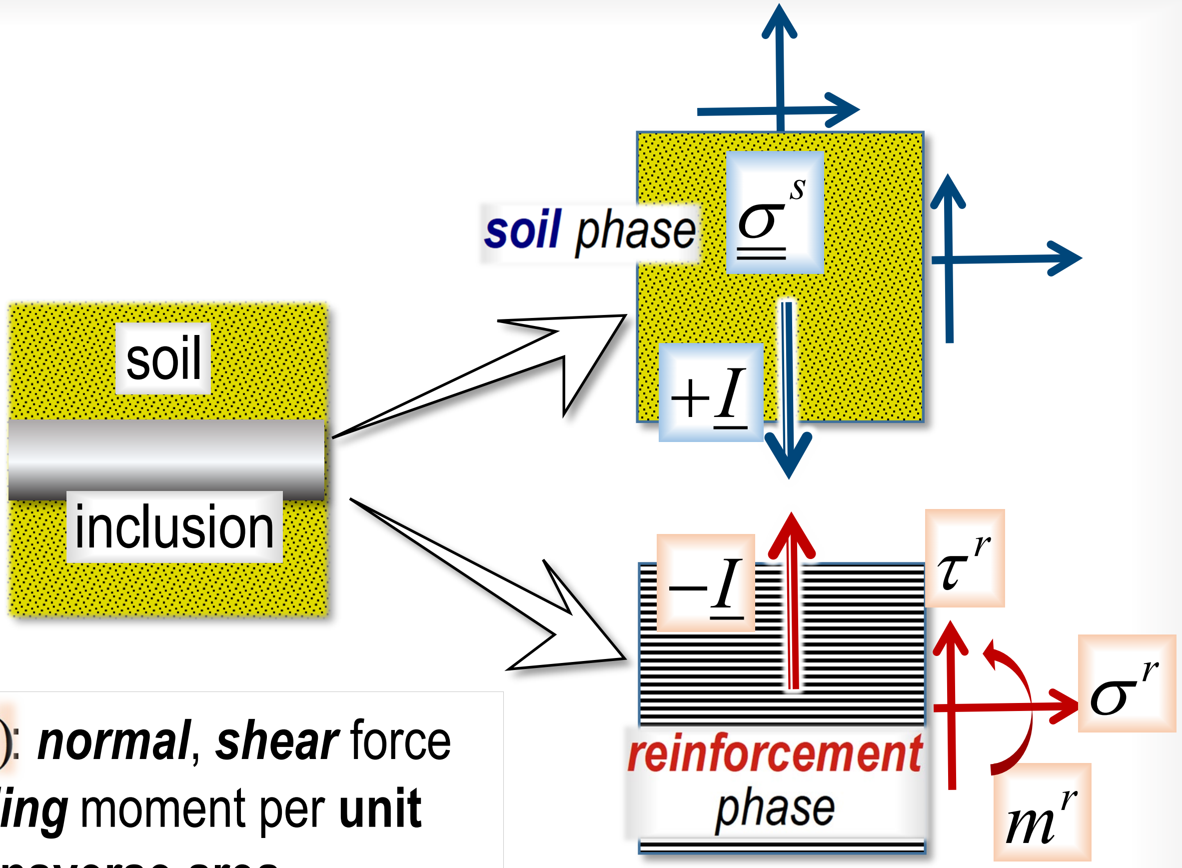
(de Buhan and Sudret, 1999; de Buhan and Hassen, 2010, …)

Description of the ***reinforced soil*** as:

- the *superposition* of two ***geometrically coïncident*** continua (***«phases»***)

- …in **mutual interaction.**

$\underline{I}$: ***interaction*** force ***volume*** *density*



(σ^r, τ^r, m^r): ***normal***, ***shear*** force and ***bending*** moment per **unit transverse area**

Yield Design analysis of *reinforced soil structures* as ***multiphase*** systems

$$Q \leq Q^{+}_{\text{mult.}}$$

$$\Updownarrow$$

soil ***reinforcement***

$$\exists \left\{ \boxed{\underline{\underline{\sigma}}^{s}} ; \boxed{\underline{I}} ; \boxed{\left(\sigma^{r}, \tau^{r}, m^{r}\right)} \right\}$$

interaction

Distributions of ***internal stresses***…

…**Statically Admissible** with Q

(*equilibrium equations* + *boundary conditions* expressed on *each phase* **separately**)

…and satisfying the different ***strength conditions***

soil phase: $f^{s}(\underline{\underline{\sigma}}^{s}) \leq 0$

Mohr-Coulomb

Interaction criterion

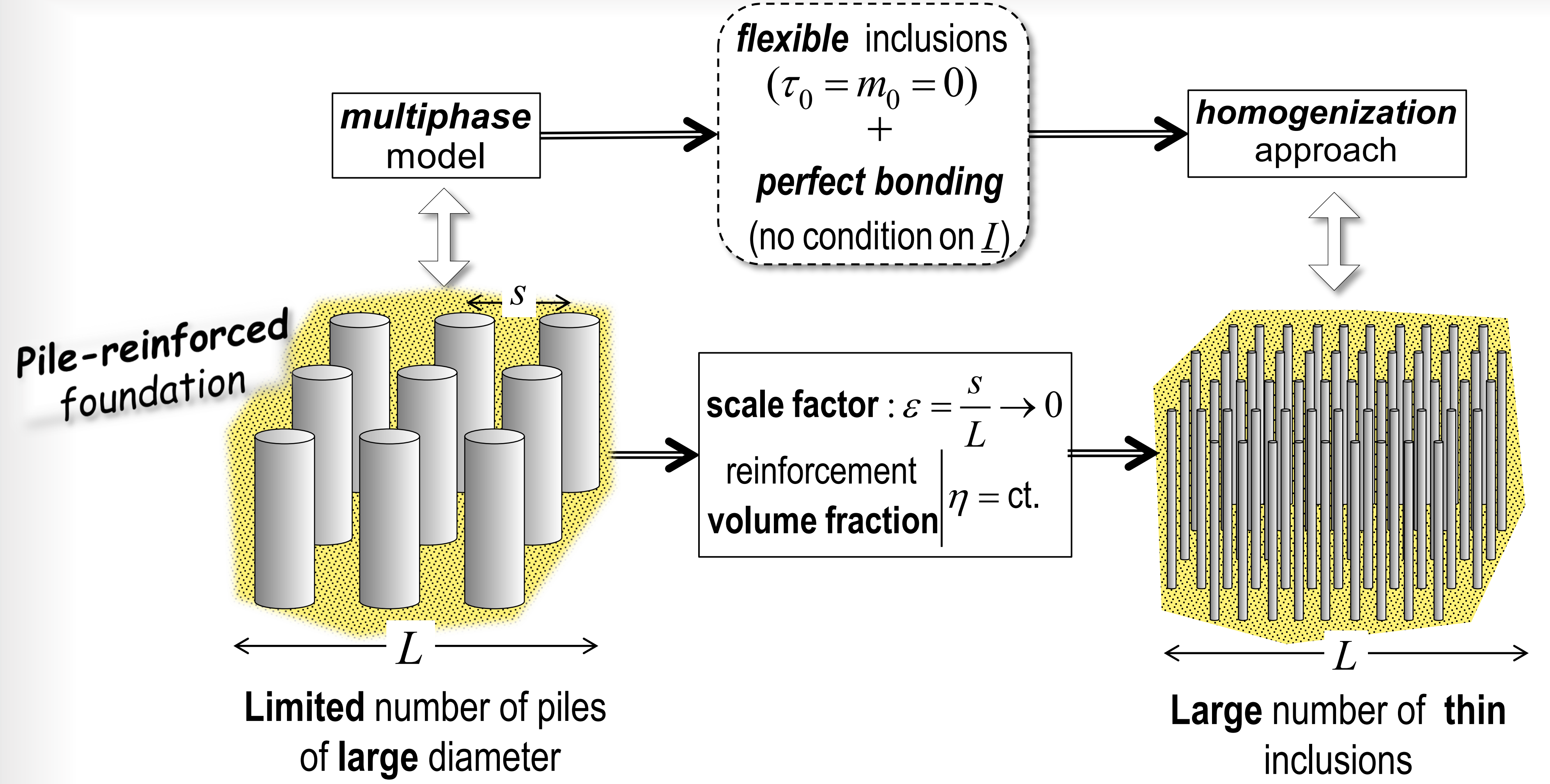
$$f^{I}(\underline{I}) \leq 0$$

reinforcement phase:

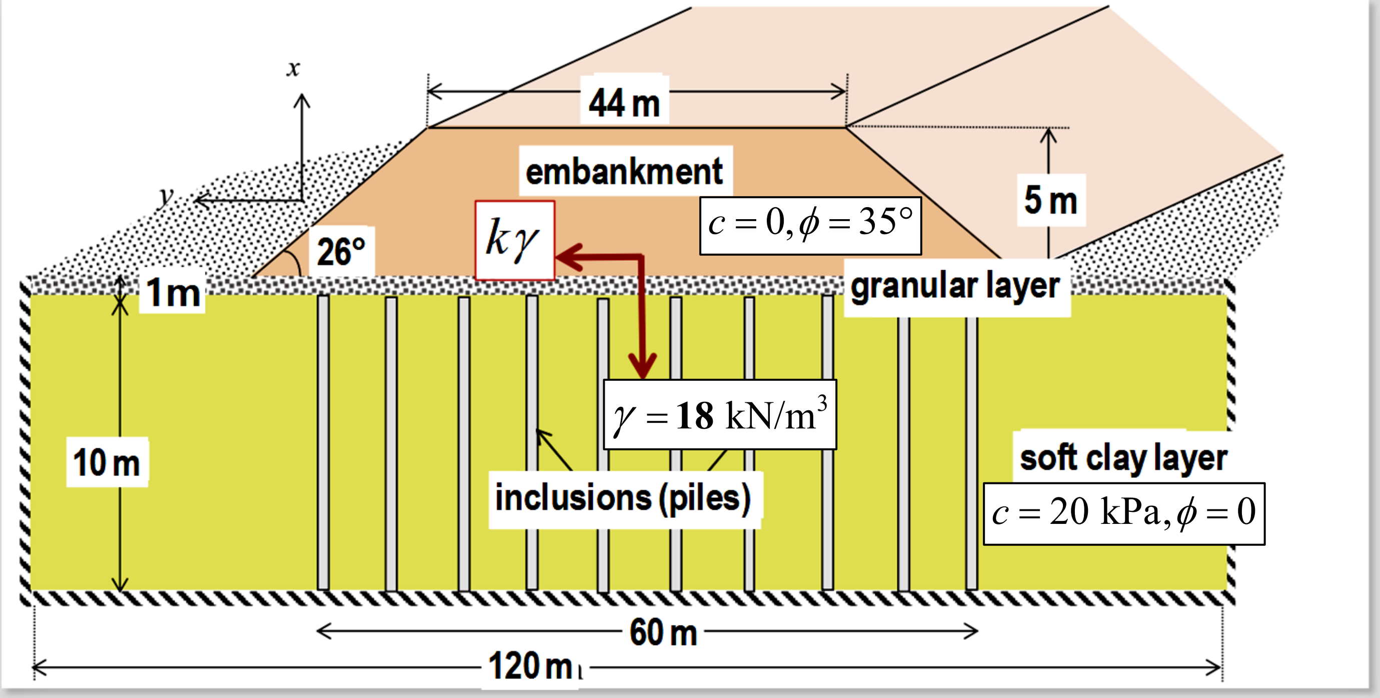
$$\left(\frac{\sigma^{r}}{\sigma_0}\right)^2 + \left(\frac{\tau^{r}}{\tau_0}\right)^2 + \left|\frac{m^{r}}{m_0}\right| - 1 \leq 0$$

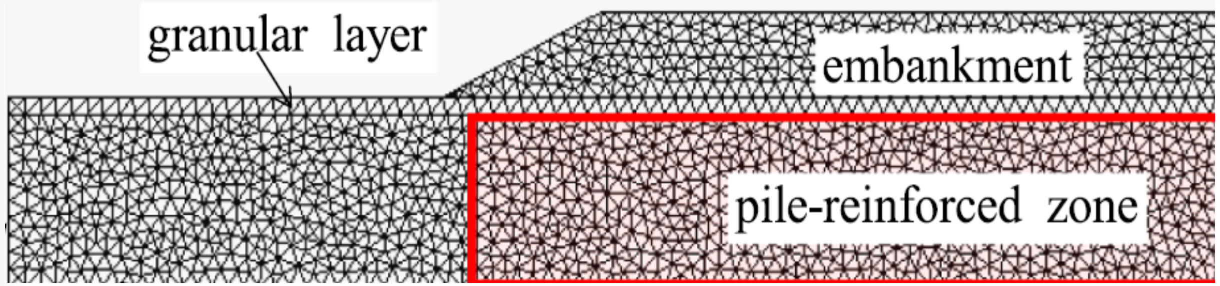
(σ_0, τ_0, m_0) : *normal*, *shear* and *bending* **strengths** per **unit transverse area**

The ***homogenization method*** as a particular case of the ***multiphase model***



Application to the **stability analysis** of a **piled embankment** under **earthquake loading**





Seismic loading characterized by the ***non-dimensional coefficient k***

Stability

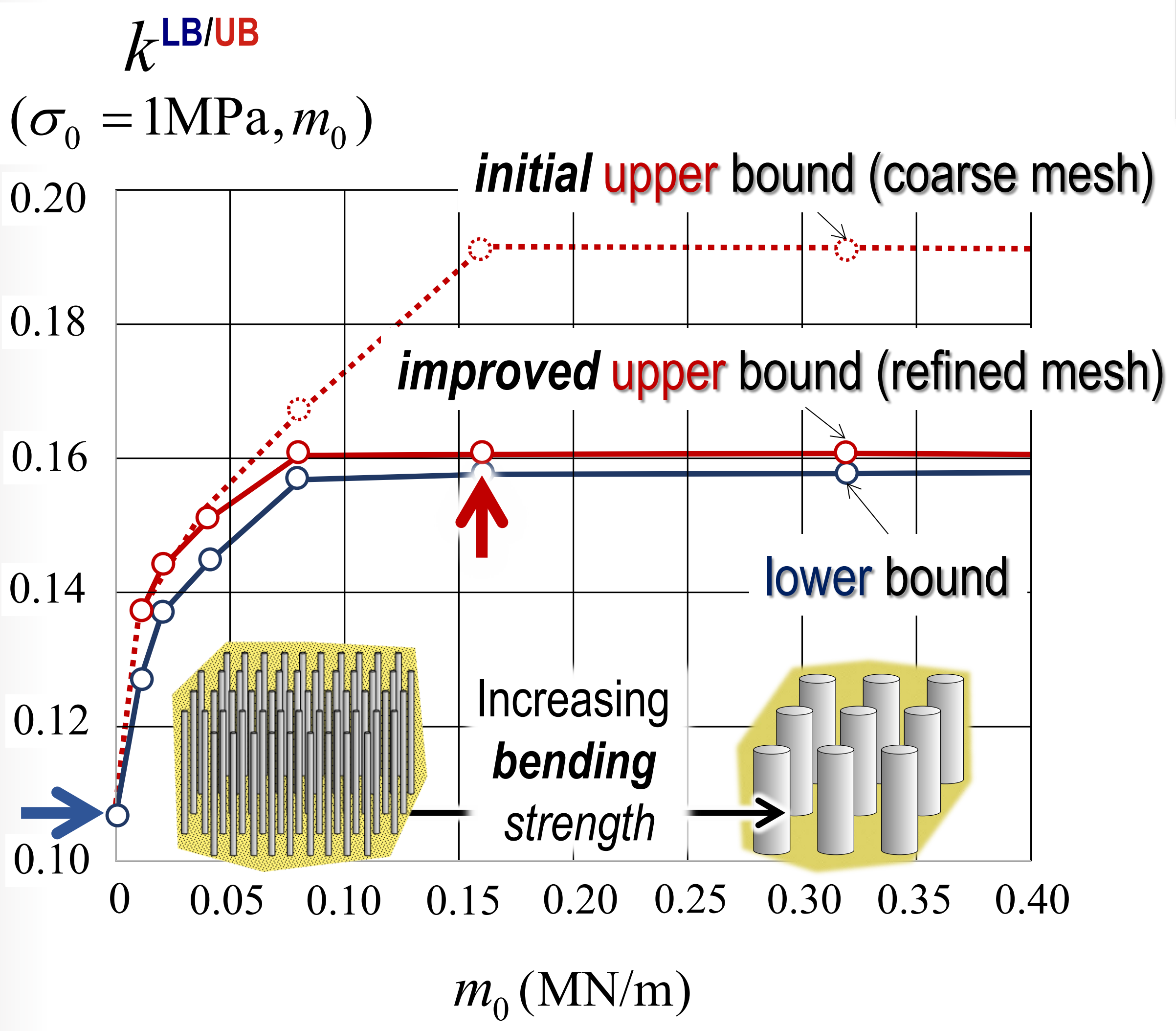
$\Updownarrow$

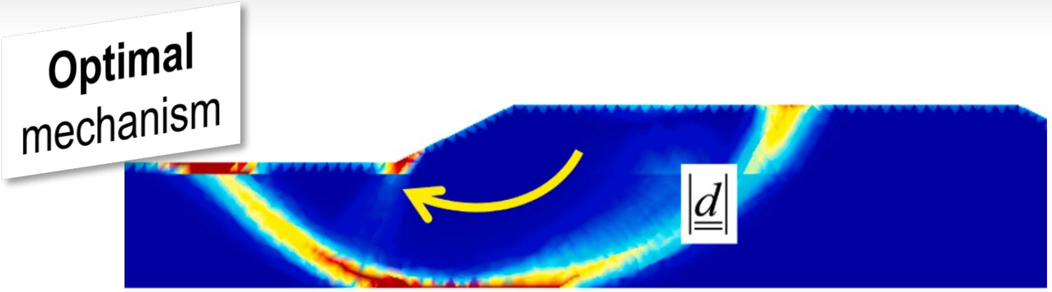
$k \leq k^+$

Derivation of ***upper*** and ***lower bound*** estimates for ***k*⁺,** using:

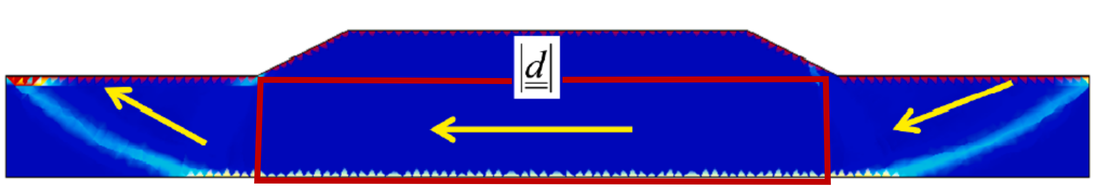
- a ***finite element*** formulation,
- **convex optimization** procedures.

Results for *perfectly bonded* inclusions (Hassen et al., 2021)

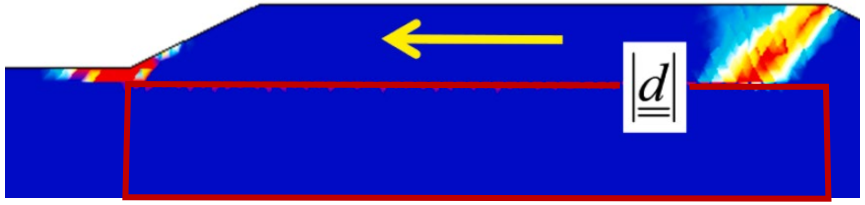




Unreinforced $k^{LB/UB} \cong$ 0.04



Flexible inclusions $k^{LB/UB} \cong$ 0.107



Bending resistant inclusions

$k^{LB/UB} \cong$ 0.16

Concluding remarks

The implementation of the ***Yield Design*** approach on *reinforced soil structures* reveals two situations:

➢ Only a *very small number* of inclusions are placed in the soil following *no regular arrangement:*

- The **mixed modelling approach** appears to be the most suitable way of dealing with the stability analysis of such structures, predominantly in the context of the *upper bound kinematic* approach.
- Mechanisms with ***failure surfaces*** can still be used taking the *resisting contribution of the reinforcements* into account in an appropriate manner.

➢ The most frequently encountered situation when a *large number* of *regularly distributed* inclusions are involved**:**

- The implementation of a **multiphase model**, perceived as an **improved homogenization method**, allows to obtain ***accurate bounds*** on the ***ultimate bearing capacity*** of this type of reinforced soil structures.
- The model may easily incorporate the ***shear/bending strength*** characteristics of the reinforcement as well as a specific ***failure condition*** at the ***soil-inclusion*** *interface*.
- It is also fully applicable to other kinds of ***constitutive behavior*** of the reinforced soil components (elastoplasticity, elastodynamics: example of *settlement/dynamic stiffness* of a piled foundation)