



COMITÉ FRANÇAIS DE MÉCANIQUE
DES SOLS ET DE GÉOTECHNIQUE



ACADEMIE
DES SCIENCES
INSTITUT DE FRANCE

ACADEMIE
DES TECHNOLOGIES
POUR UN PROGRÈS RAISONNÉ, CHOISI ET PARTAGÉ



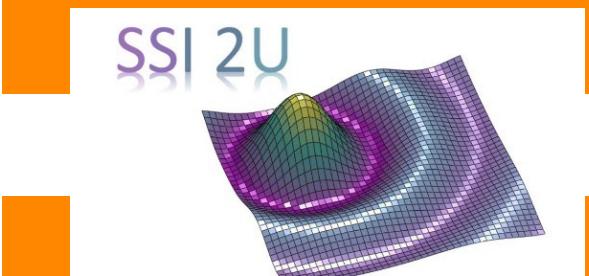
Charles-Augustin COULOMB - A geotechnical tribute

Paris, september 25 & 26, 2023



From "De la pression des terres et des revêtements" to the seismic analysis of retaining structures

L. Callisto
University of Rome La Sapienza



Shaping a World of Trust

E S S A I

principles included in the *Essai*

relevance in the seismic design of earth retaining structures

Mémoires de Mathématique et de Physique,
présentés à l'Académie Royale des Sciences, et lus
dans ses Assemblées le 10 mars et le 2 avril, 1773

*Sur une application des règles de Maximis & Minimis
à quelques Problèmes de Statique, relatifs à
l'Architecture.*

Par M. COULOMB, Ingénieur du Roi.

INTRODUCTION.

Ce Mémoire est destiné à déterminer, autant que le mélange du Calcul & de la Physique peuvent le permettre, l'influence du frottement & de la cohésion, dans quelques problèmes de Statique. Voici une légère analyse des différens objets qu'il contient.

Après quelques observations préliminaires sur la cohésion, & quelques expériences sur le même objet, l'on détermine la force d'un pilier de maçonnerie ; le poids qu'il peut porter, pressé suivant sa longueur ; l'angle sous lequel il doit se rompre. Comme ce problème n'exige que des considérations assez simples, qui servent à faire entendre toutes les autres parties de cet Essai, tâchons de développer les principes de sa solution.

Si l'on suppose un pilier de maçonnerie coupé par un plan incliné à l'horizon, en sorte que les deux parties de ce pilier soient unies dans cette section, par une cohésion donnée, tandis que tout le reste de la masse est parfaitement solide, ou lié par une adhérence infinie ; qu'ensuite on charge ce pilier d'un poids : ce poids tendra à faire couler la partie supérieure du pilier sur le plan incliné, par lequel il touche la partie inférieure. Ainsi, dans le cas d'équilibre, la portion de la pesanteur, qui agit parallèlement à la section, sera exactement égale à la cohérence. Si l'on remarque actuellement, dans le cas de l'homogénéité, que l'adhérence du pilier est réellement égale

stability of a masonry column

- equilibrium

dans le cas d'équilibre...

- strength compatibility

les deux parties de ce pilier soient unies
dans cette section, par une cohésion donnée

- optimisation

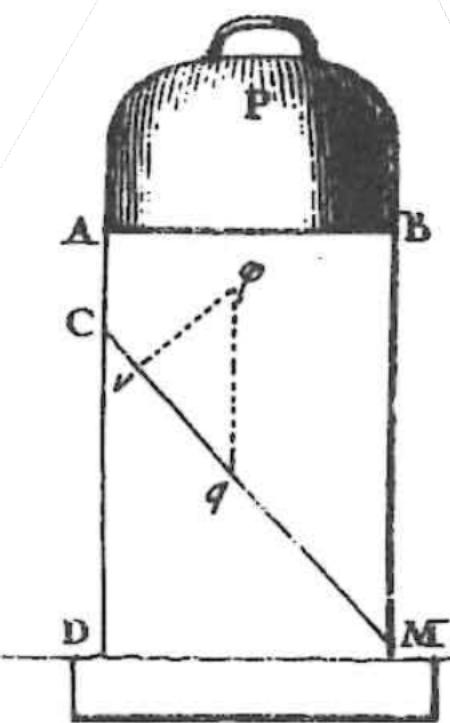
... ce qui aura lieu lorsque on déterminera P , tel
qu'il soit un *minimum*...

Résistance des Piliers de Maçonnerie.

Soit un pilier homogène de maçonnerie, que je suppose Fig. 5. d'abord quarré, chargé d'un poids P ; l'on demande la direction de la ligne CM , suivant laquelle ce pilier se rompra, & la pesanteur du poids nécessaire pour cette rupture.

Je suppose ici que l'adhérence oppose une égale résistance, soit que la force soit dirigée parallèlement ou perpendiculairement au plan de rupture, conformément à la première & deuxième expérience. Je suppose encore le pilier d'une matière

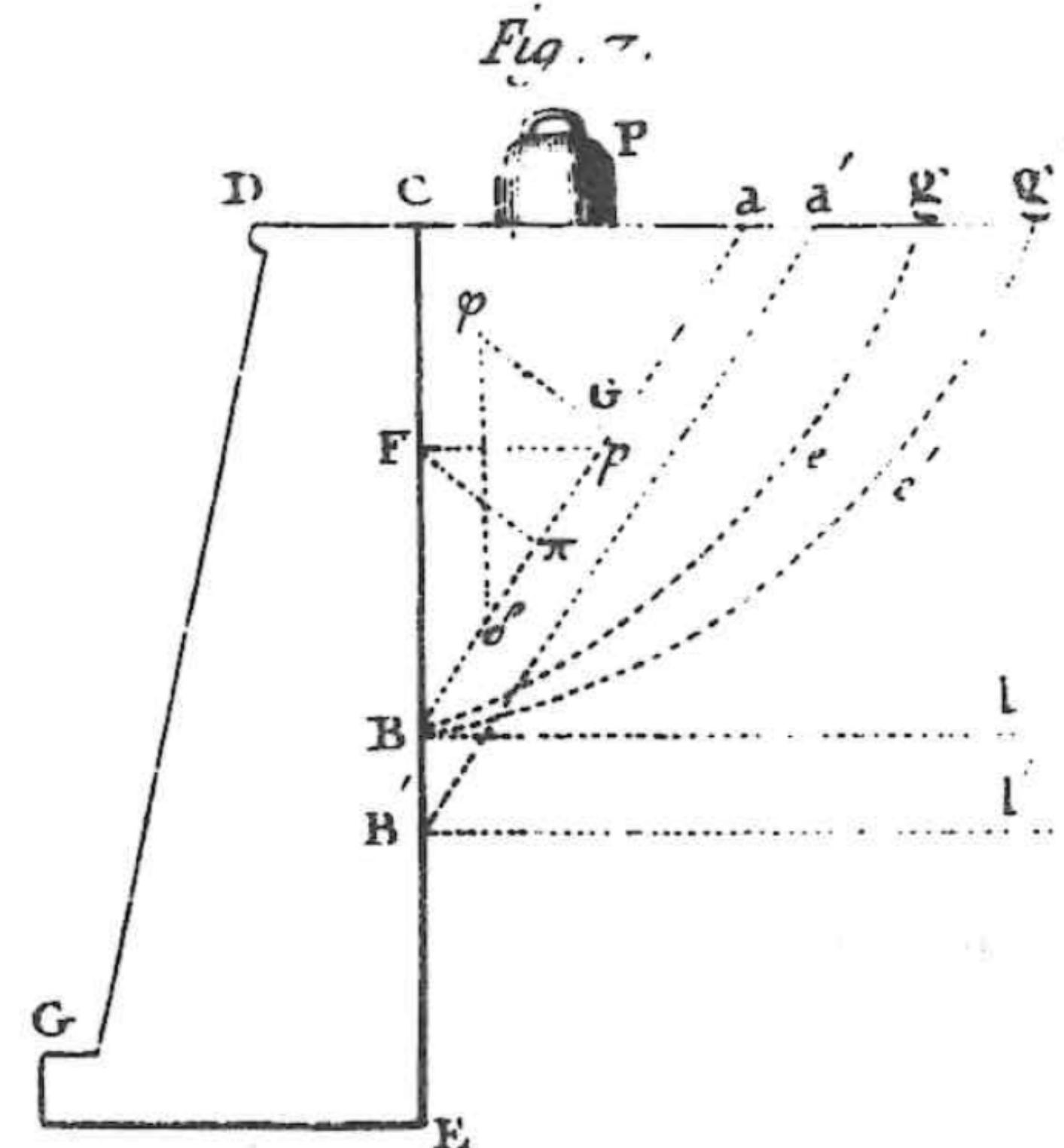
Fig. 5.



analysis of retaining walls

(...) la méthode est absolument la même.

(...) pour avoir la pression d'une surface de terre contre un plan vertical, il faut trouver parmi toutes les surfaces (...) celle qui (...) exigerait, pour son équilibre, d'être soutenue par une force horizontale qui fut un *maximum*

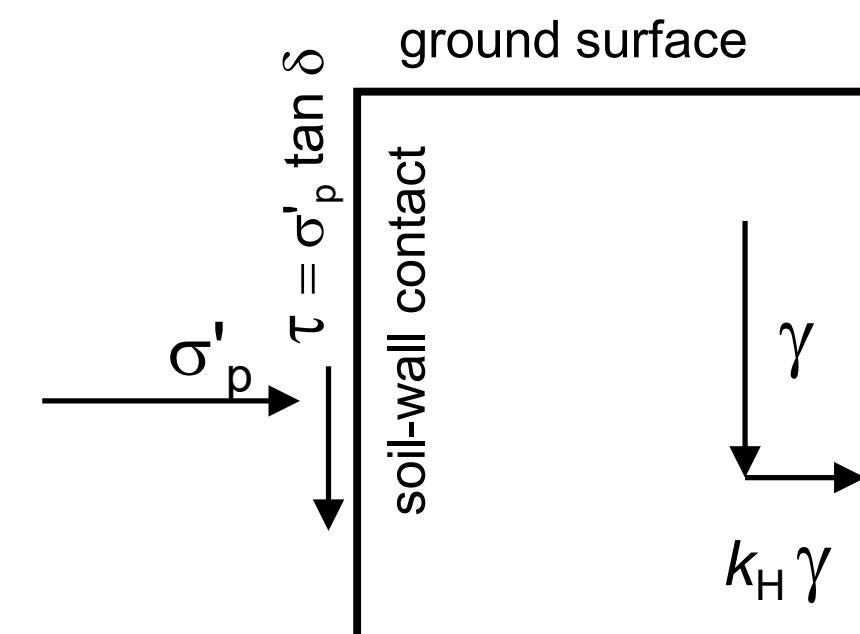
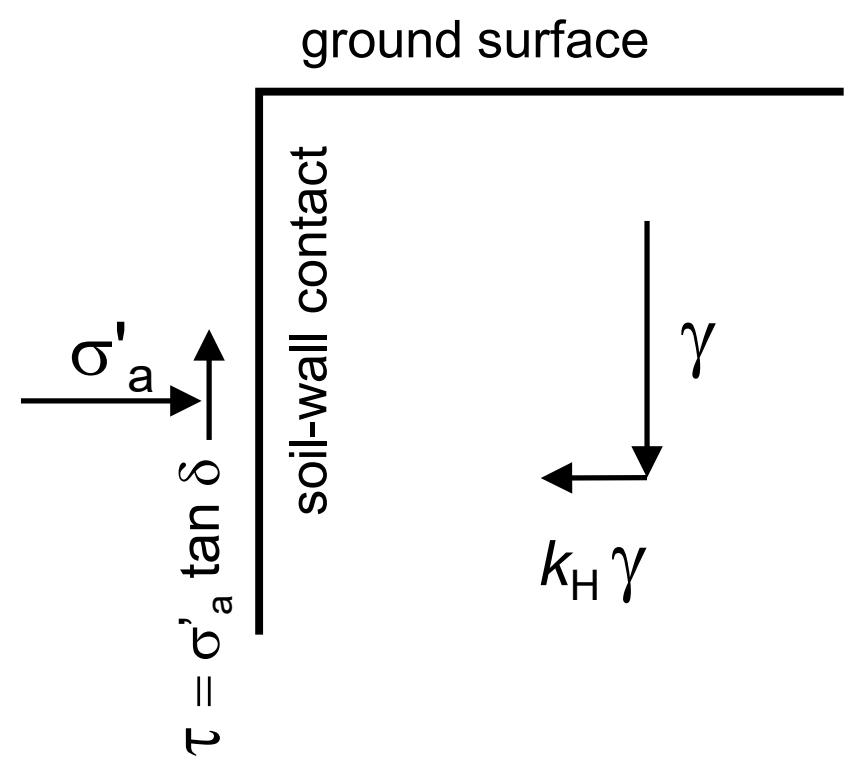


equilibrium, strength, and optimisation

→ derive forces under working load conditions

lower bound solution

- equilibrium of stress field
- strength compatibility
- optimisation



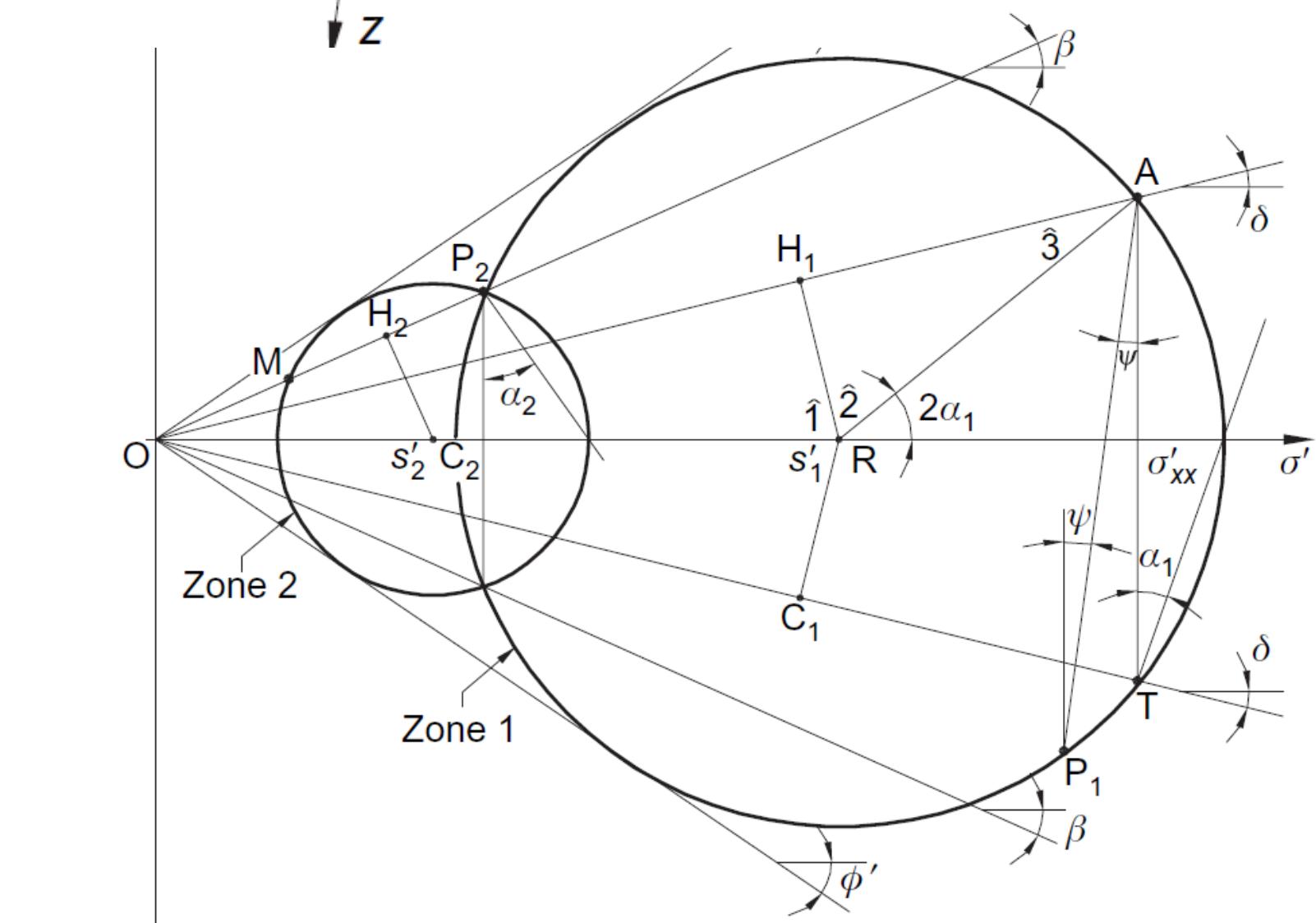
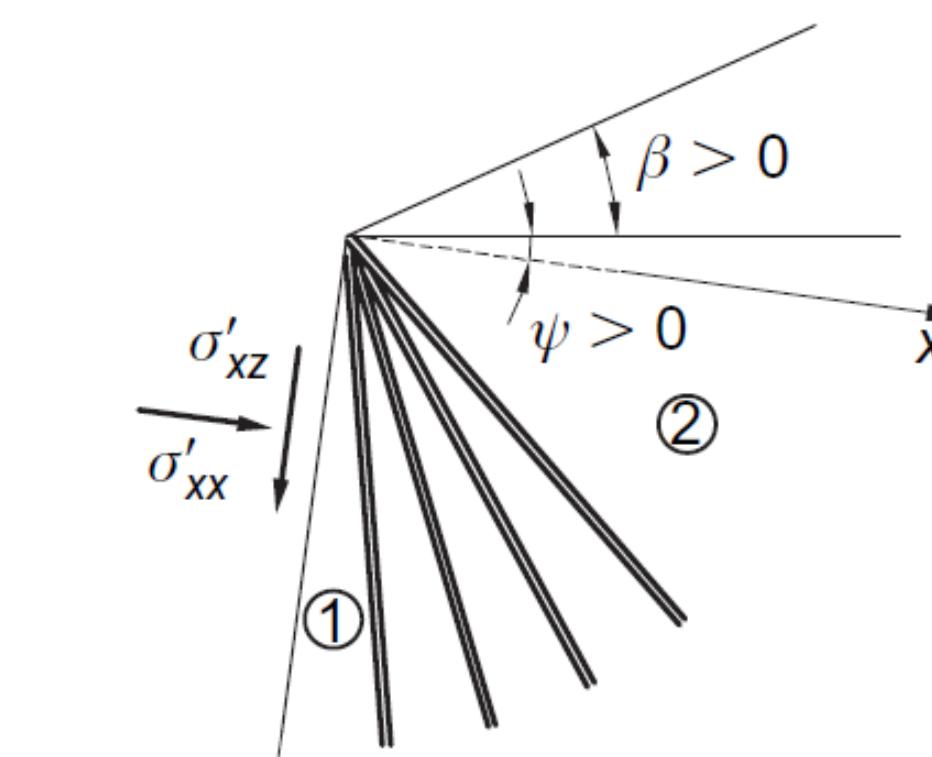
$$K_{ap} = \left[\frac{\cos \delta}{\cos \theta \pm \sqrt{\sin^2 \phi' - \sin^2 \theta}} \cdot \left(\cos \delta \mp \sqrt{\sin^2 \phi' - \sin^2 \delta} \right) \right] \frac{1}{\cos \theta} \cdot e^{\mp 2\psi \tan \phi'}$$

$$2\psi = \arcsin \left(\frac{\sin \delta}{\sin \phi'} \right) \mp \arcsin \left[\frac{\sin \theta}{\sin \phi'} \right] \mp \delta + \theta$$

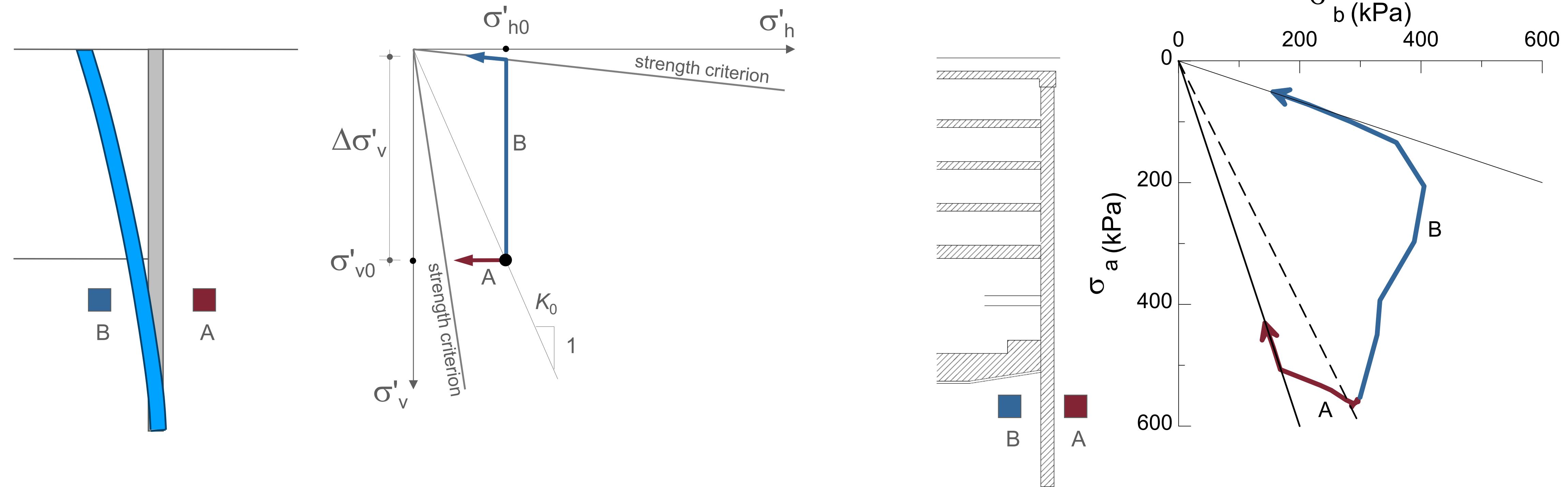
TECHNICAL NOTE

Lower-bound approach for seismic passive earth resistance

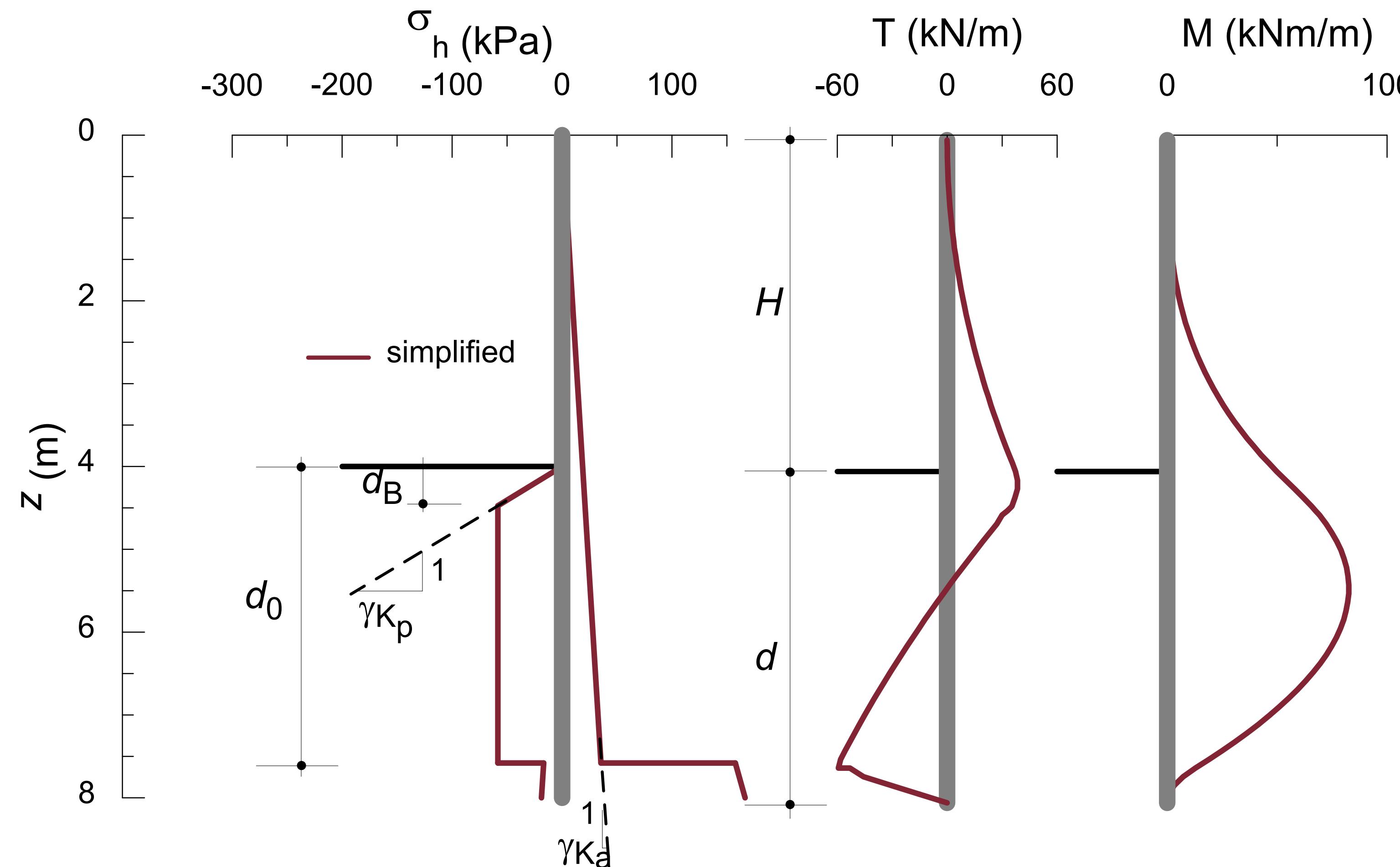
R. LANCELLOTTA*



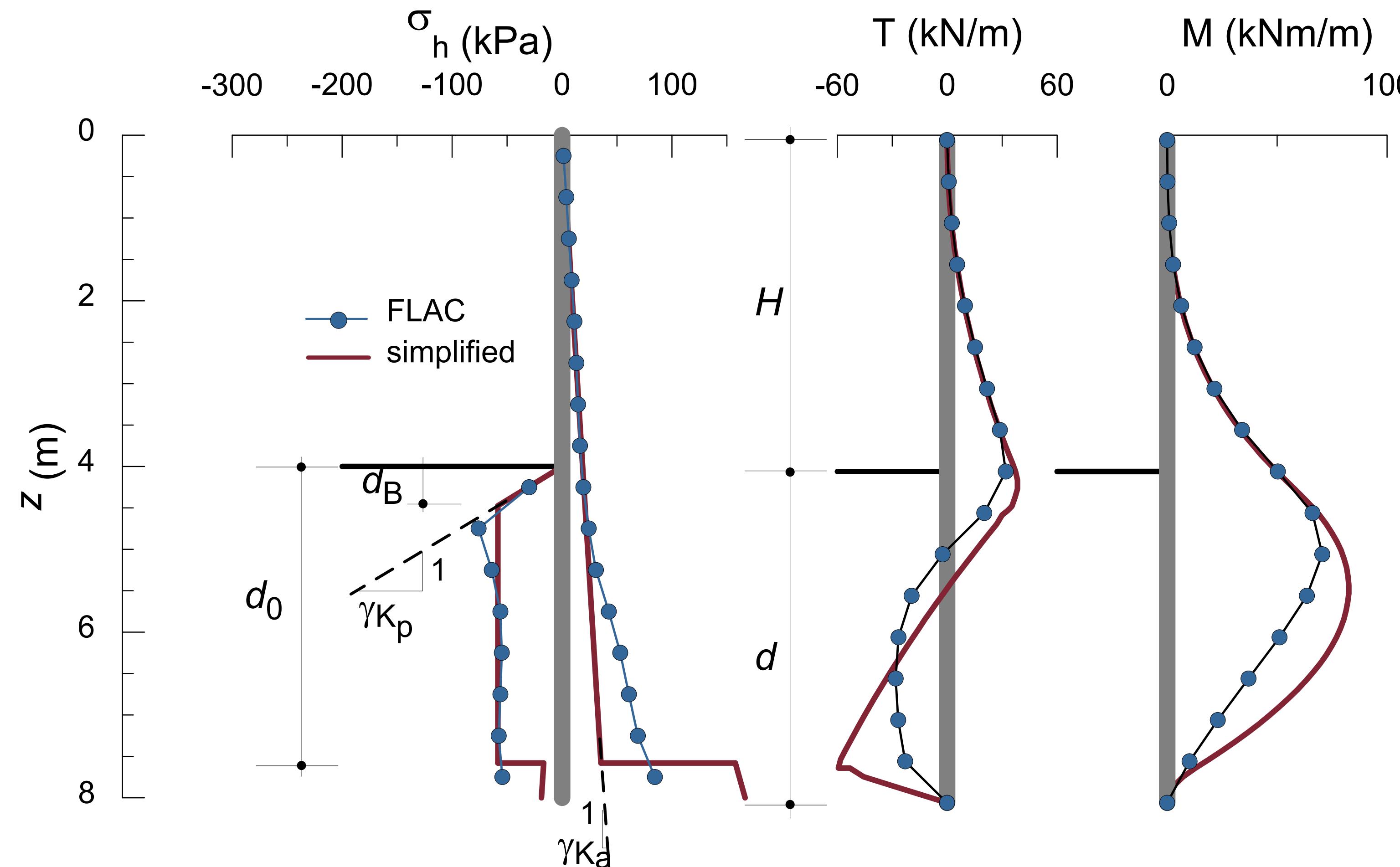
stress paths induced by an excavation



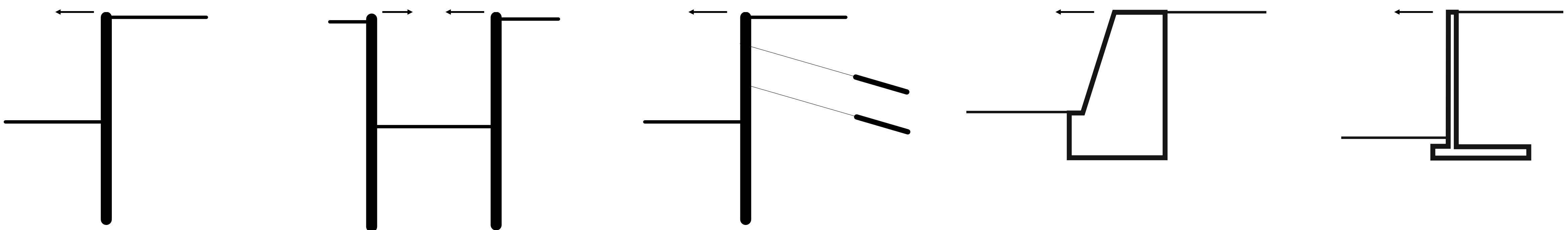
simplified analysis of a cantilevered retaining wall



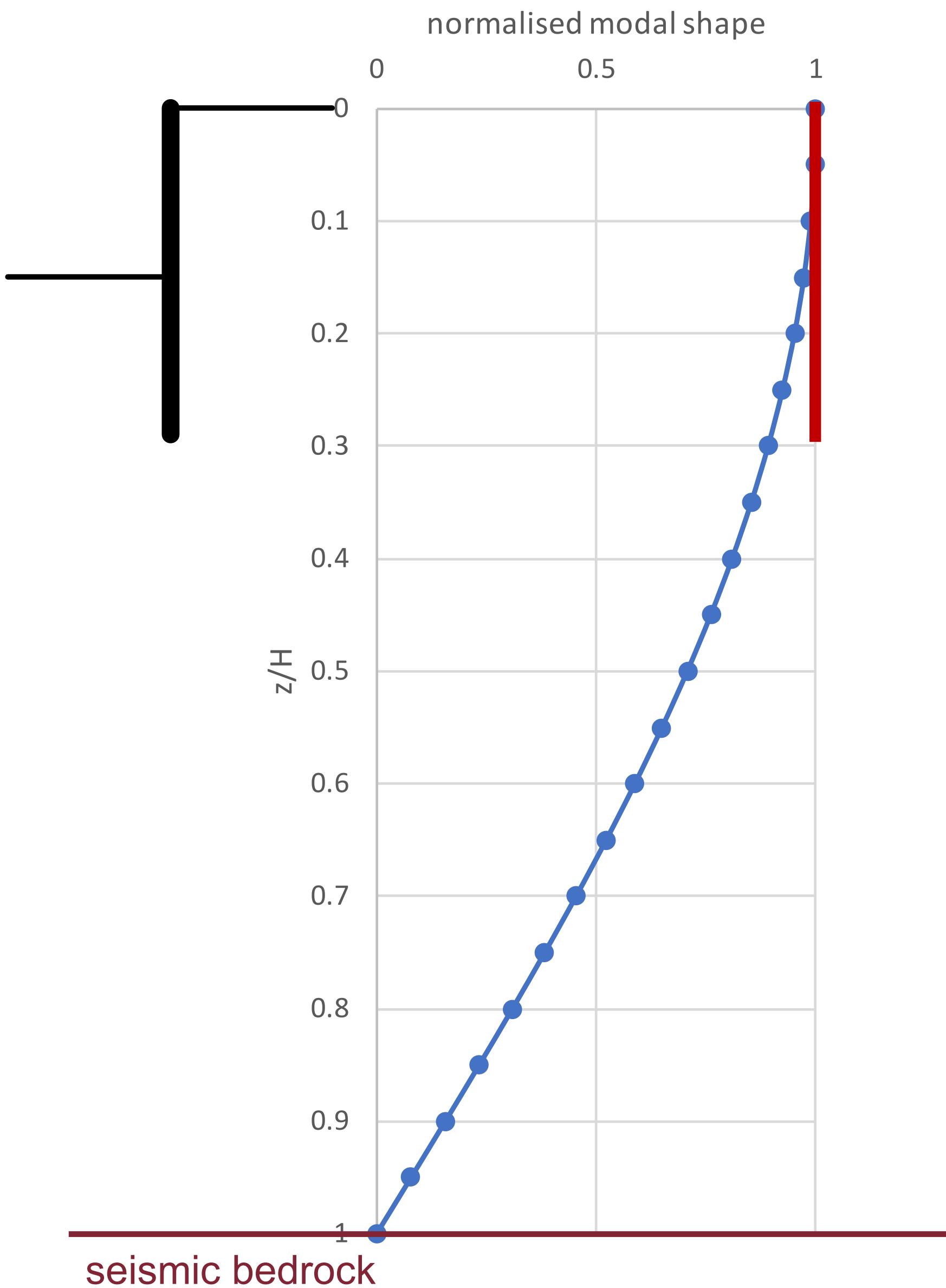
simplified analysis of a cantilevered retaining wall



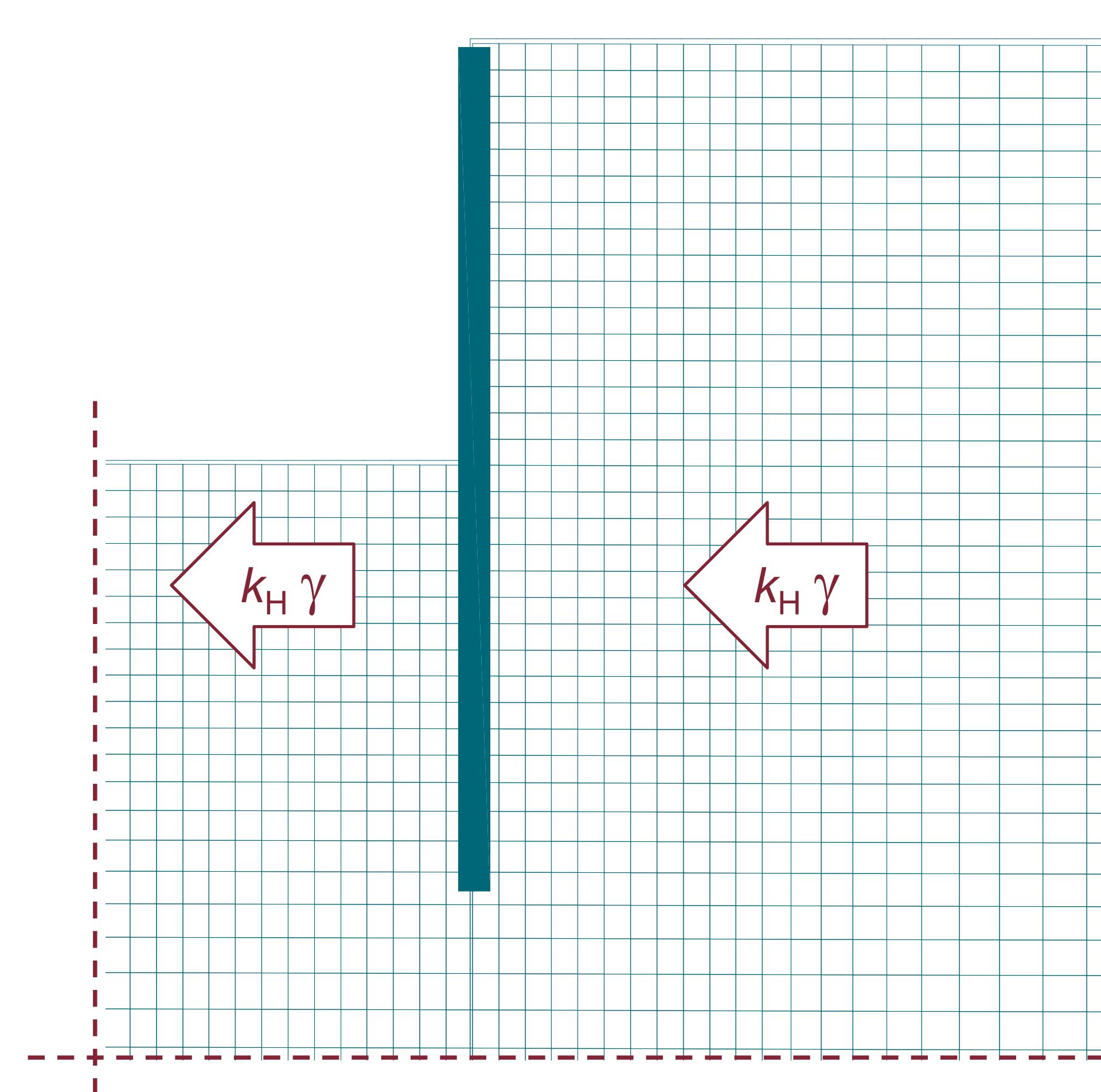
seismic design of earth retaining structures



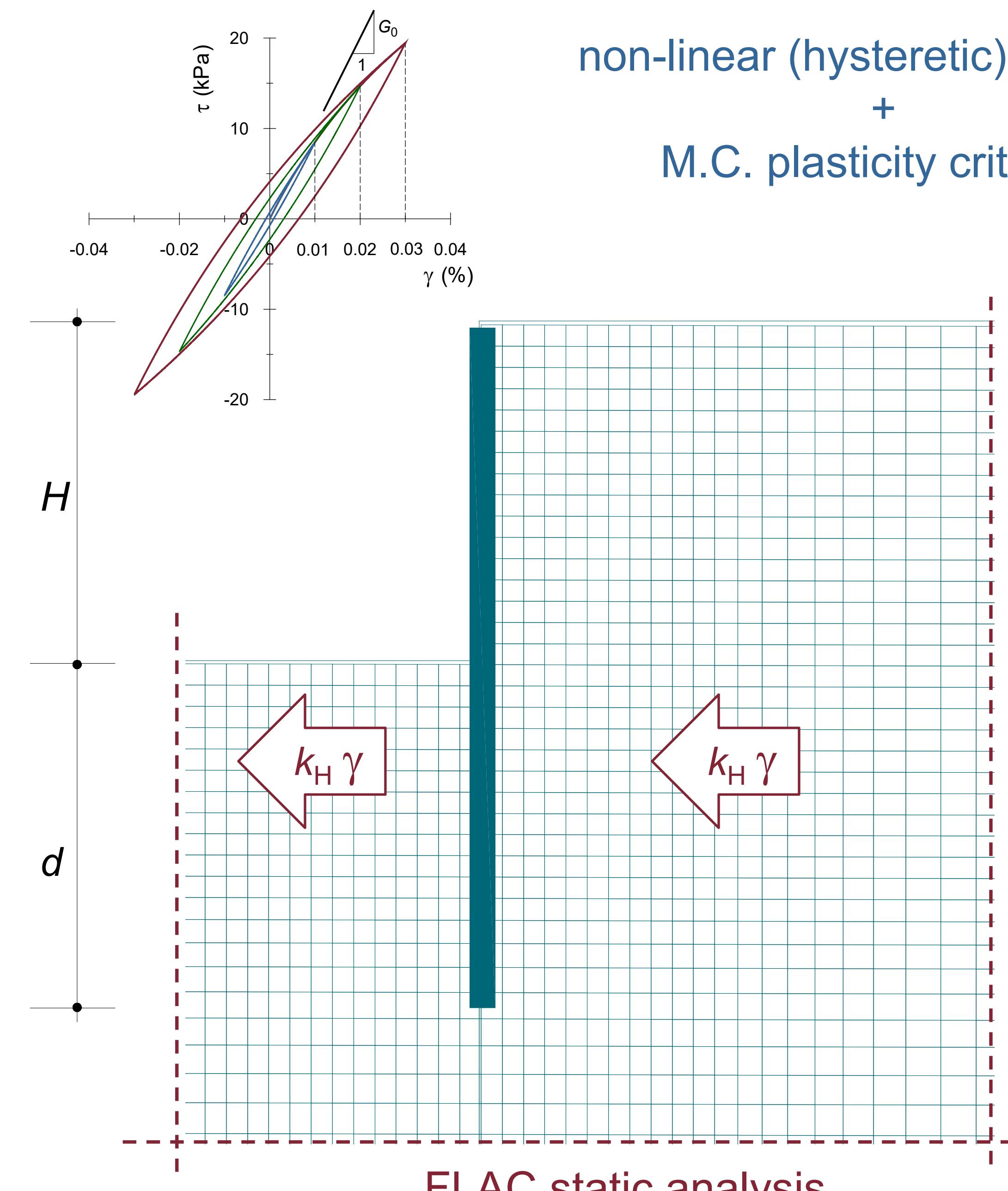
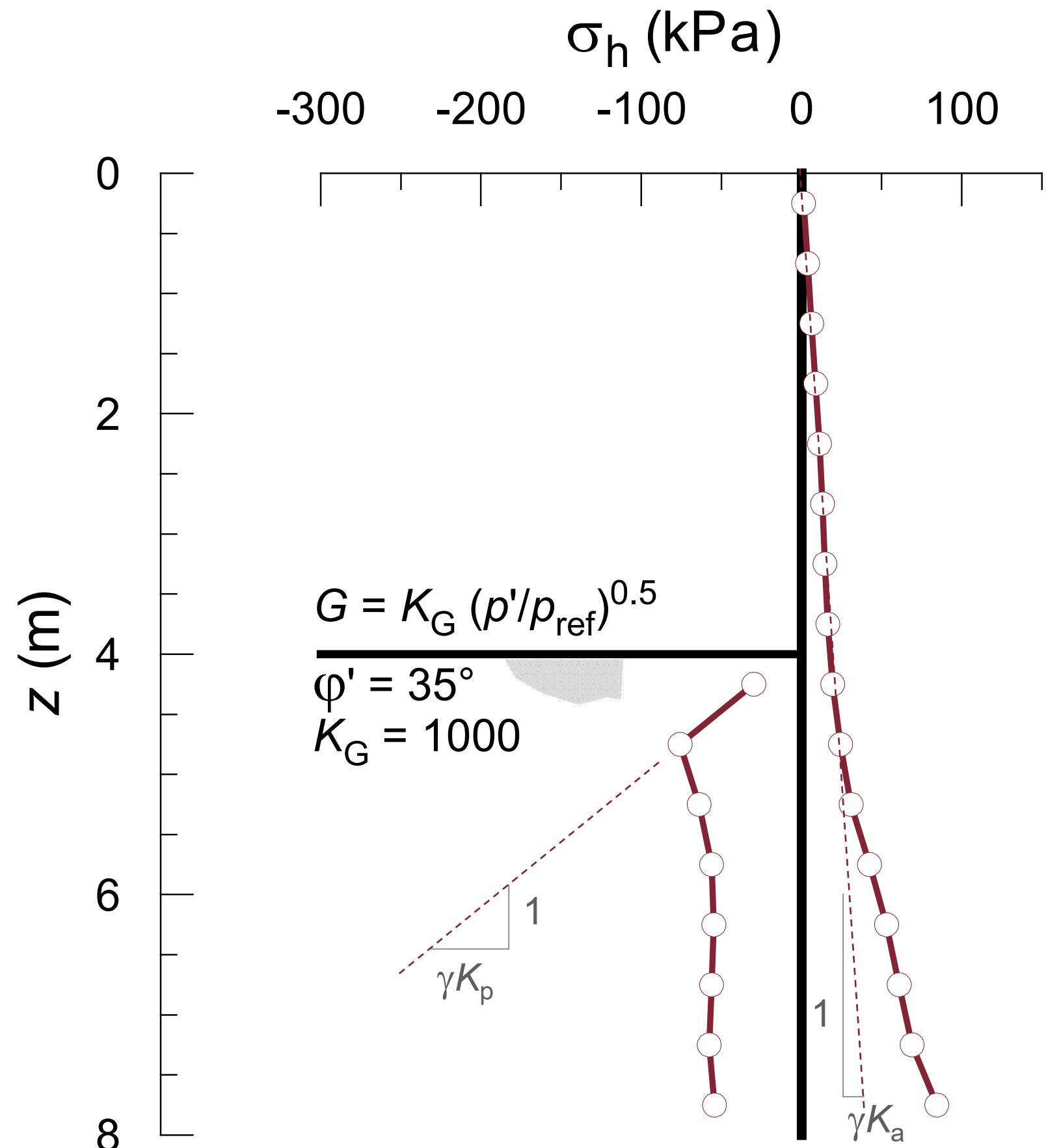
- final (permanent) displacement
- internal forces in structural members



static push-over analysis
→ uniform inertial forces

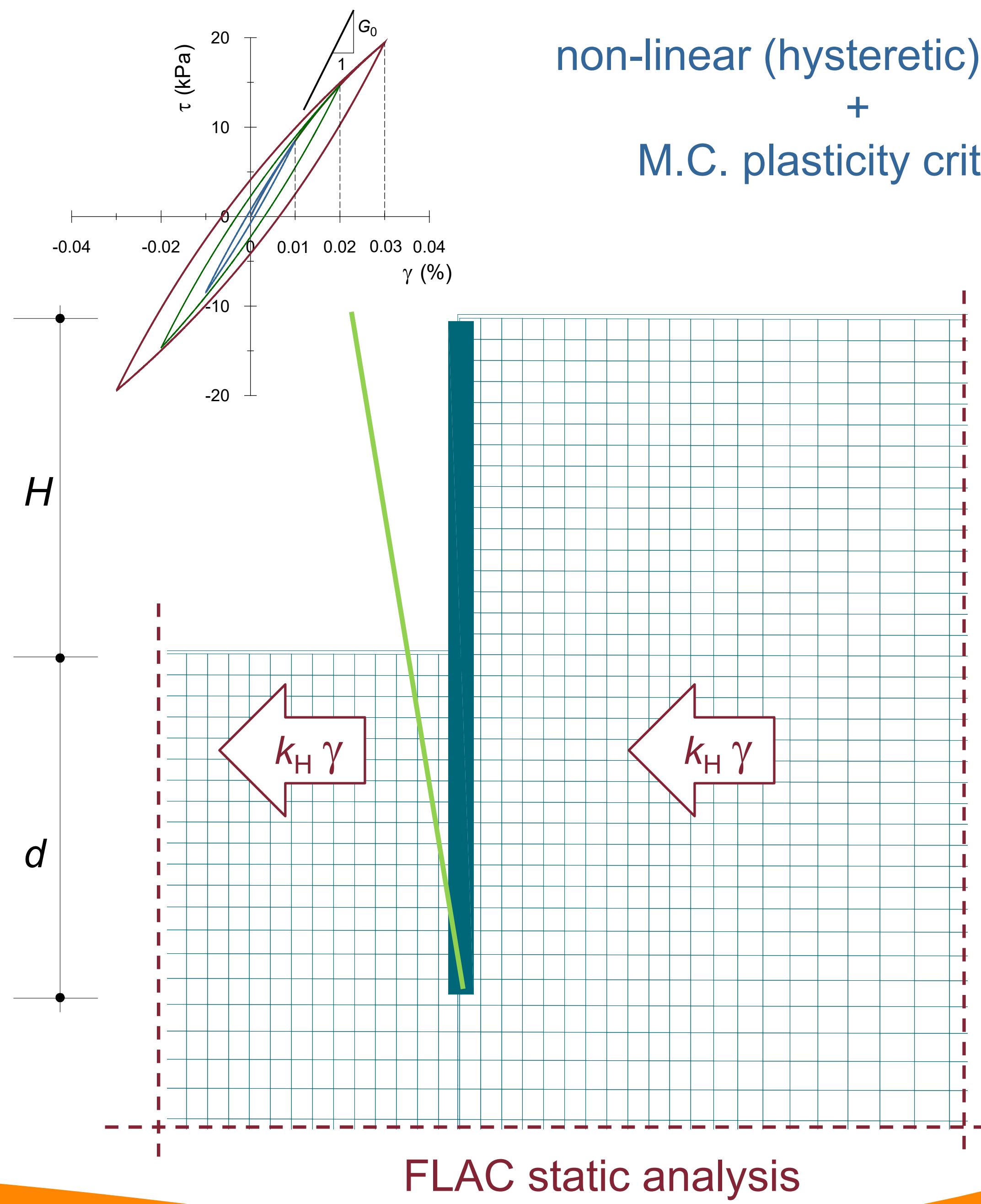
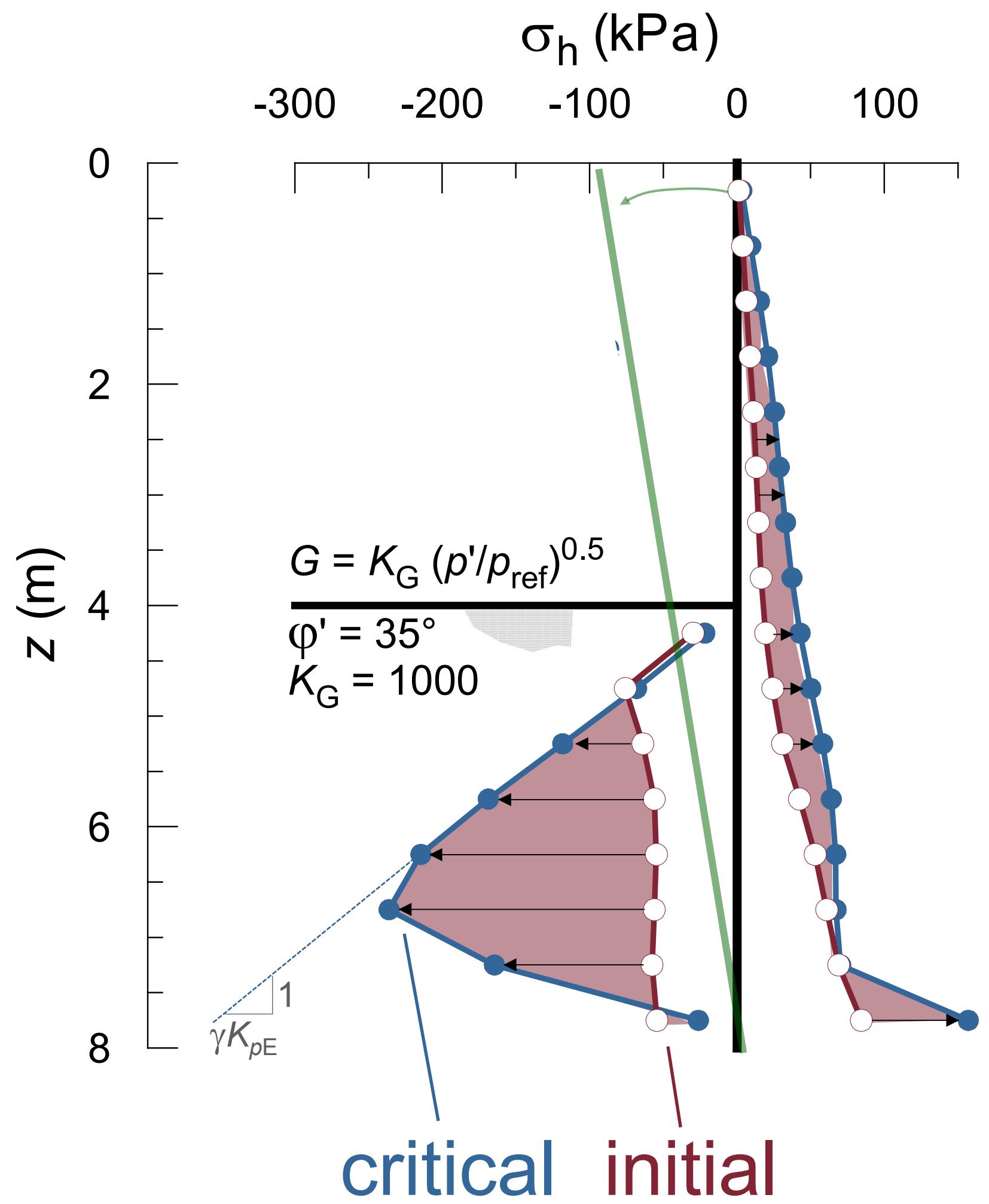


embedded cantilevered wall



non-linear (hysteretic) elasticity
+
M.C. plasticity criterion

transition to a plastic mechanism

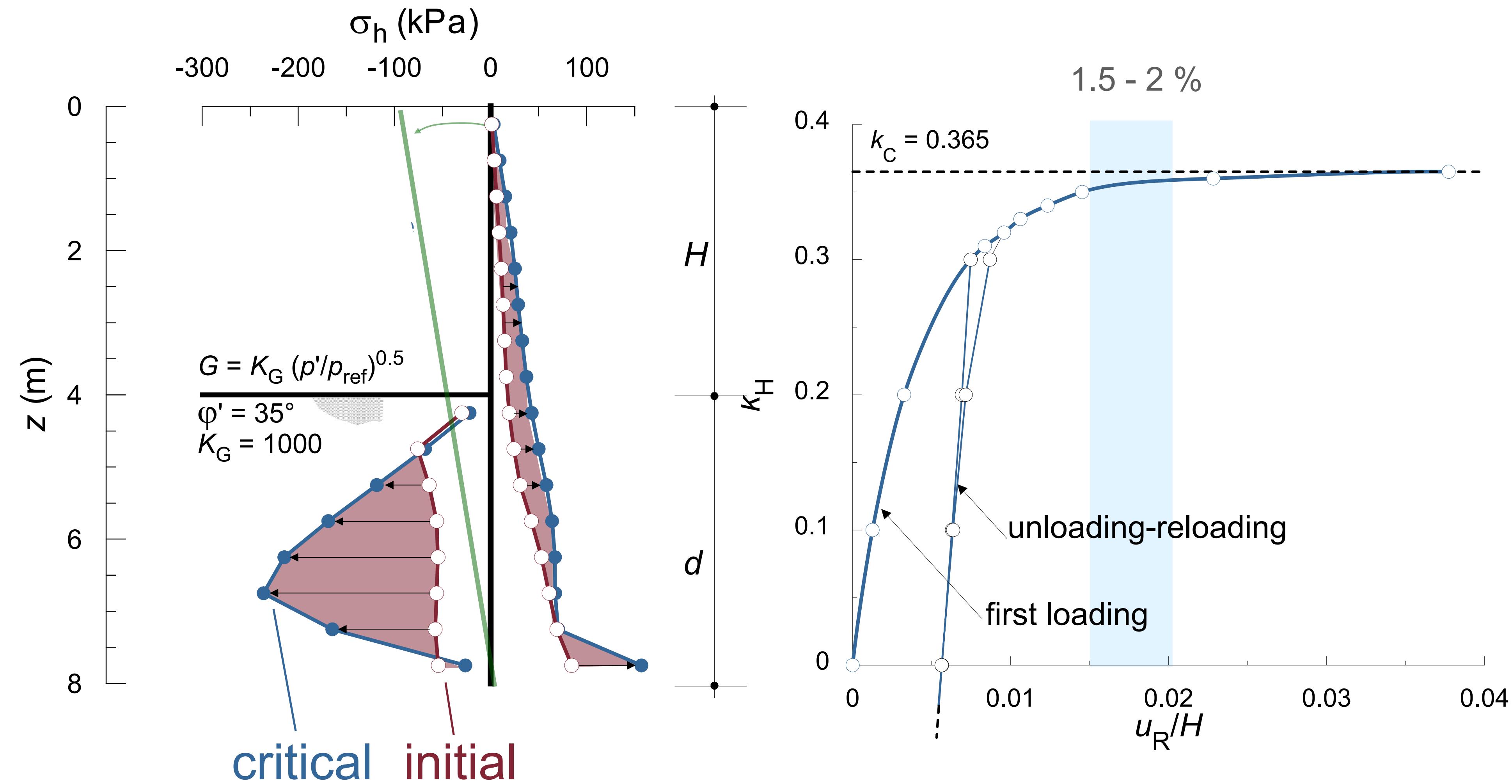


non-linear (hysteretic) elasticity
+
M.C. plasticity criterion

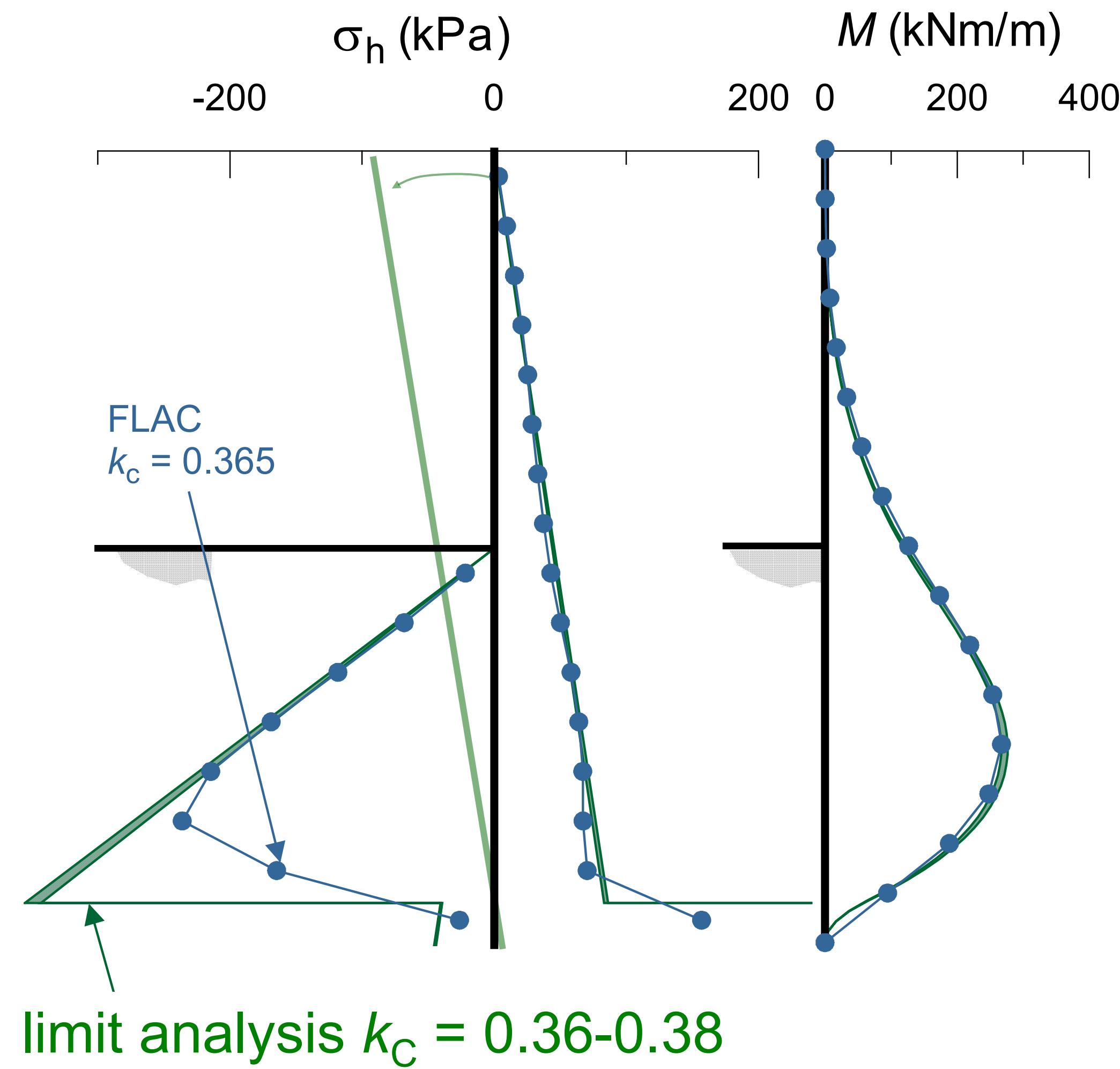
transition to a plastic mechanism



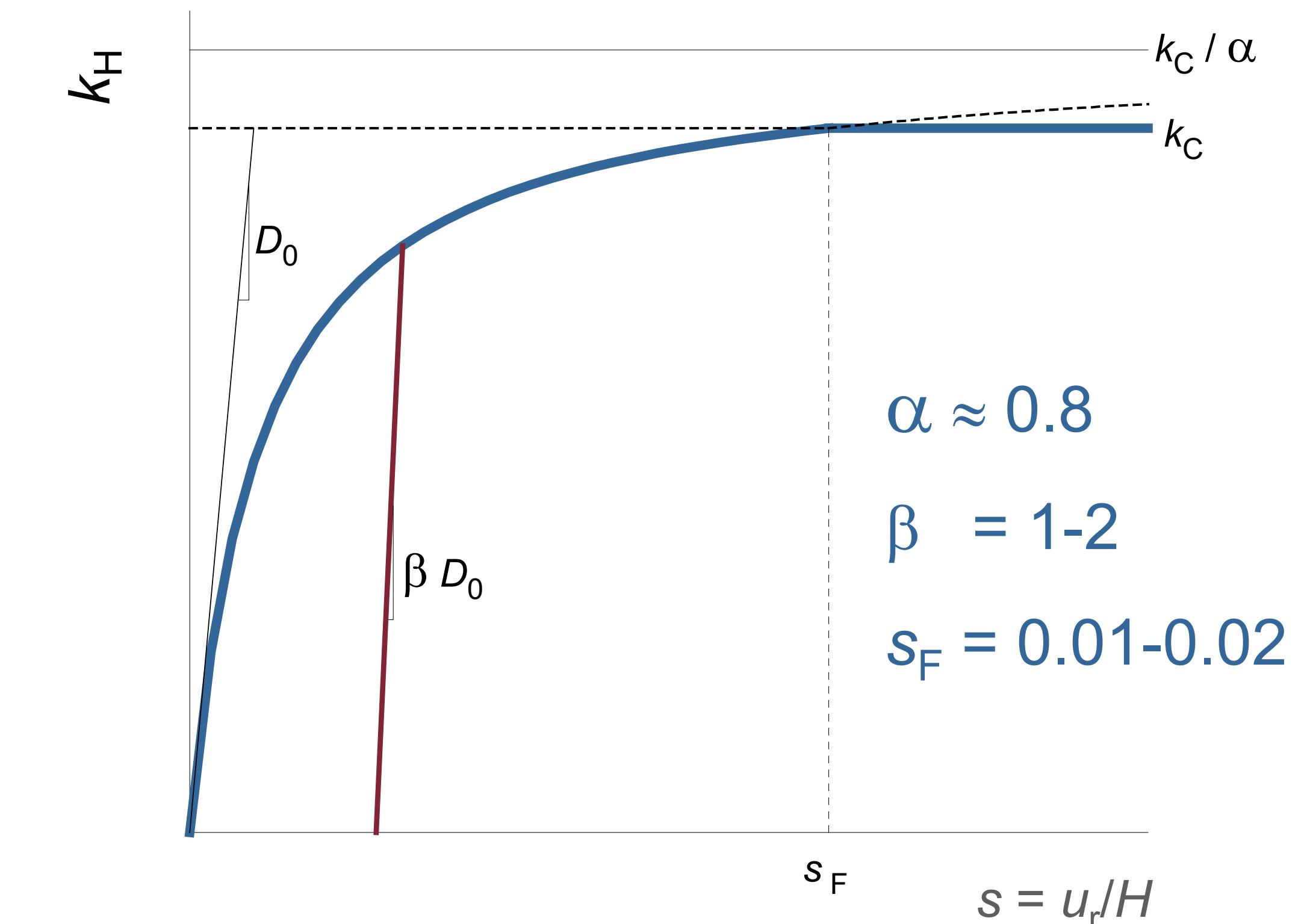
capacity curve



plastic mechanism – limit equilibrium

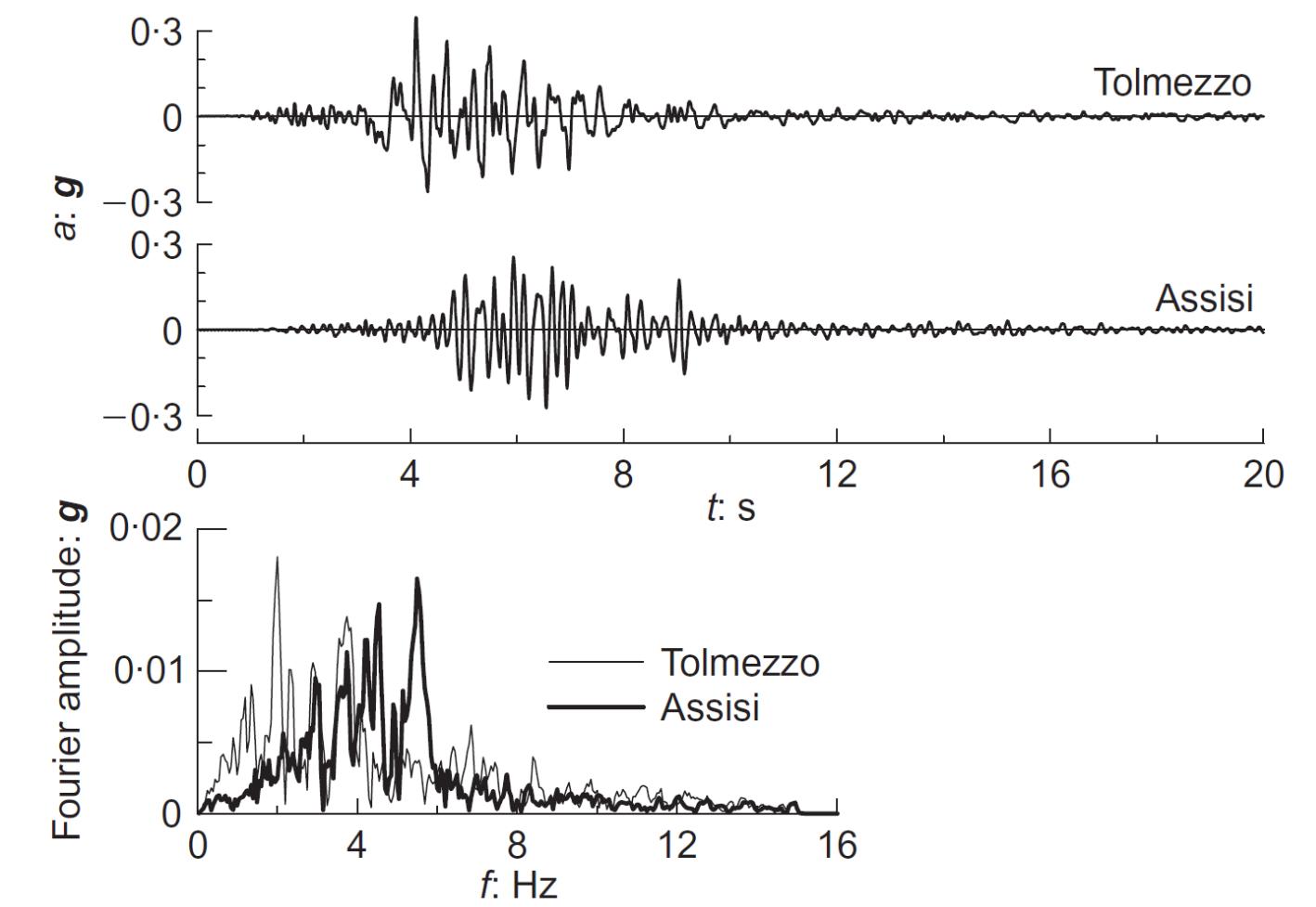
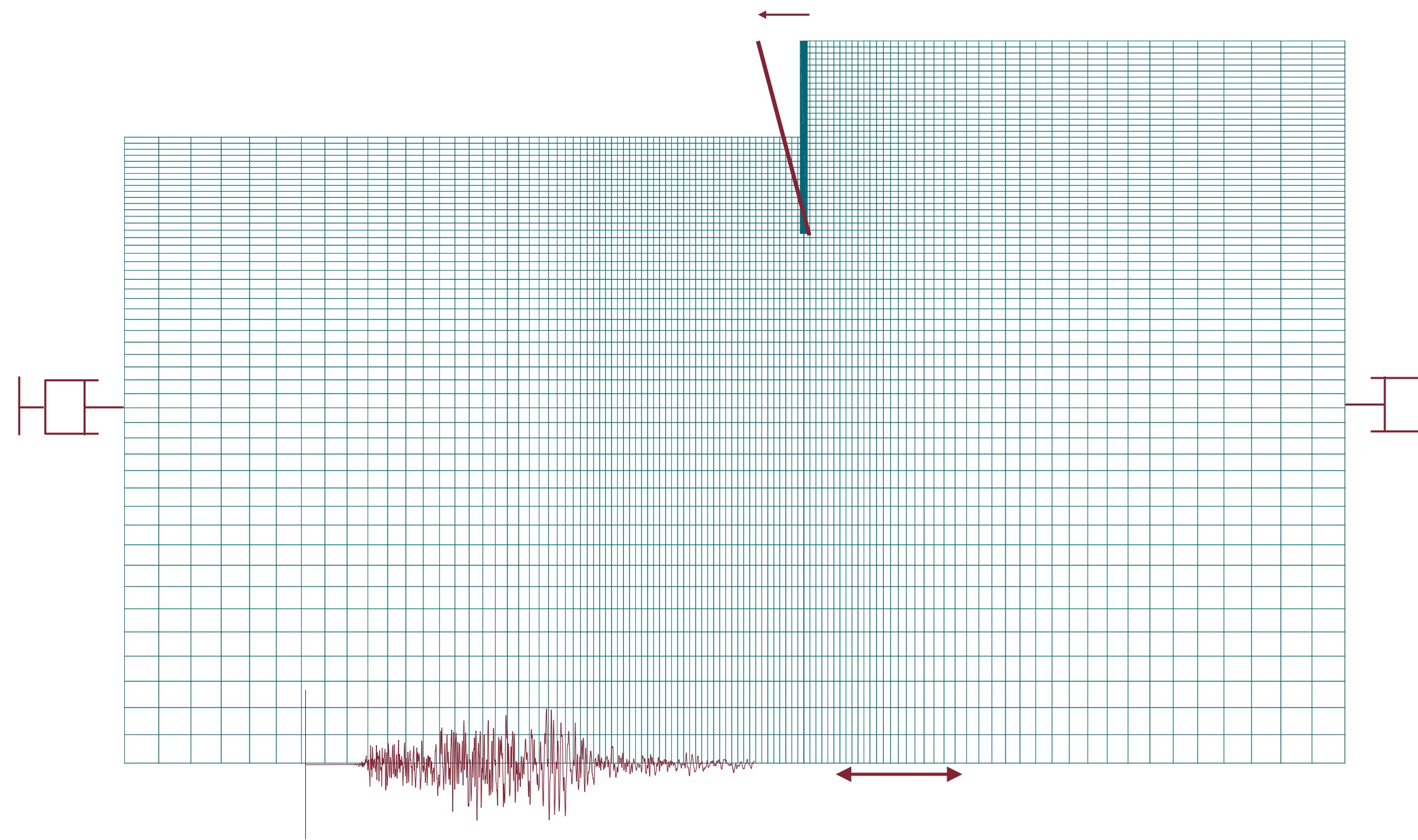


approximate capacity curve

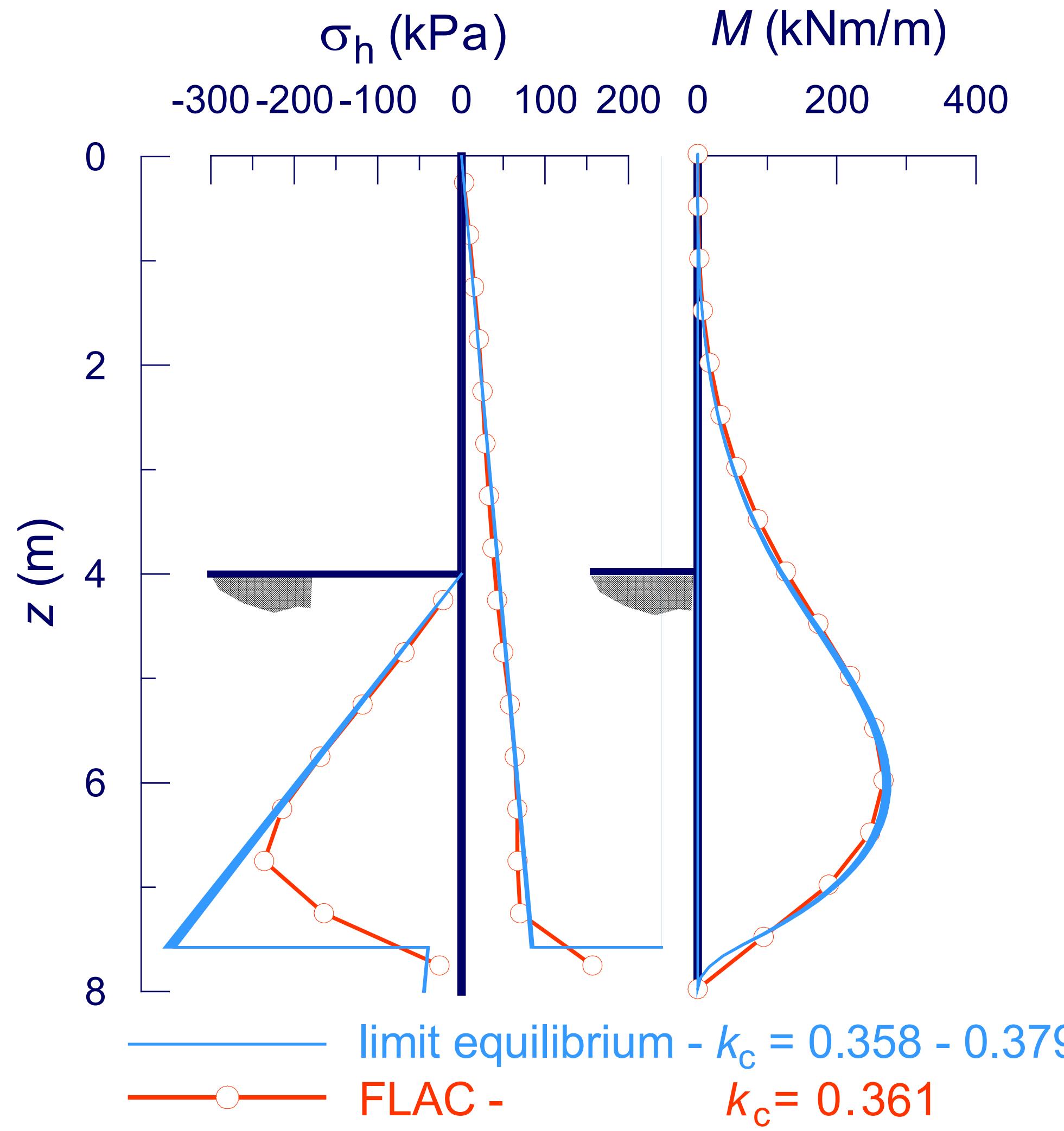


$$k_H = \frac{s k_C}{s \alpha + s_F(1-\alpha)}$$

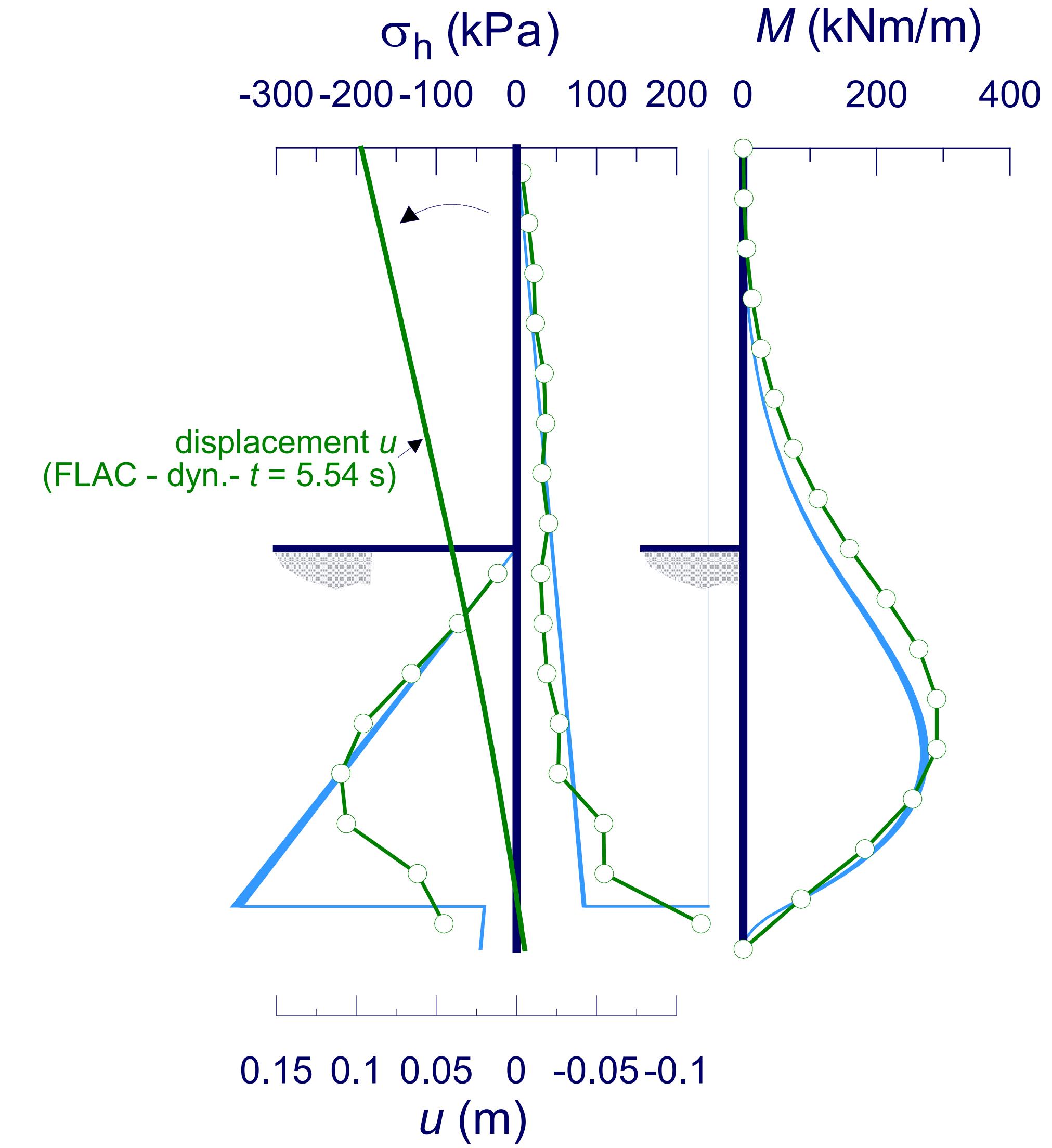
comparison with results of dynamic analyses



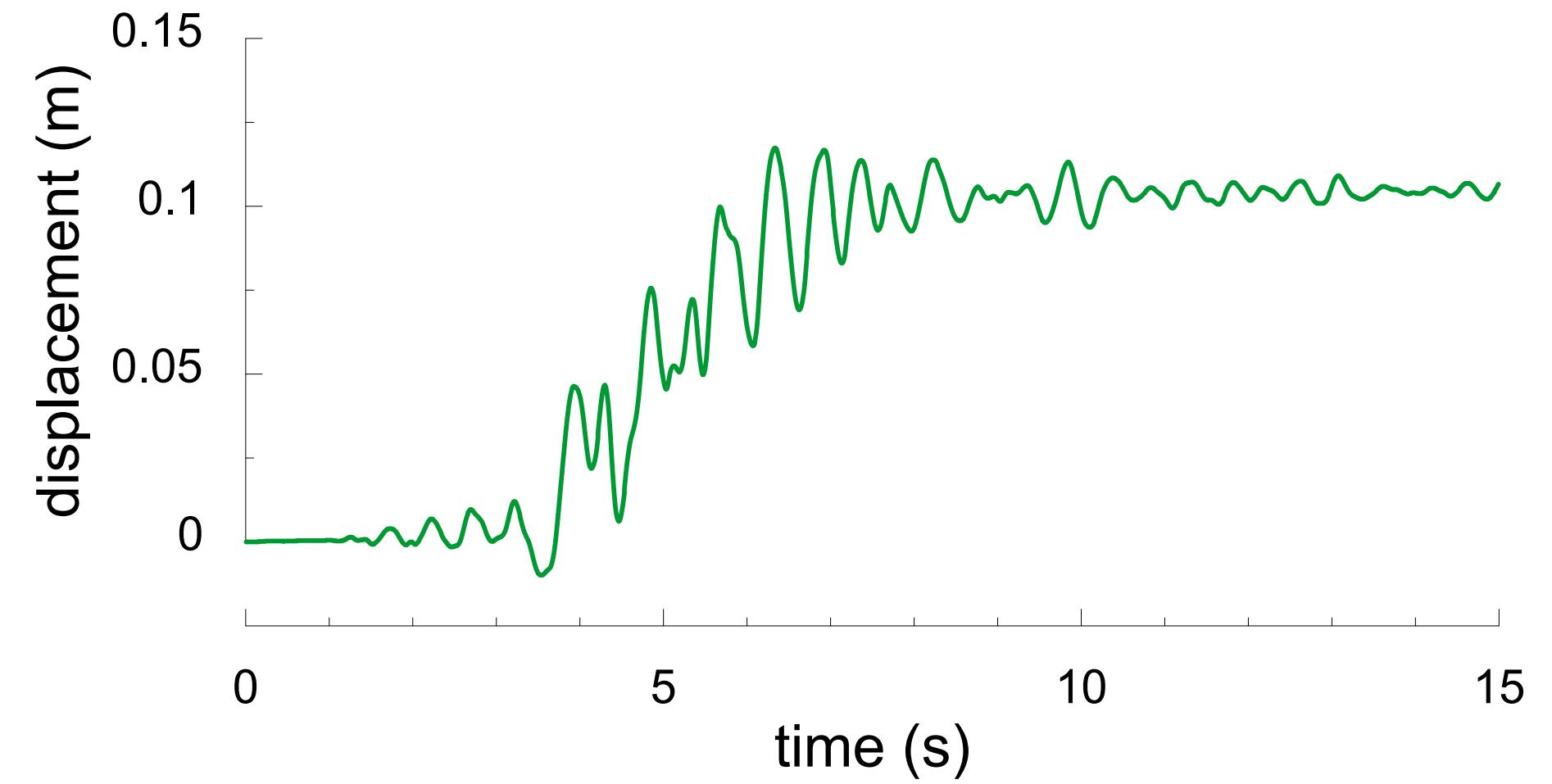
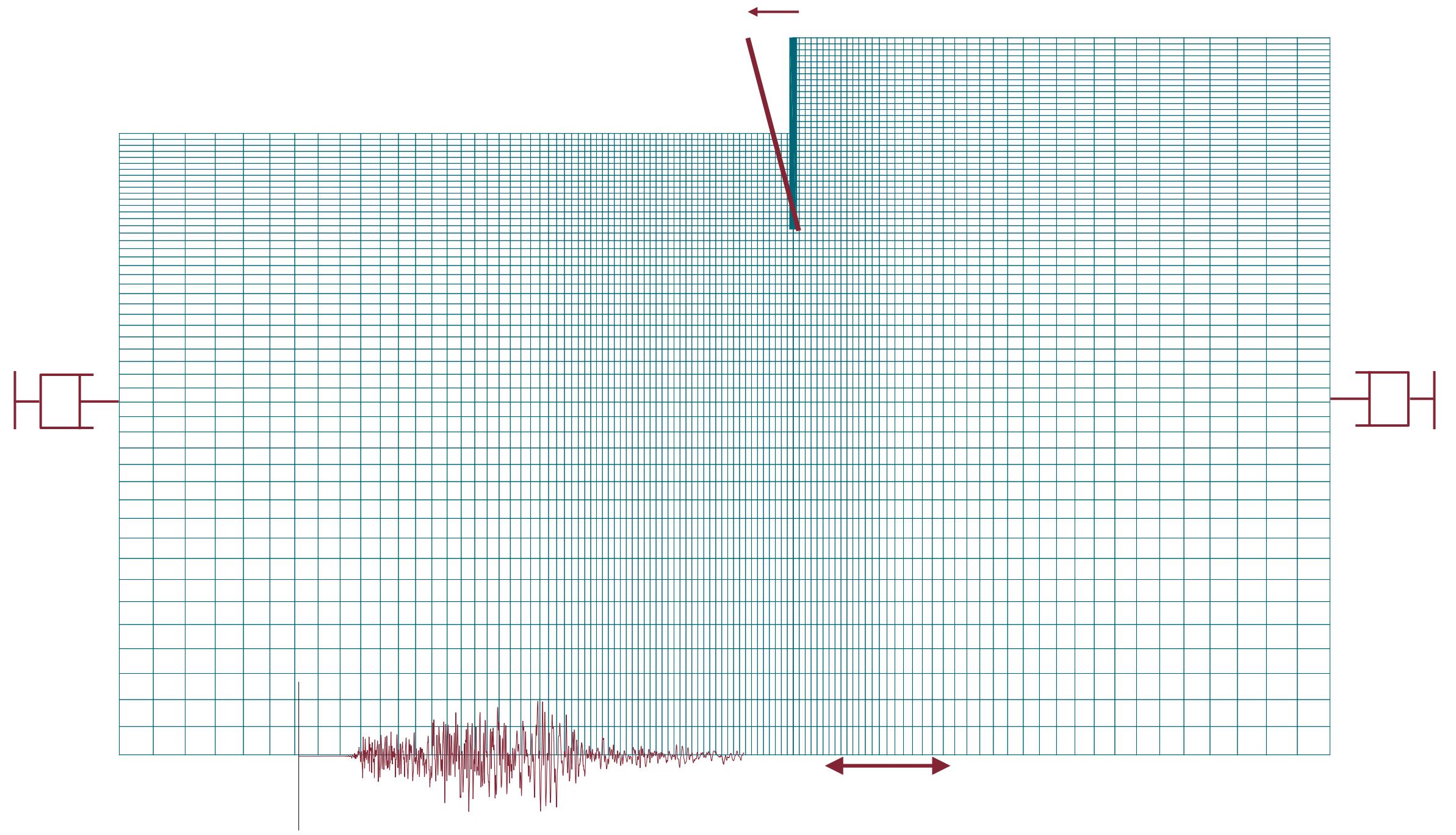
static push-over



dynamic

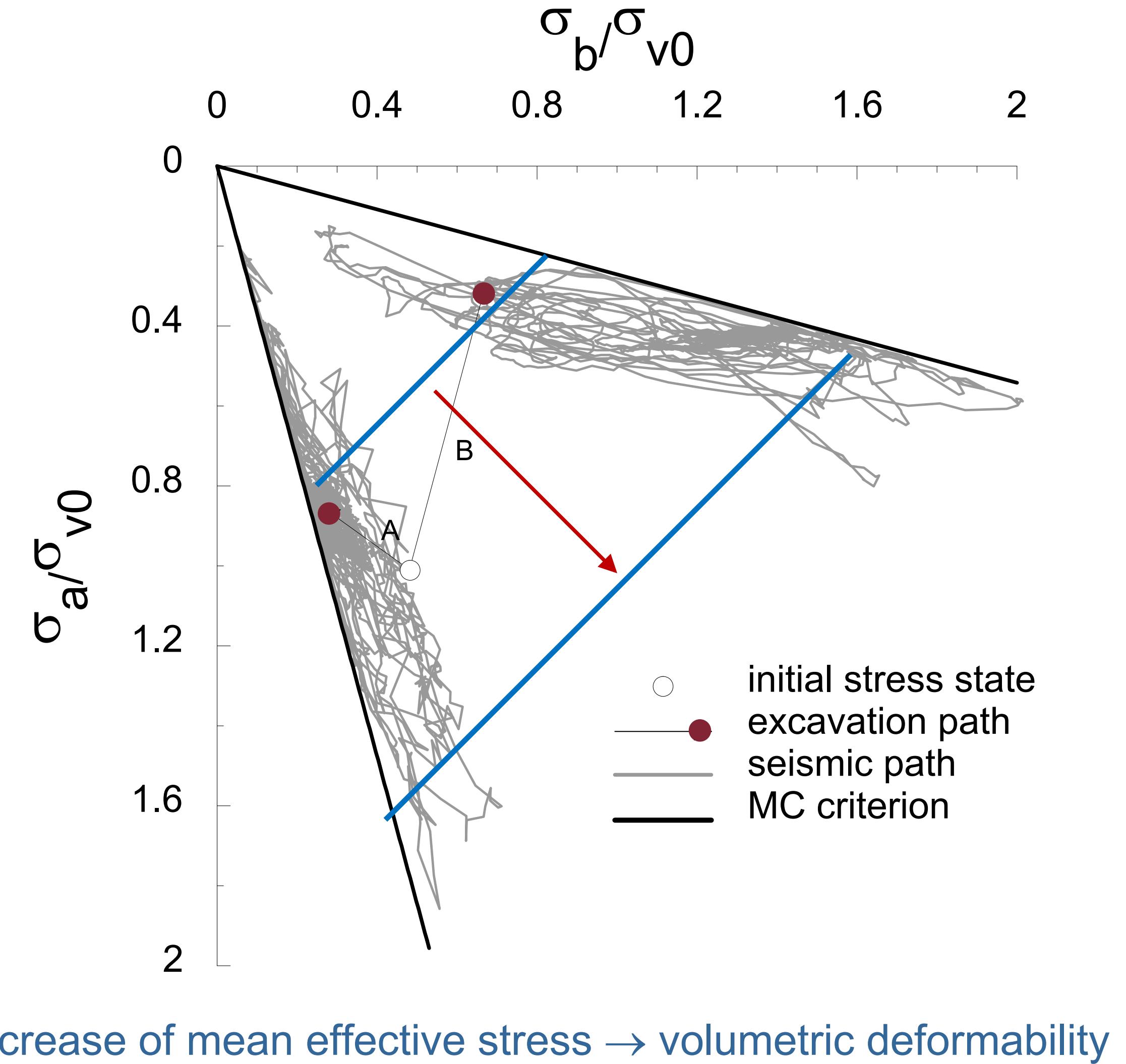
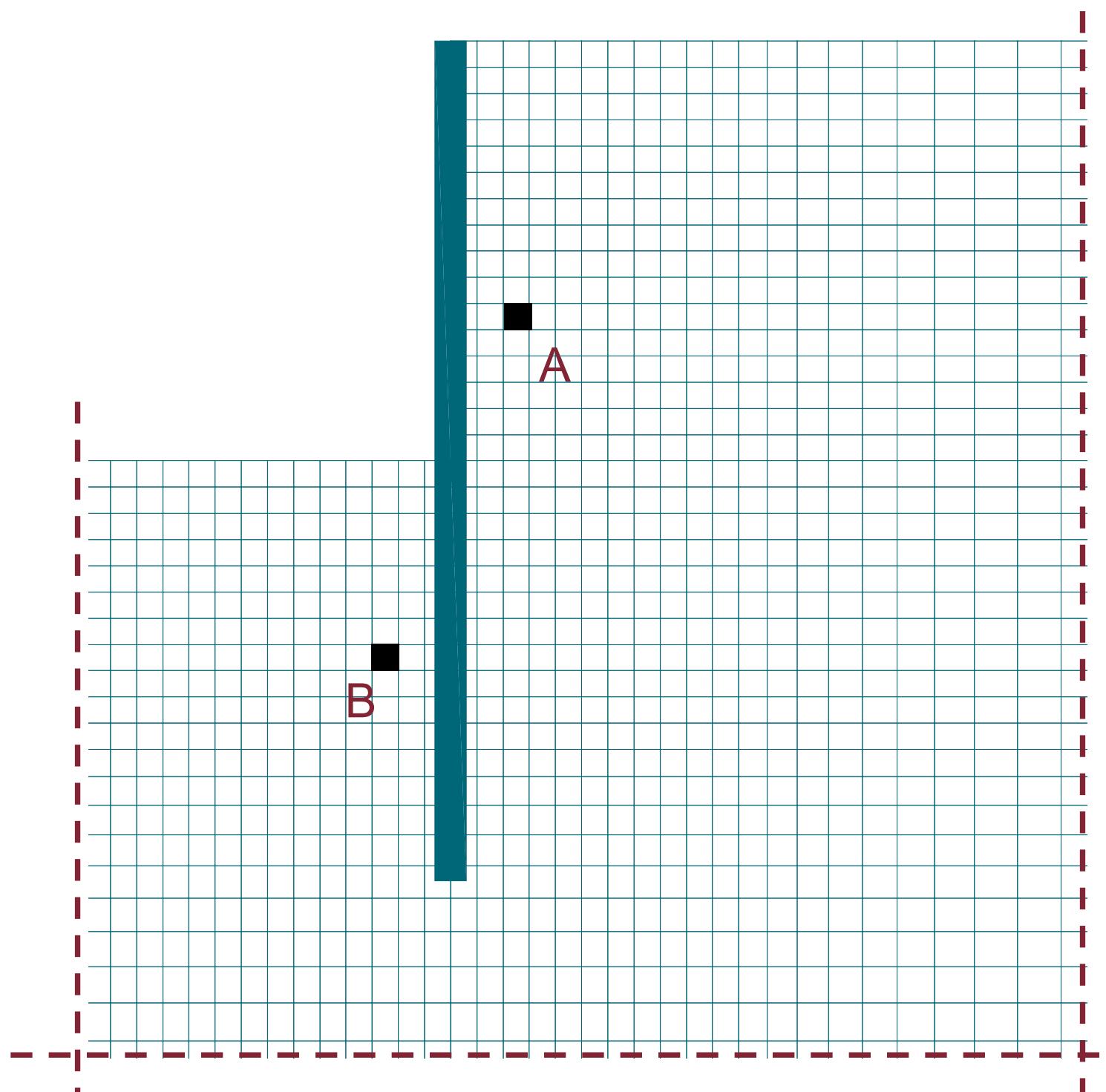


dynamic analyses yield permanent displacements



causes for permanent displacement

- activation of soil strength
- volumetric deformability

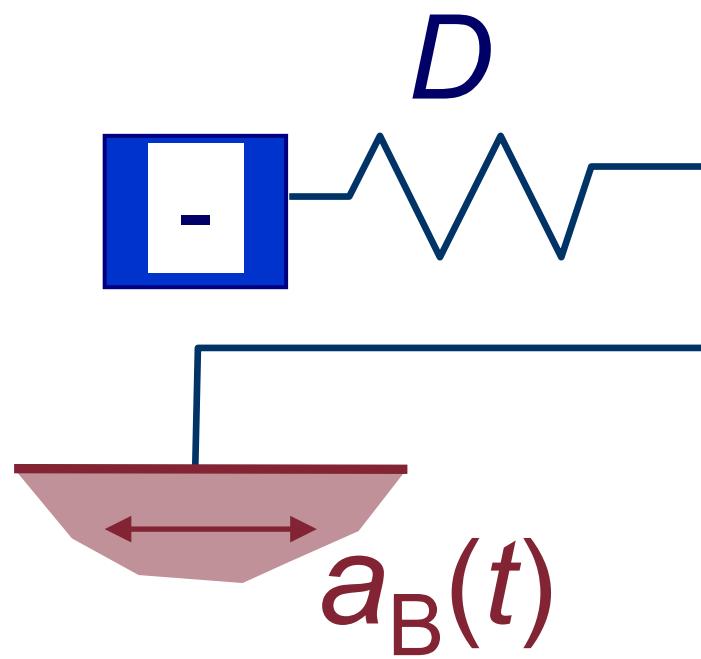


causes for permanent displacement

described at macro-scale by the capacity curve

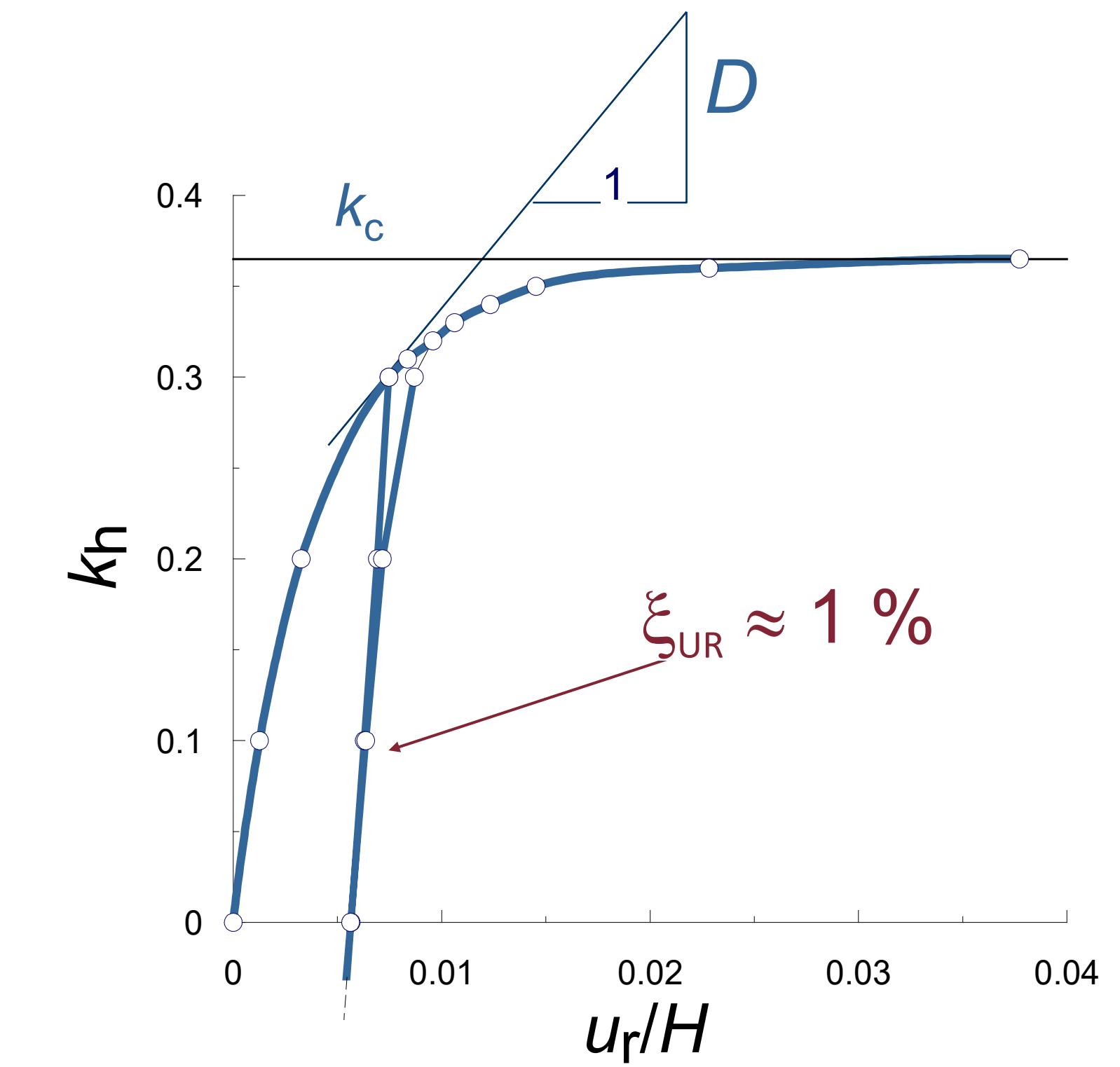
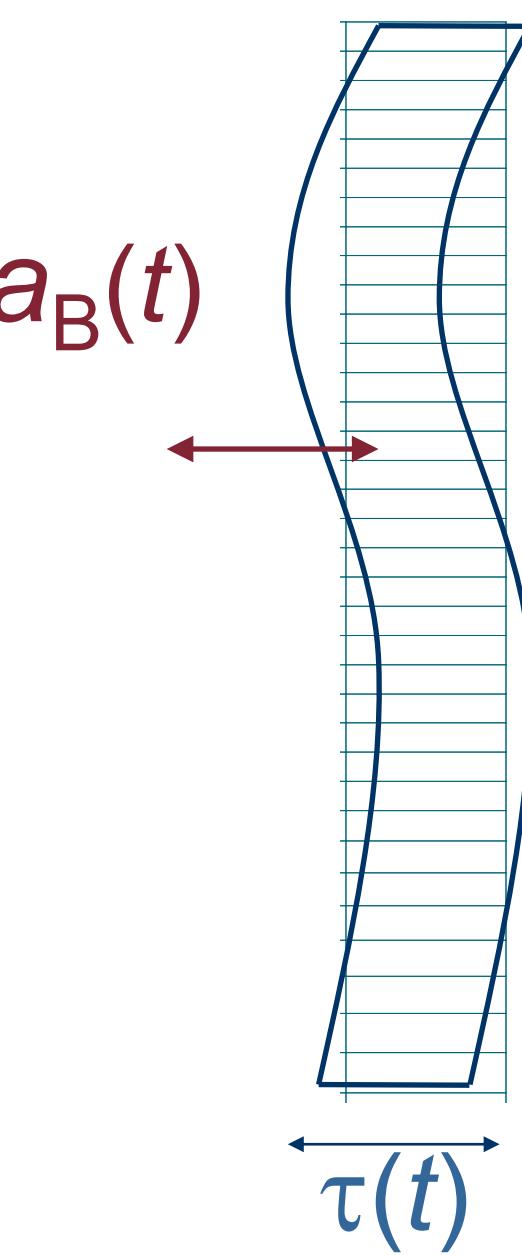
that can be used to predict the displacements

macro-element:
equivalent SDOF



equivalent
base motion

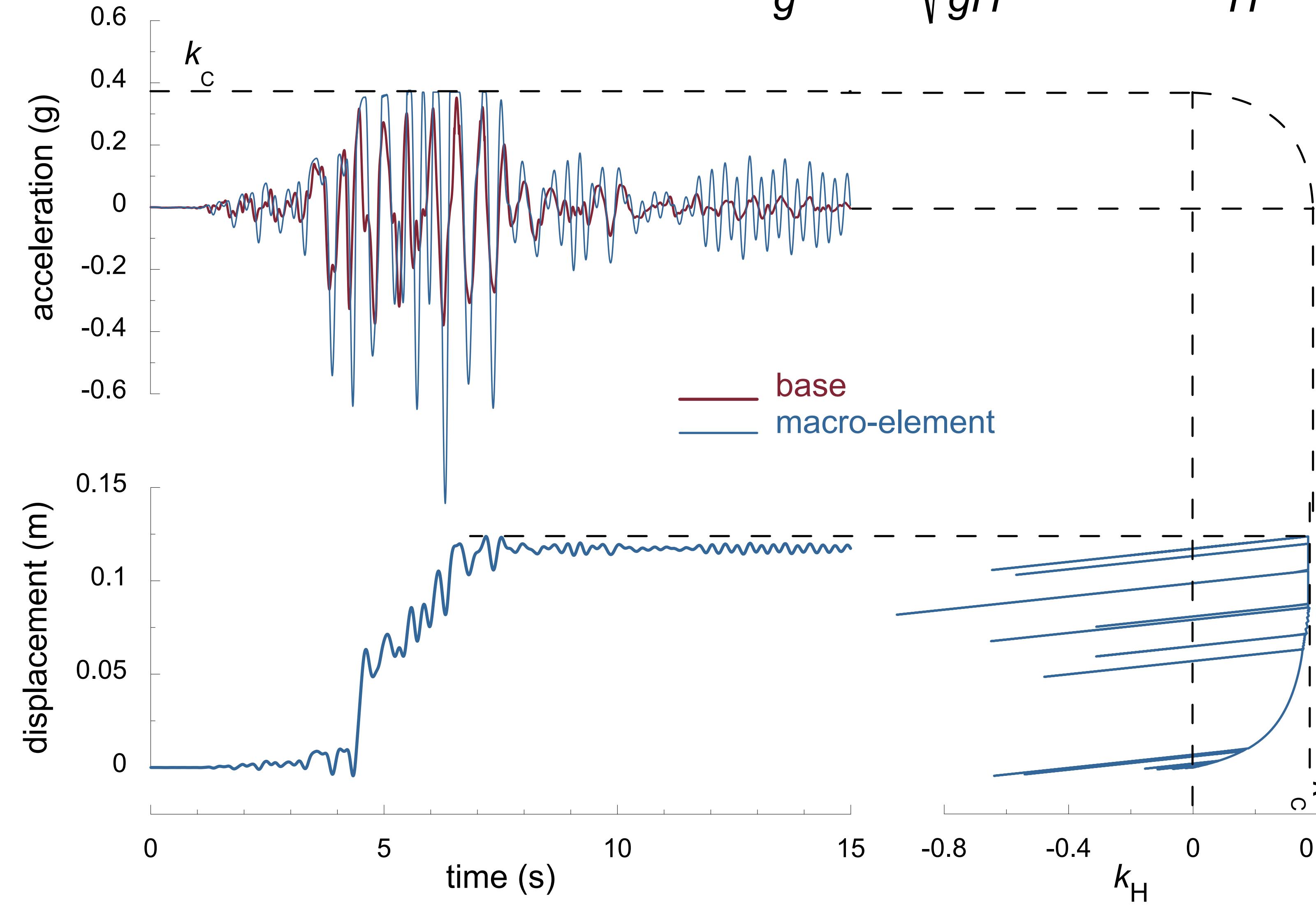
$$\frac{a_B(t)}{g} = \frac{\tau(t)}{\sigma_v}$$



$$\frac{\ddot{u}_R(t)}{g} + 2\xi\sqrt{\frac{D}{gH}}\dot{u}_R(t) + D \times \frac{u_R(t)}{H} = -\frac{a_B(t)}{g}$$

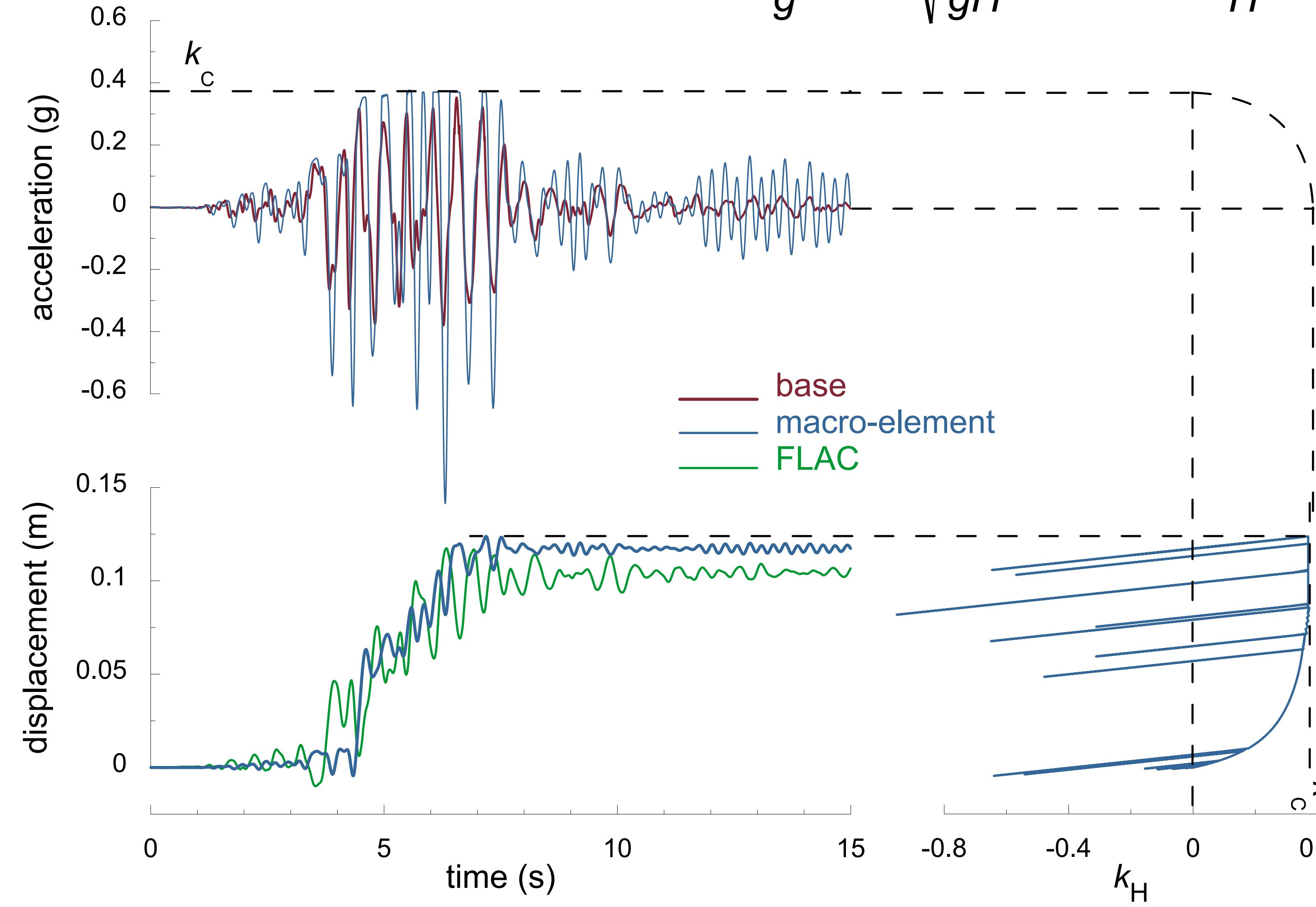
macro-element predictions

$$\frac{\ddot{u}_R(t)}{g} + 2\xi\sqrt{\frac{D}{gH}}\dot{u}_R(t) + D \times \frac{u_R(t)}{H} = -\frac{a_B(t)}{g}$$

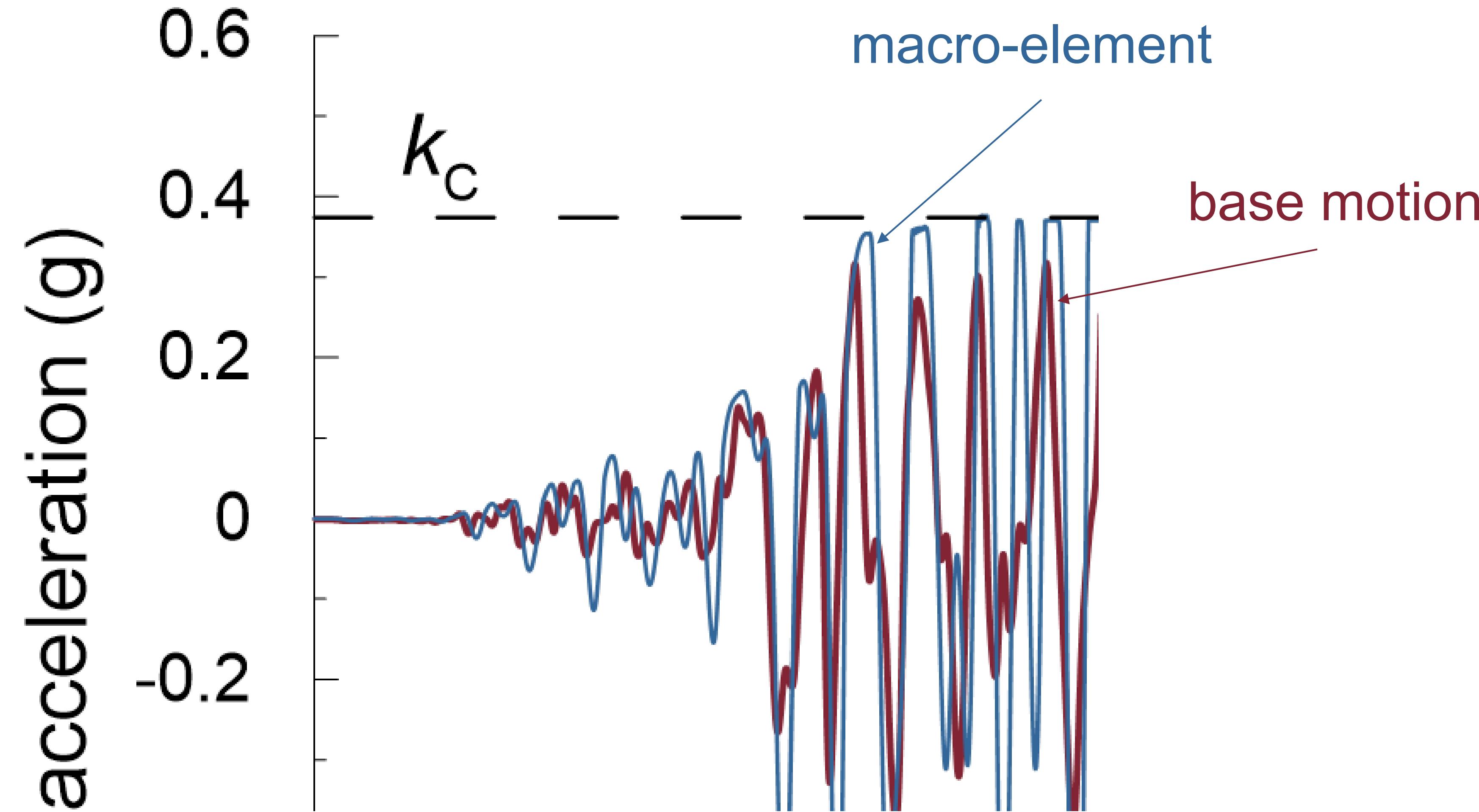


macro-element predictions

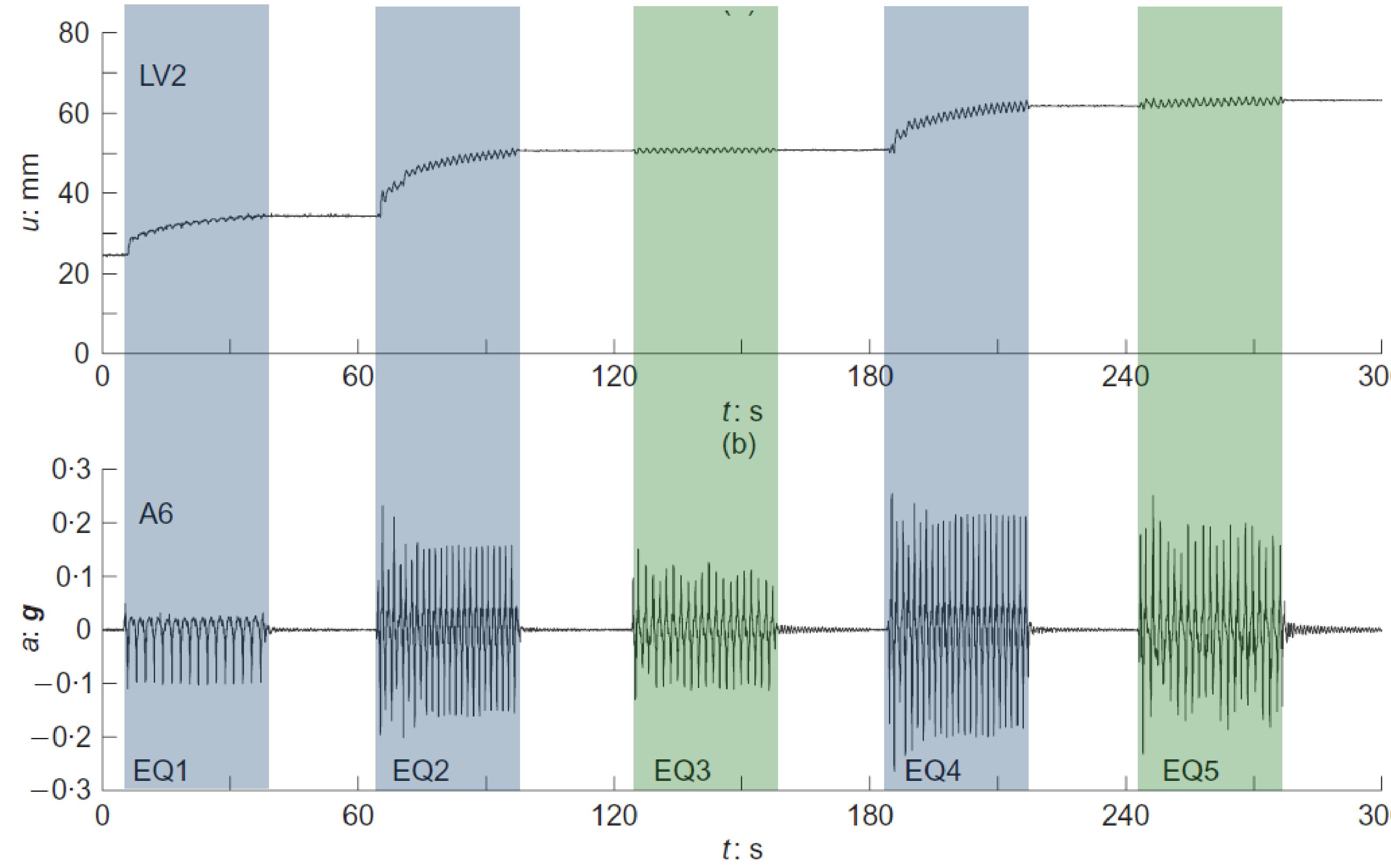
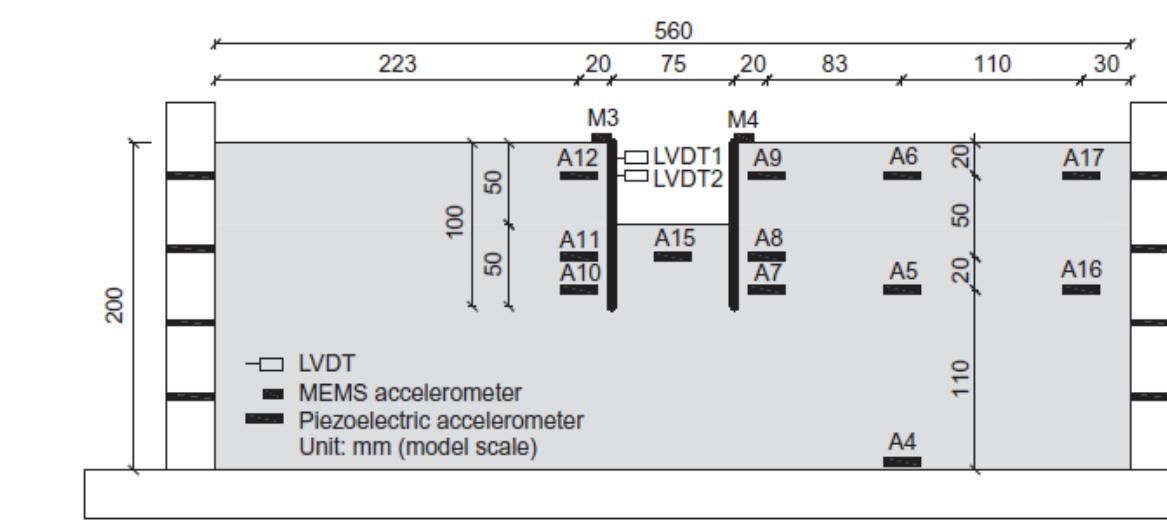
$$\frac{\ddot{u}_R(t)}{g} + 2\xi\sqrt{\frac{D}{gH}}\dot{u}_R(t) + D \times \frac{u_R(t)}{H} = -\frac{a_B(t)}{g}$$



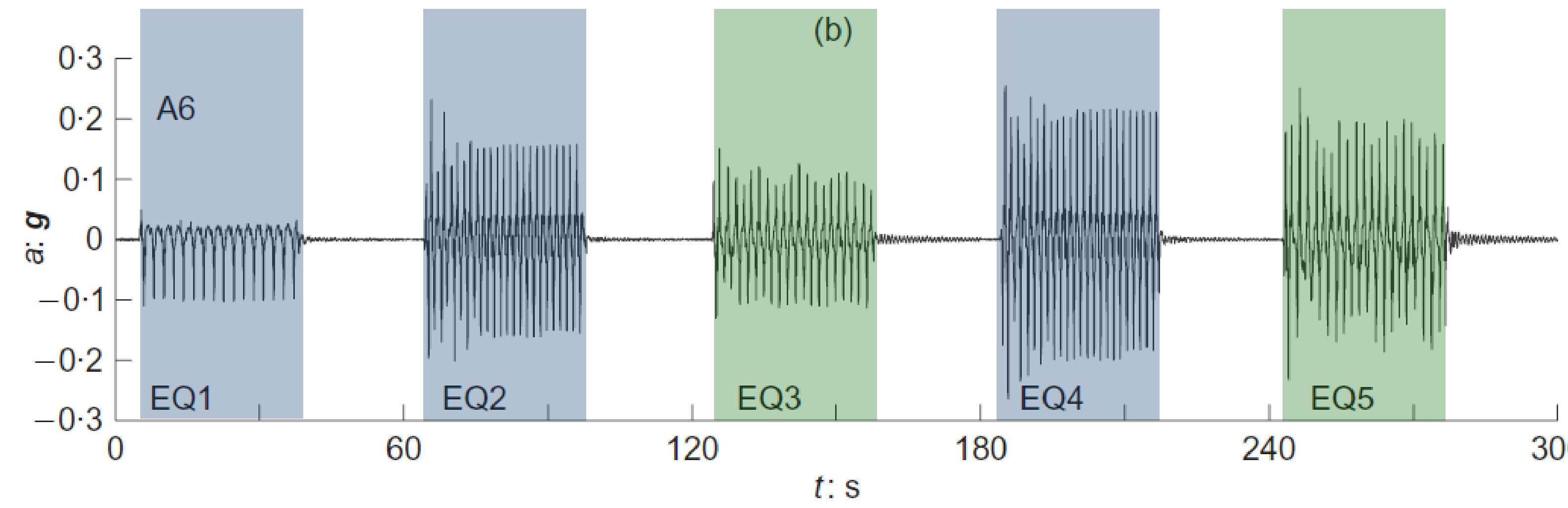
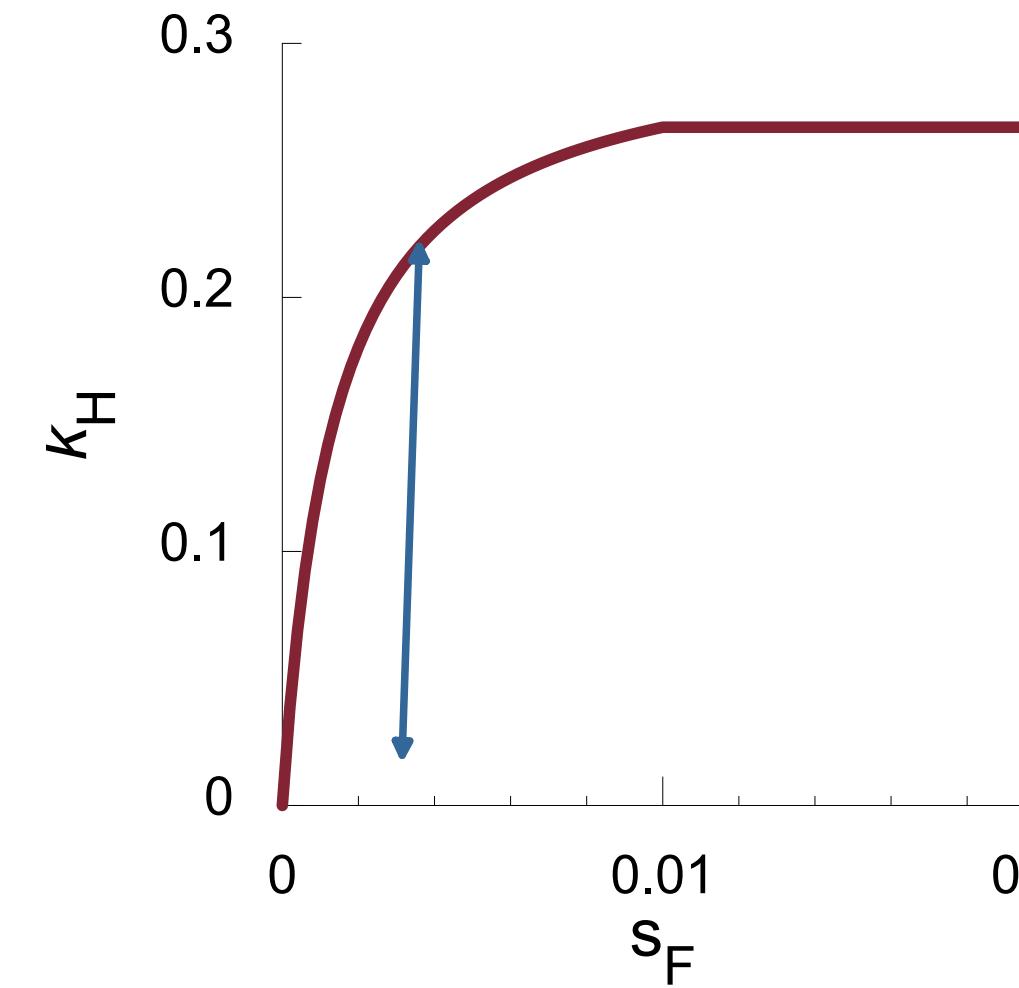
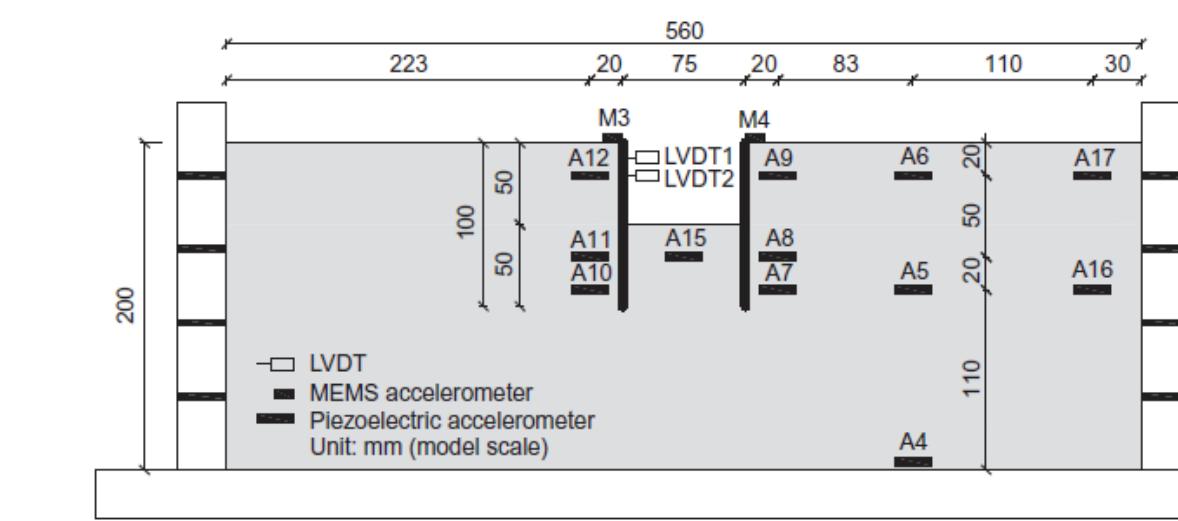
macro-element predictions



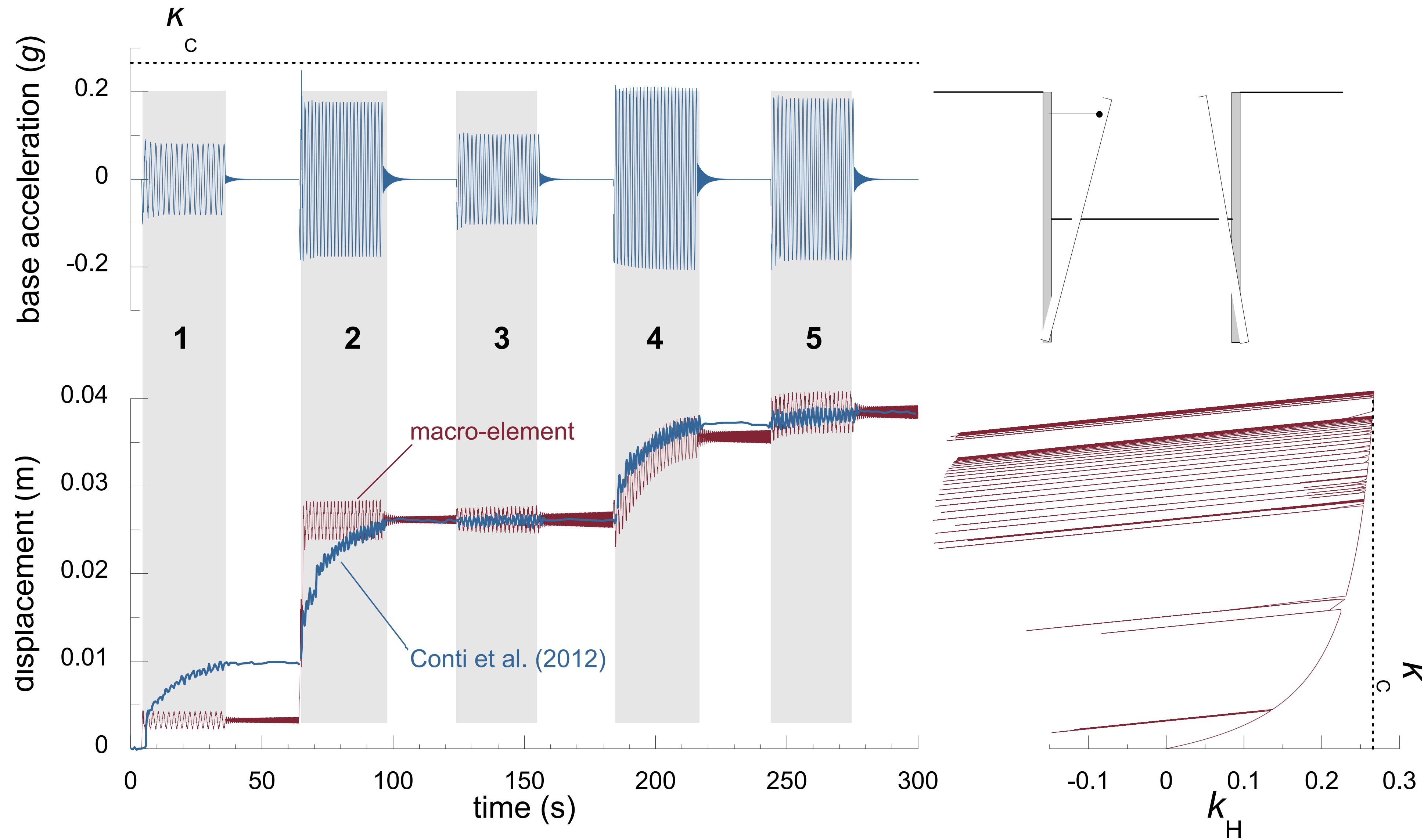
centrifuge test (Conti et al. 2012)



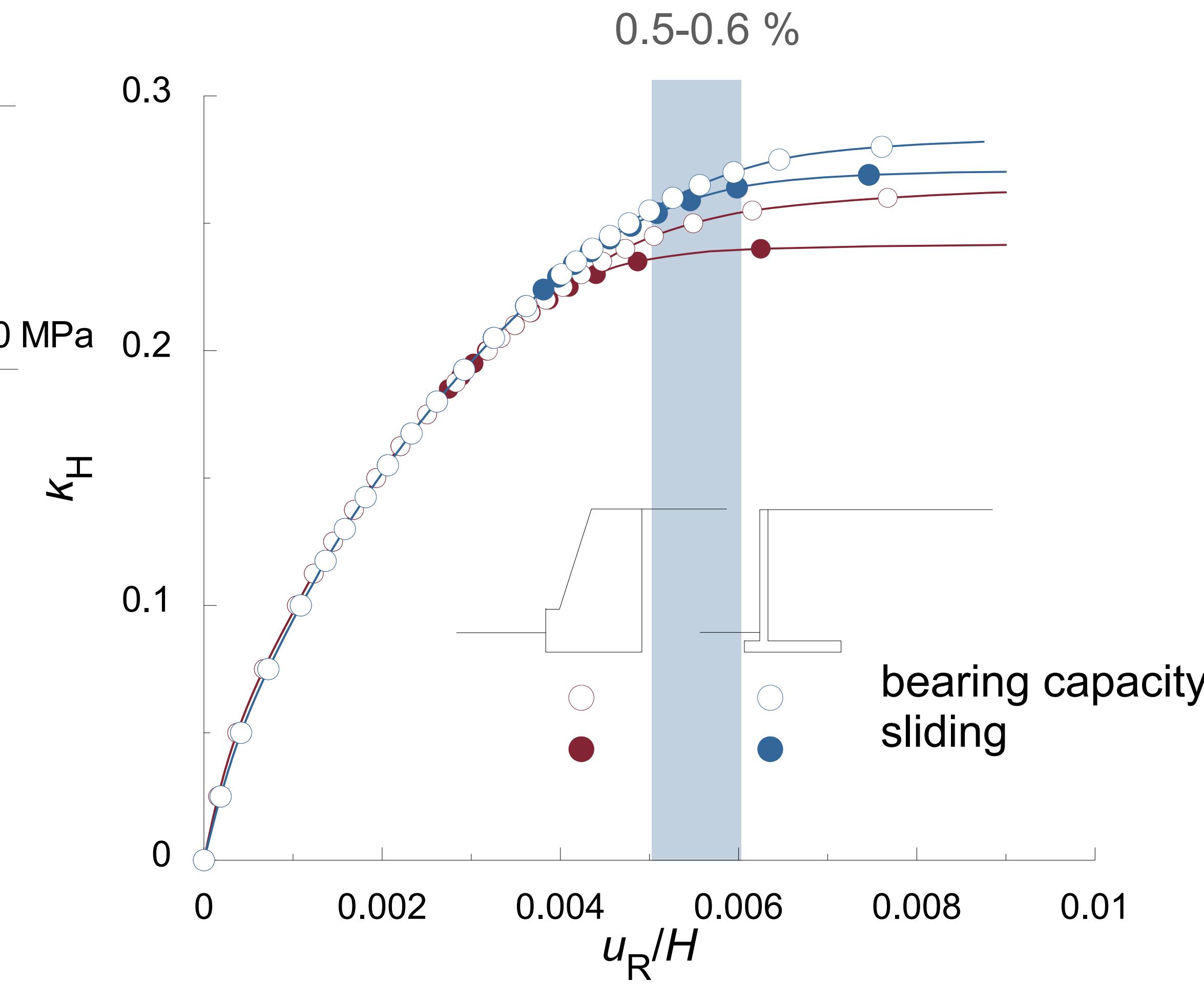
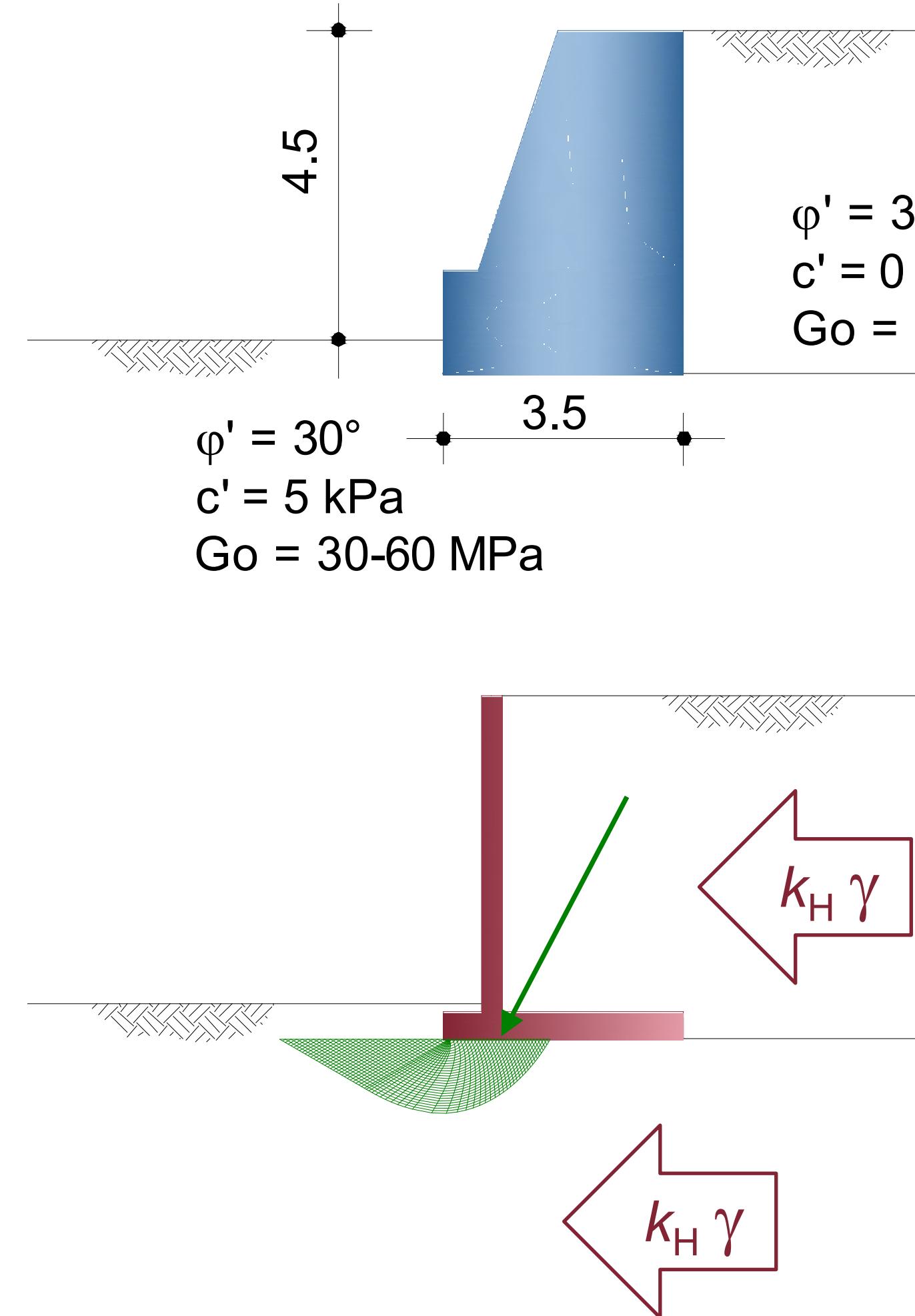
centrifuge test (Conti et al. 2012)



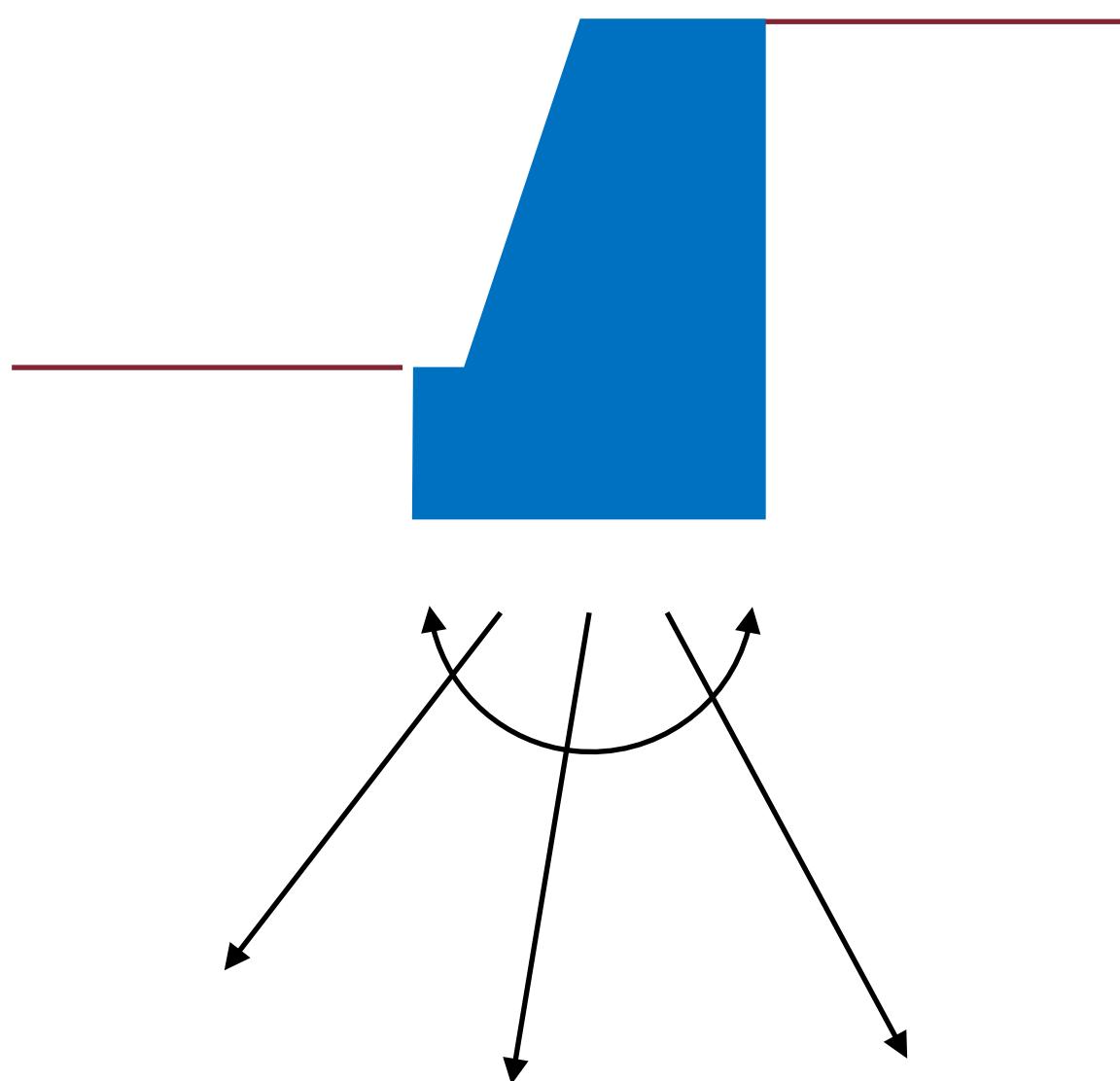
simulation of centrifuge test



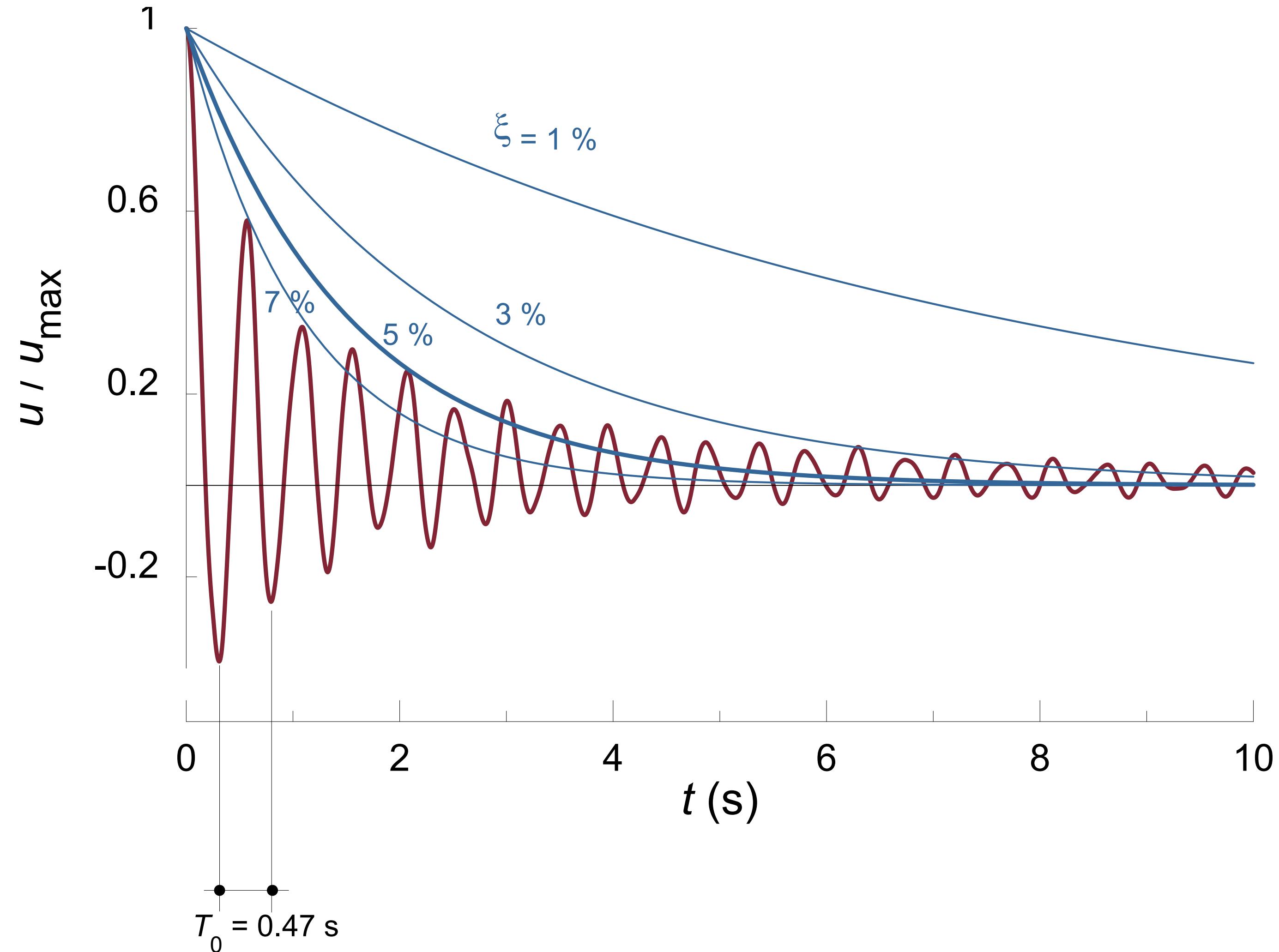
gravity walls



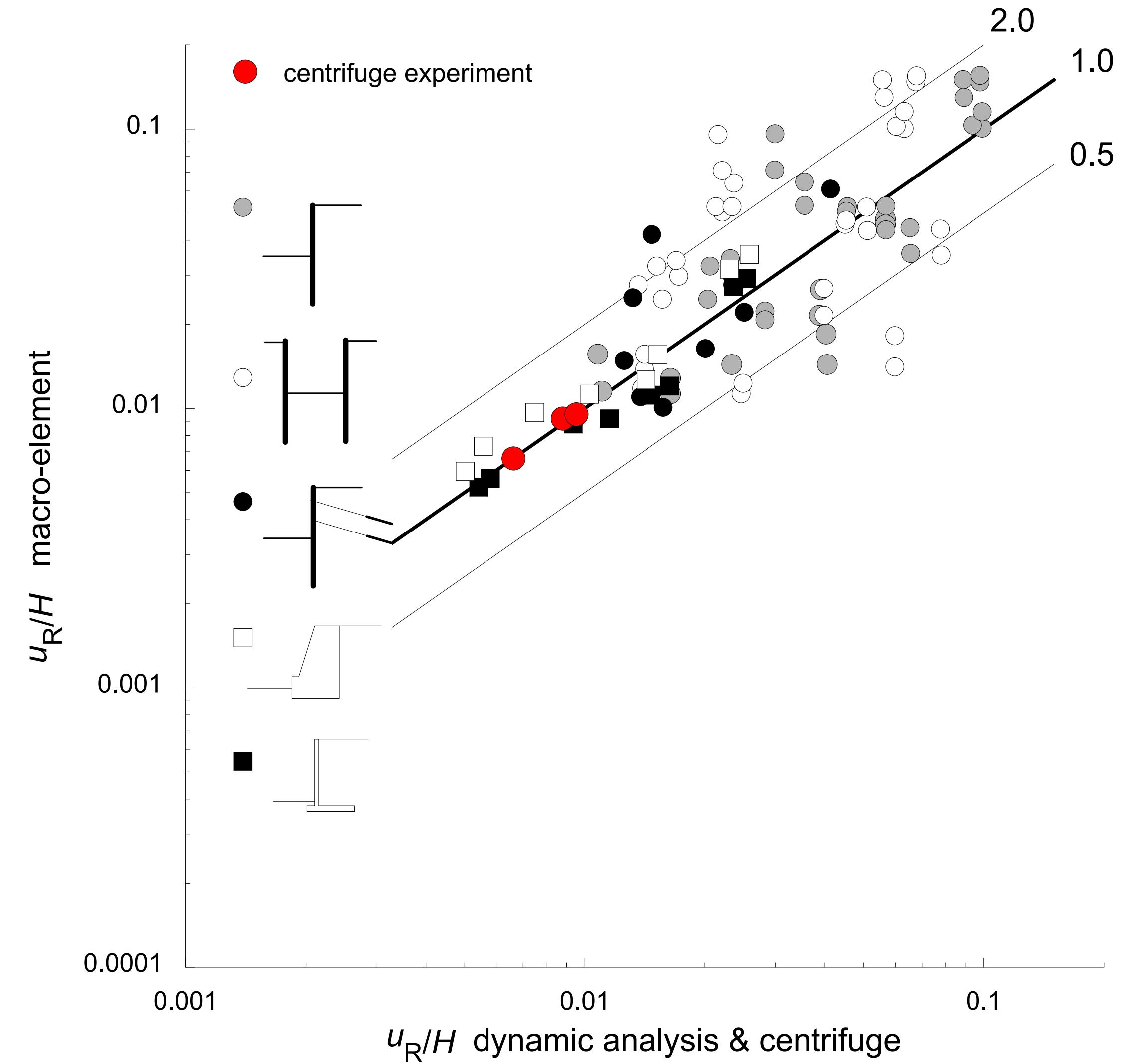
gravity walls – free vibrations



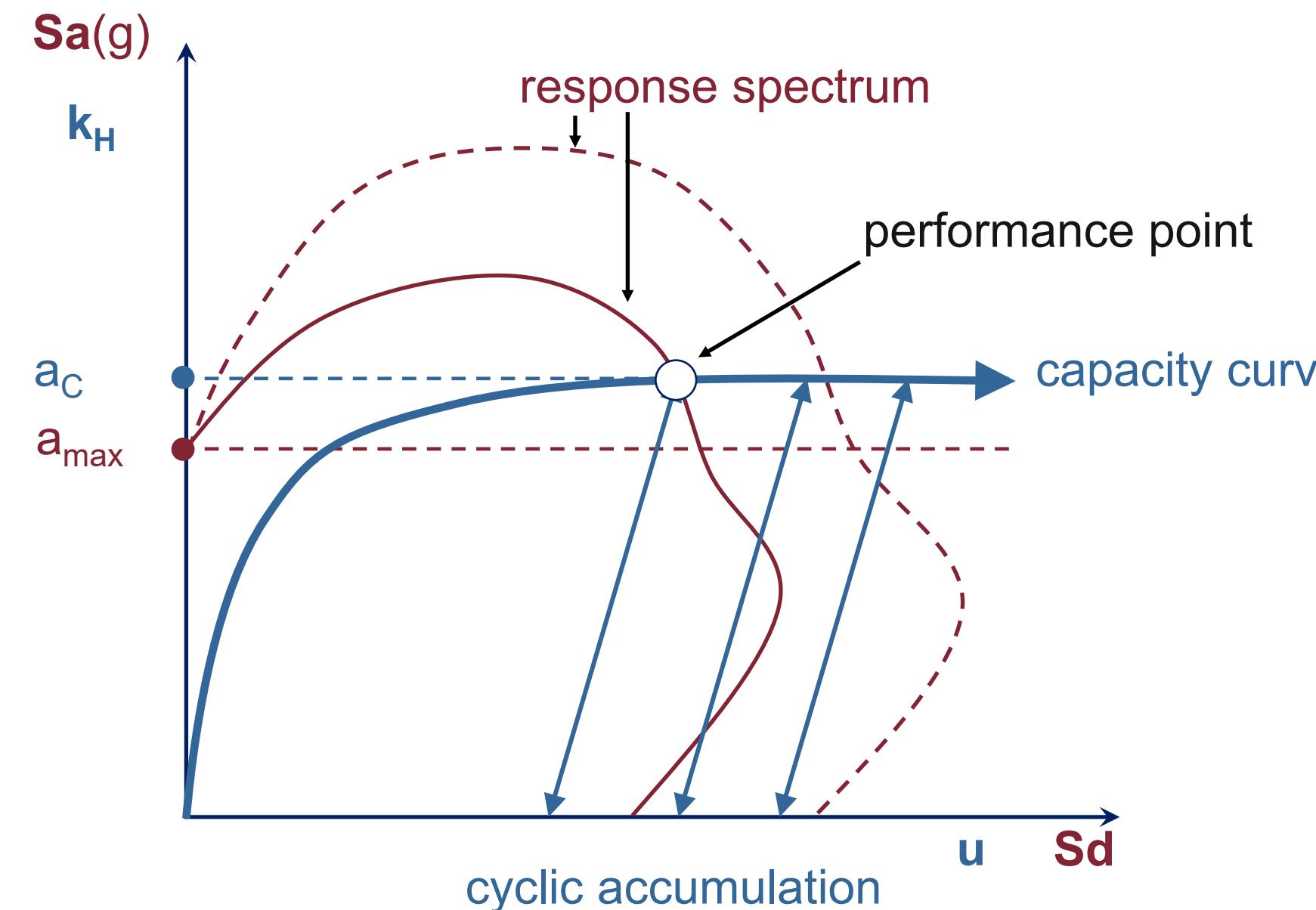
radiation damping



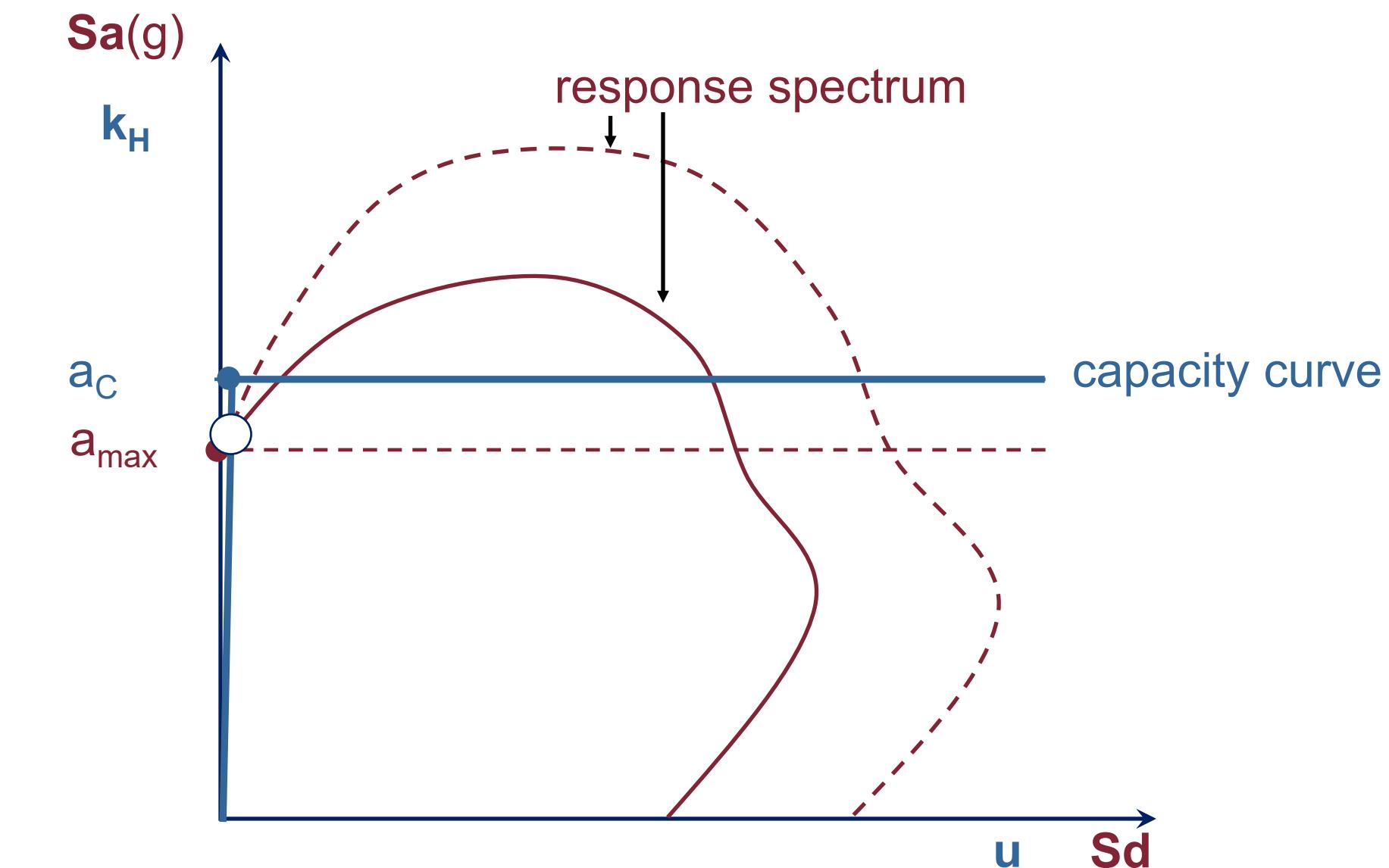
- good overall comparison
- effect of higher modes
- Nemark method is ineffective



acceleration – displacement plane



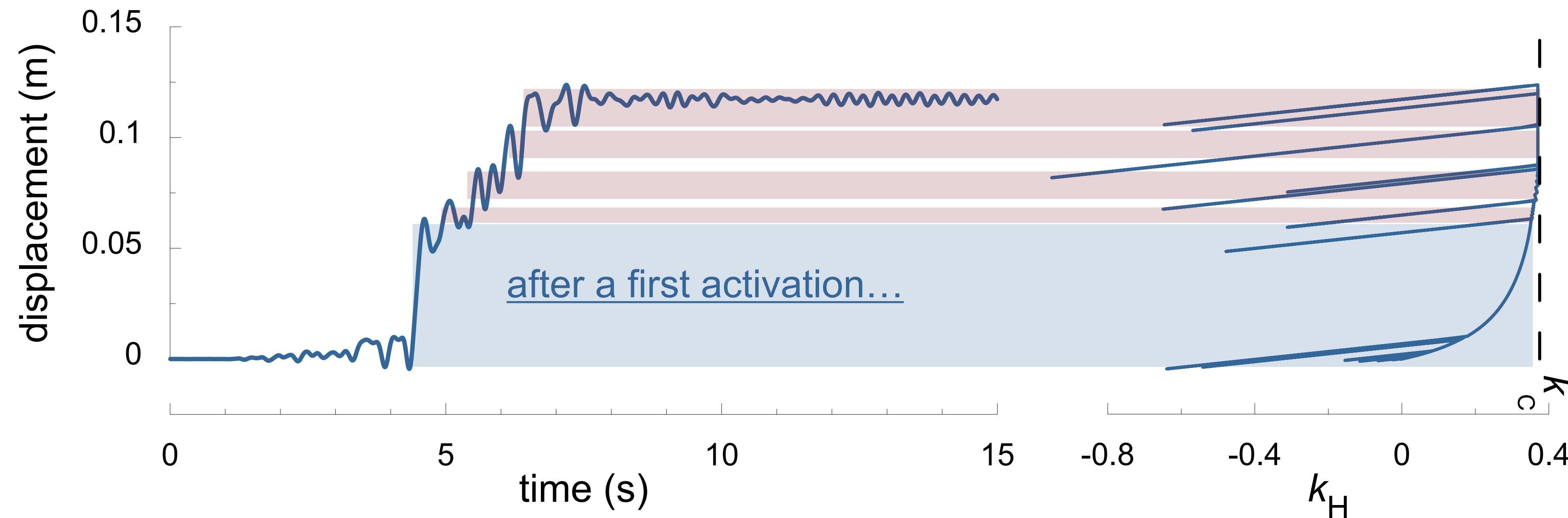
deformable system
cyclic accumulation



rigid-perfectly plastic (Newmark)
no displacements

macro-element predictons

...dynamic response is controlled by
unloading/reloading

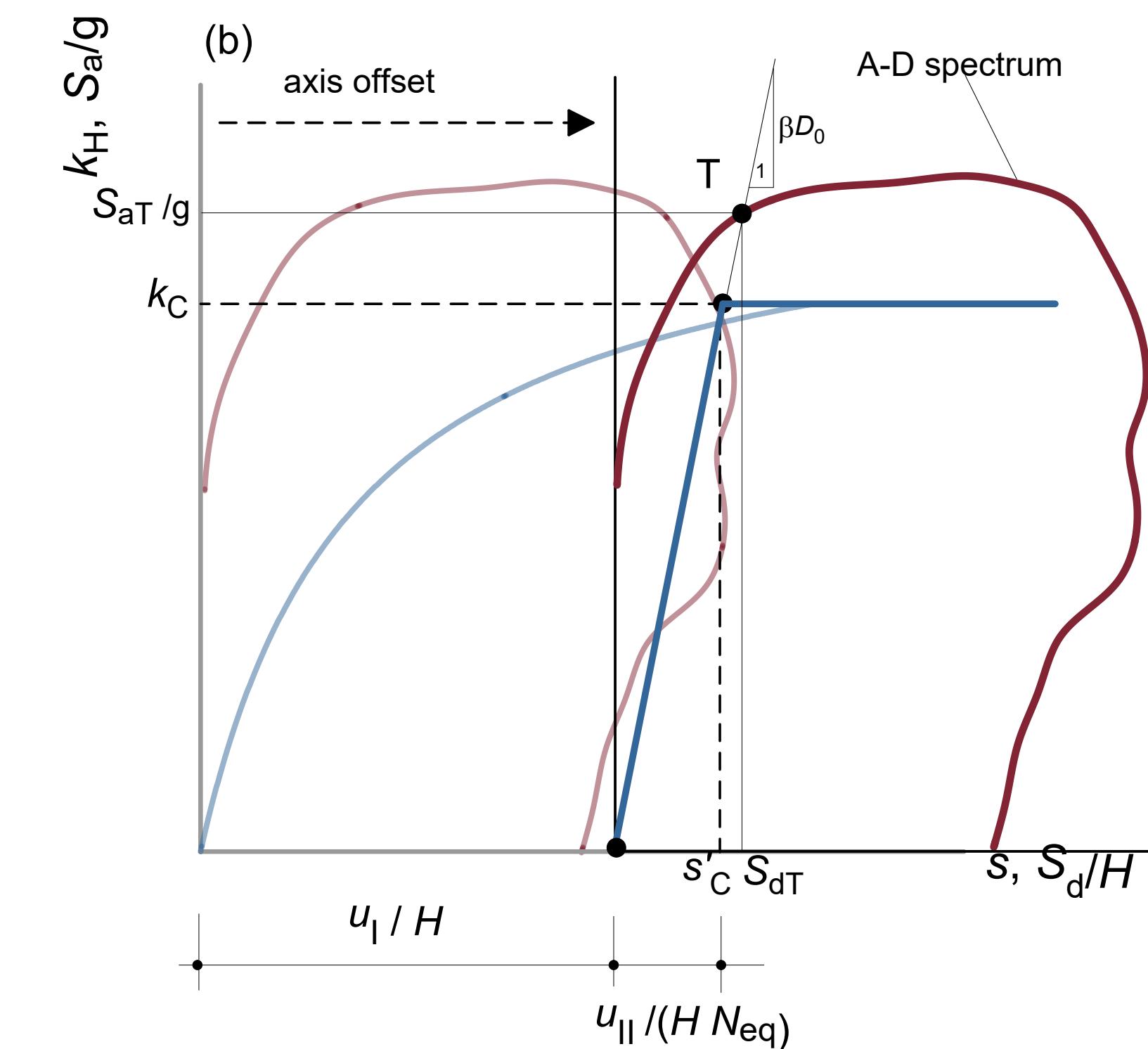
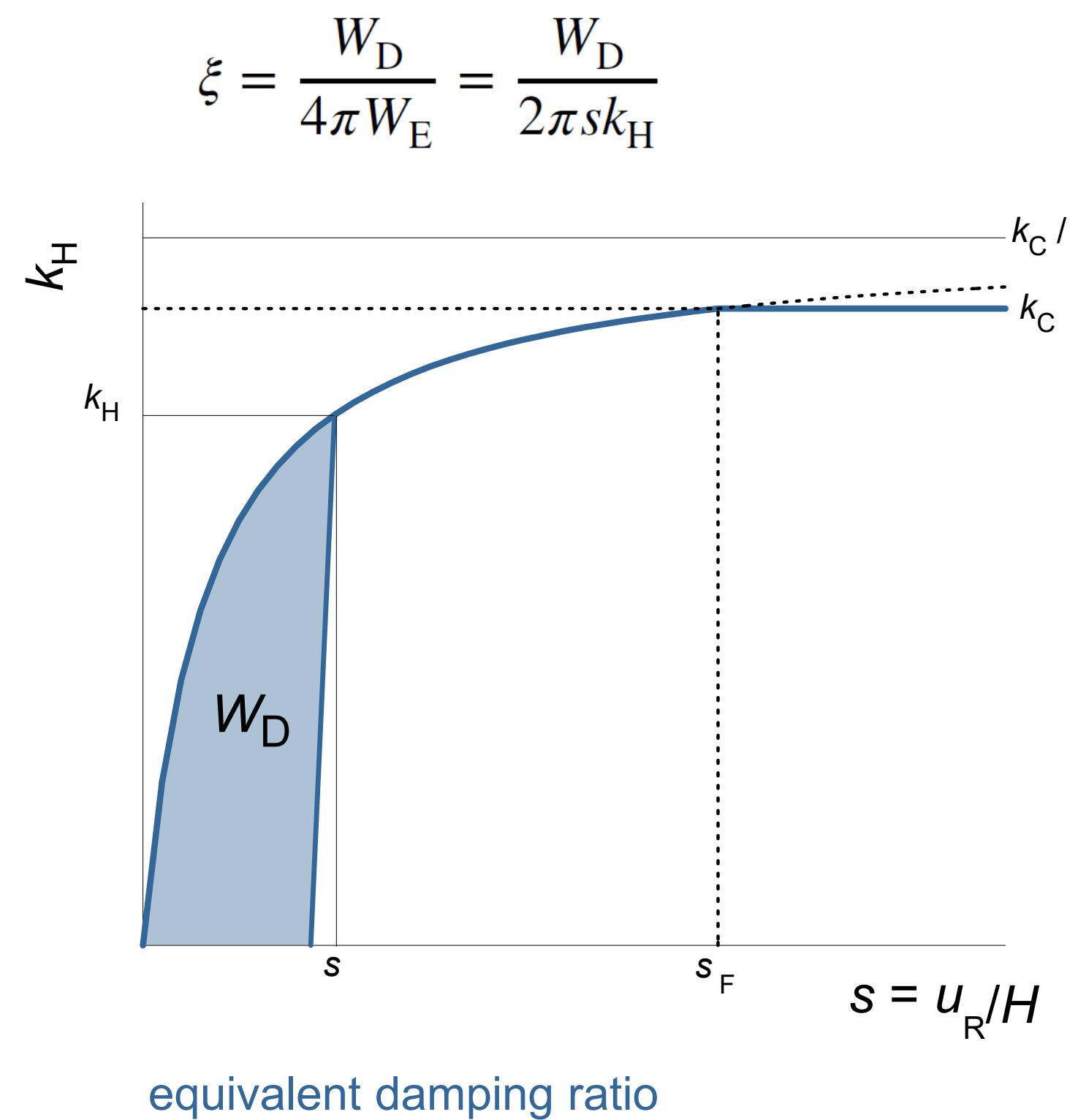
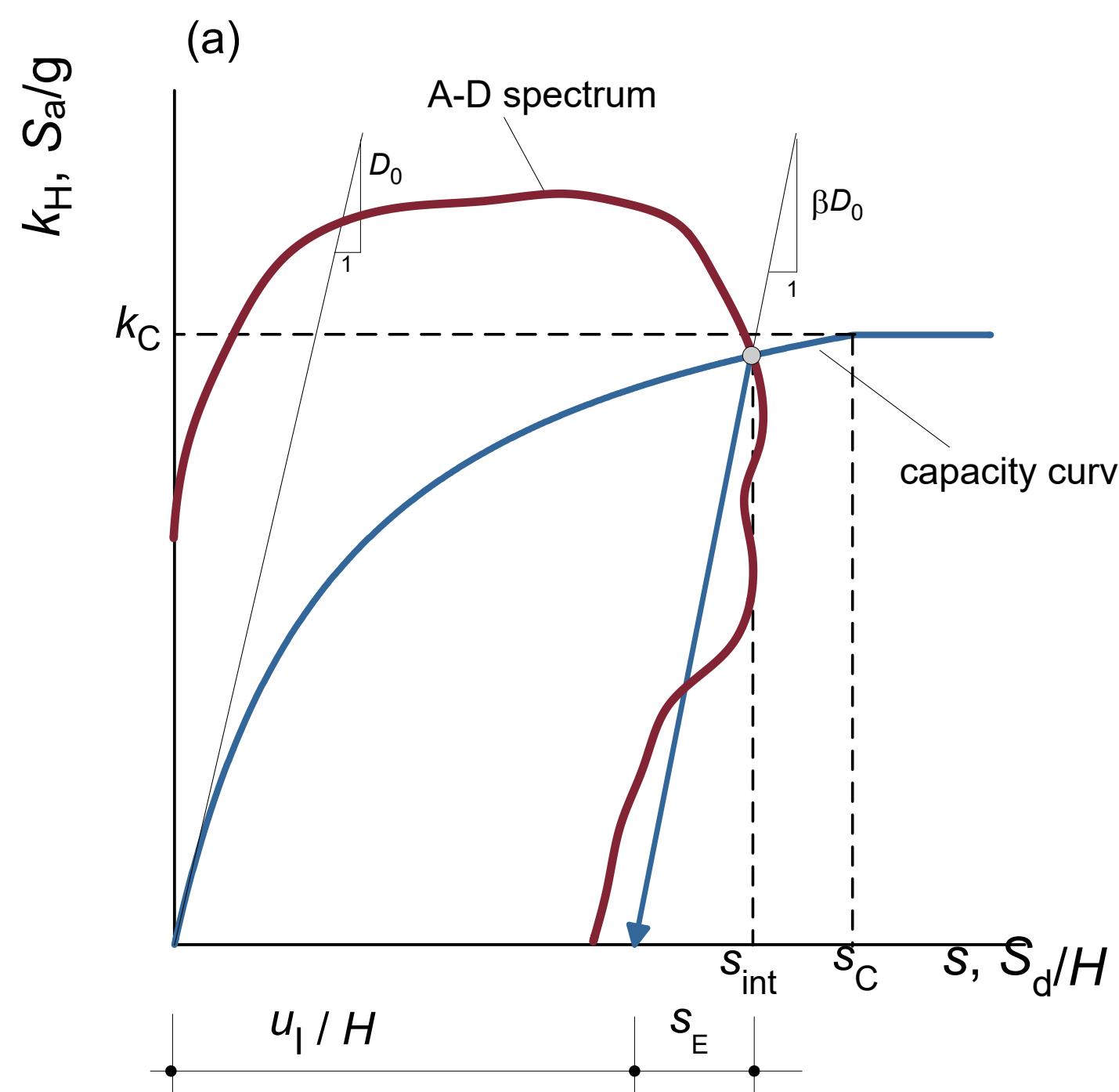


$$u = u_I + N_{eq} u_{II}$$

prediction of seismic performance

demand described by a response spectrum...

... plus equivalent number of cycles

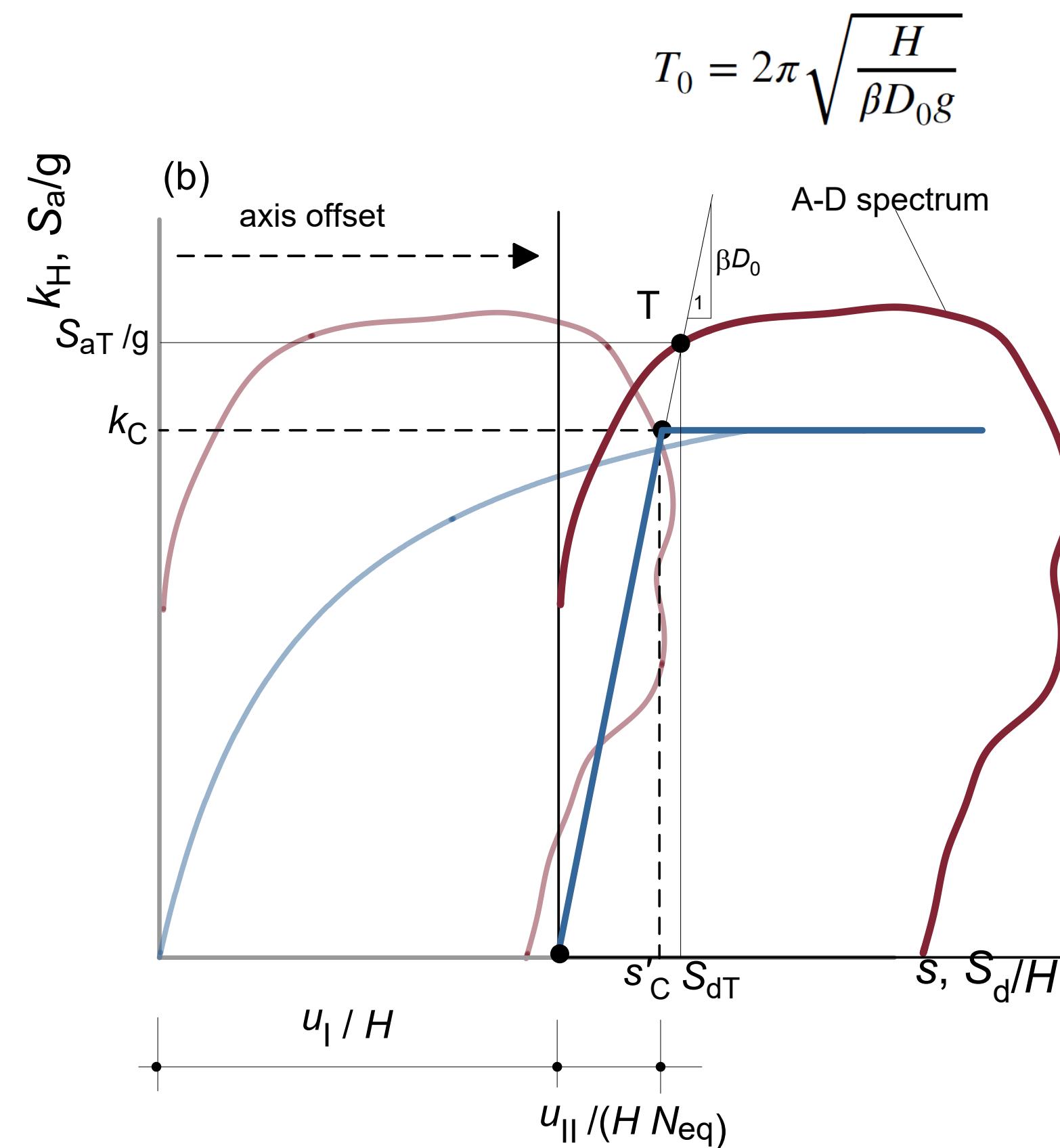


Laguardia, Gallese,
Gigliotti, Callisto (2020)

$$u = u_I + N_{eq} u_{II}$$

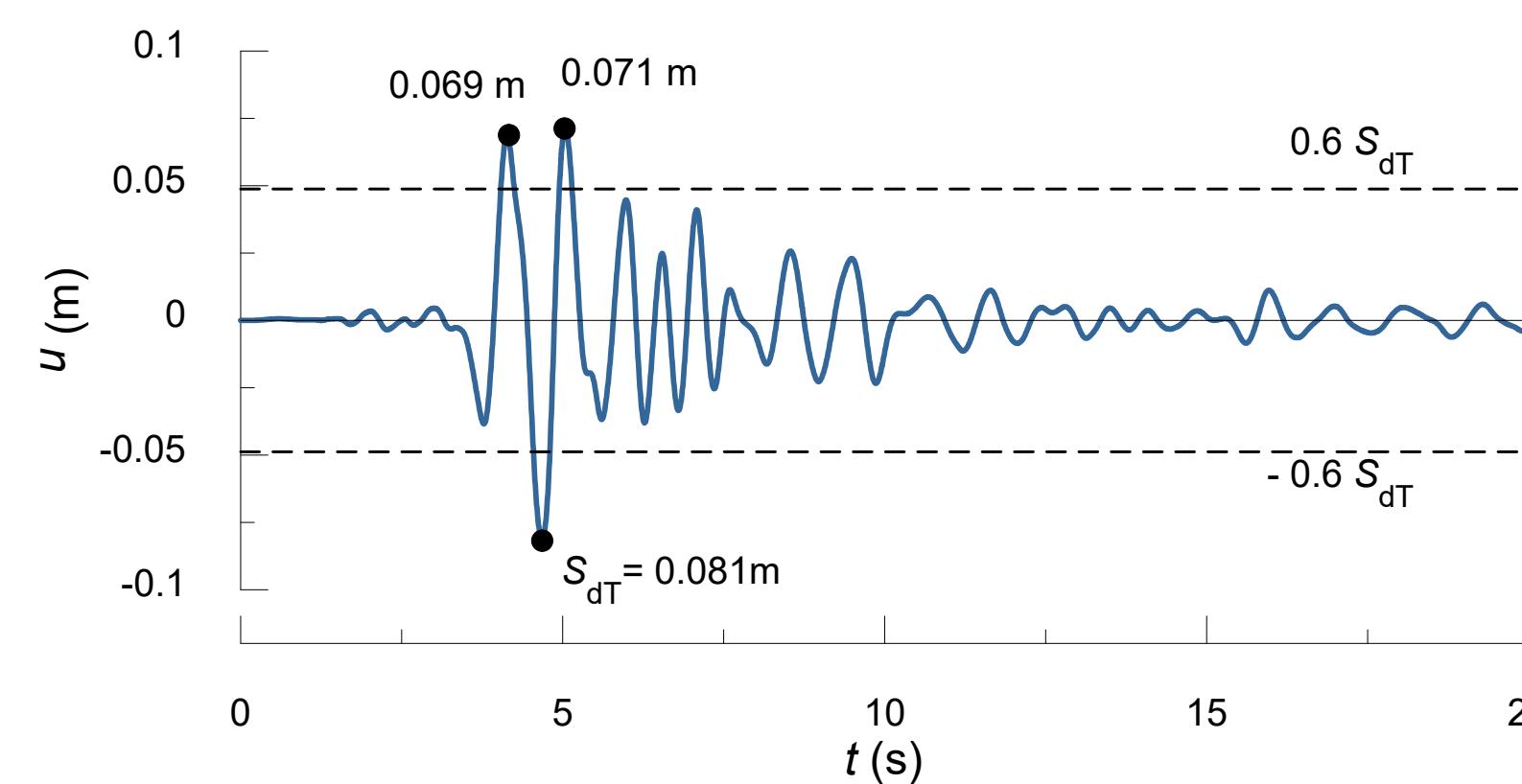
definition of equivalent number of cycles

spectrum of N_{eq}

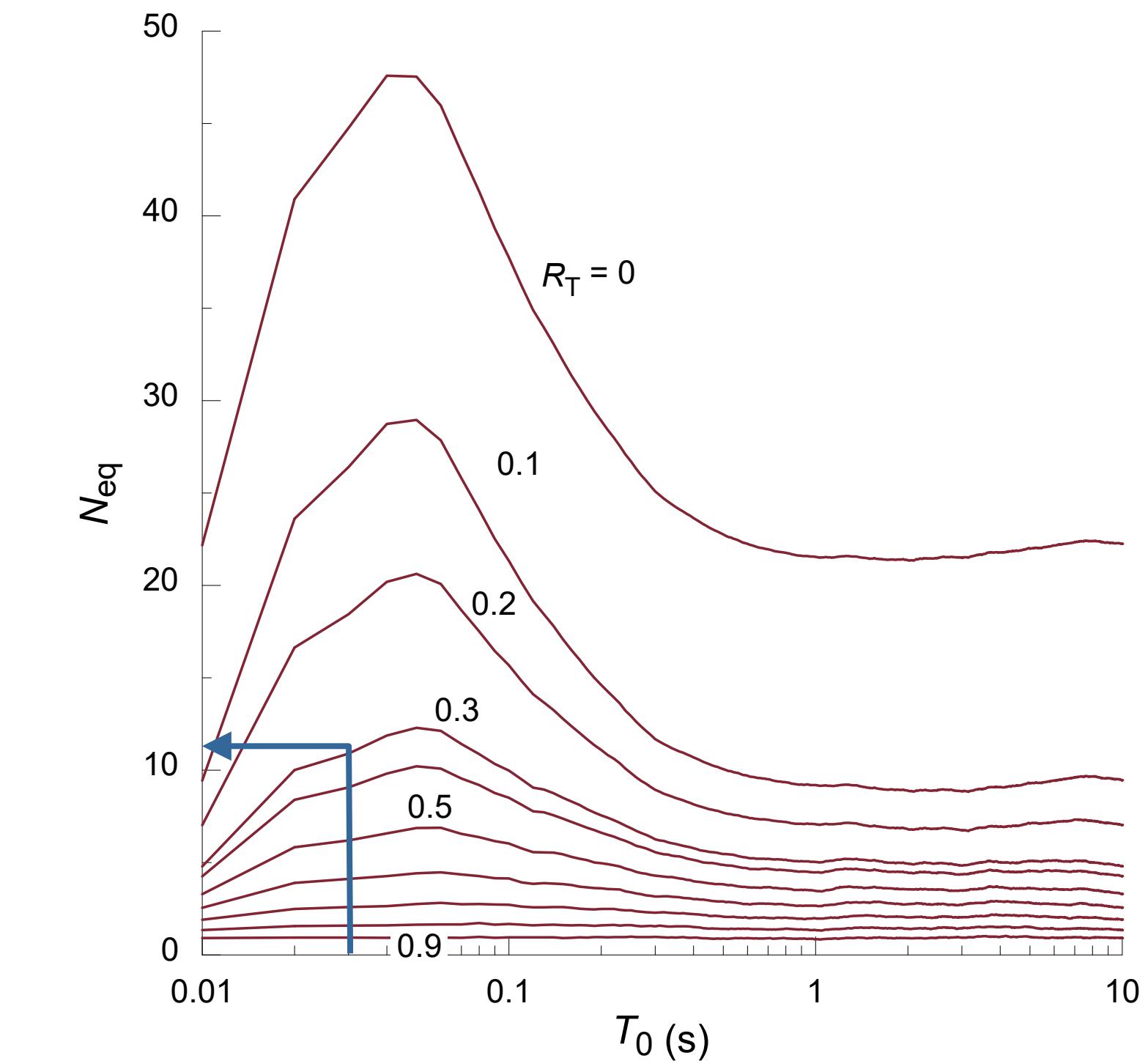


$$R_T = \frac{gk_C}{S_{aT}} = \frac{s'_C}{S_{dT}}$$

$$N_{\text{eq}} = \frac{1}{2} \sum_{i=1}^n \left[\frac{u_i}{S_{dT}} \times hsf\left(\frac{u_i}{S_{dT}} - R_T\right) \right]$$

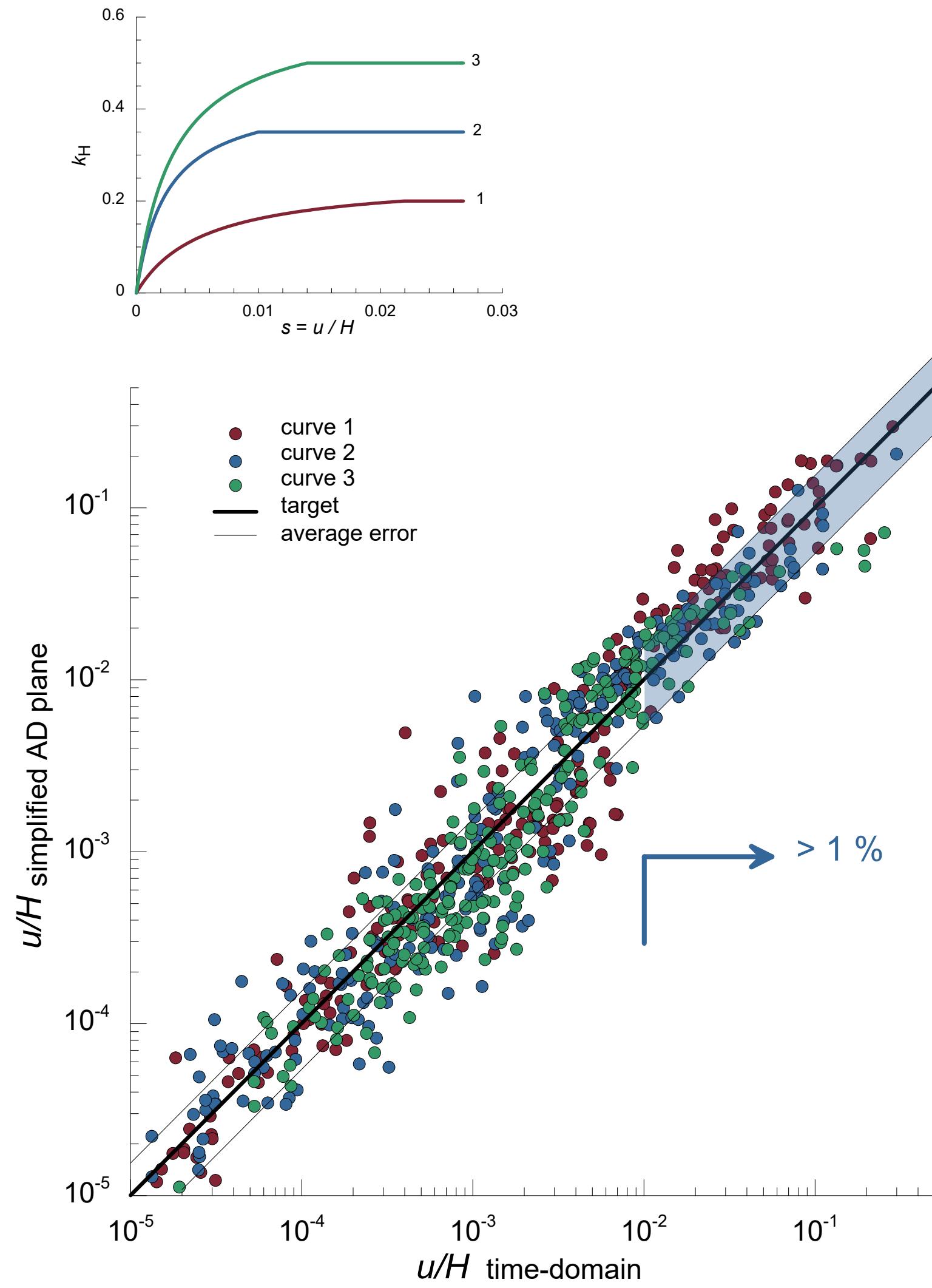


$$u = u_I + N_{\text{eq}} u_{II}$$

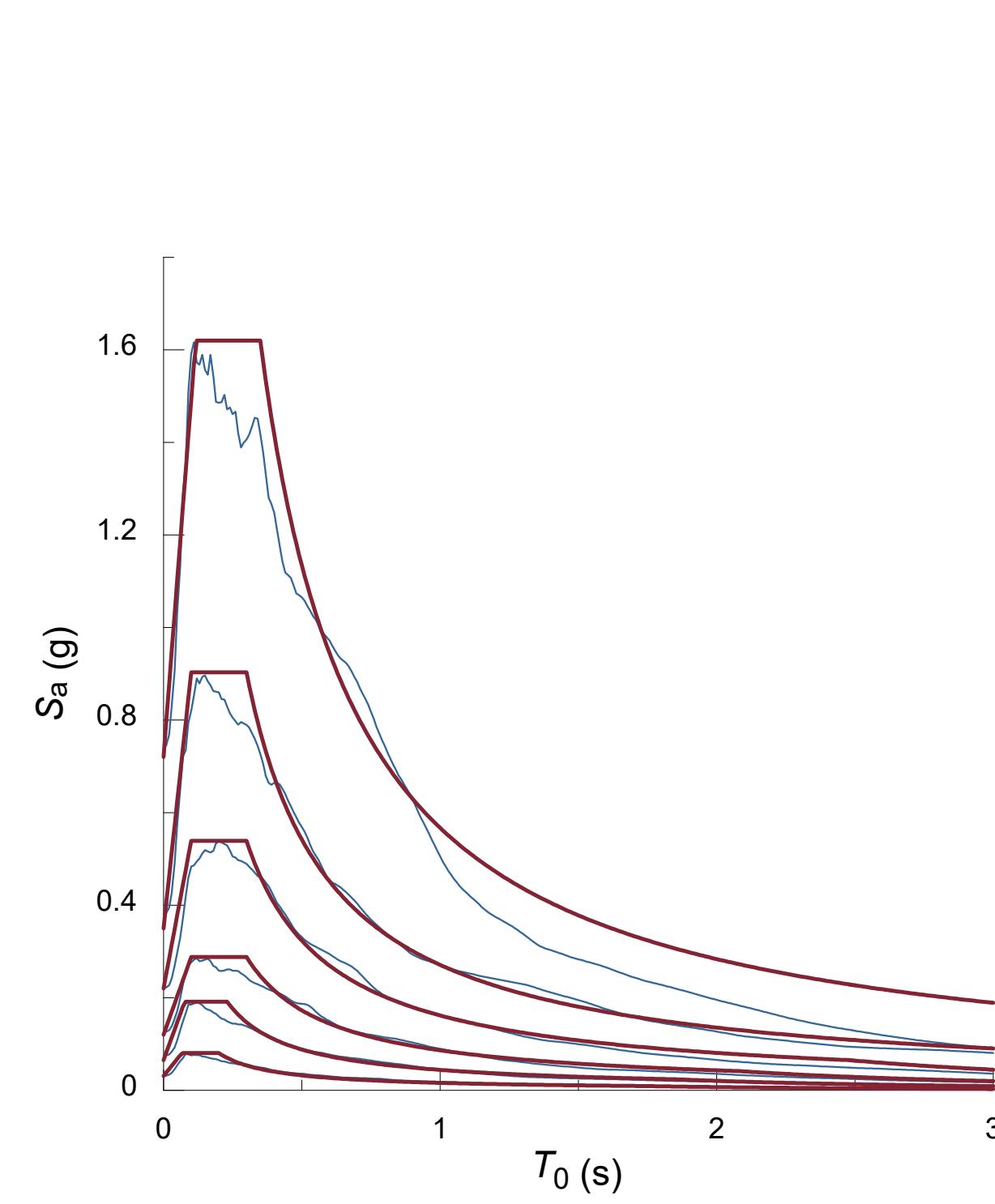


SIMBAD database
(Smerzini et al. 2014)
~ 500 time histories

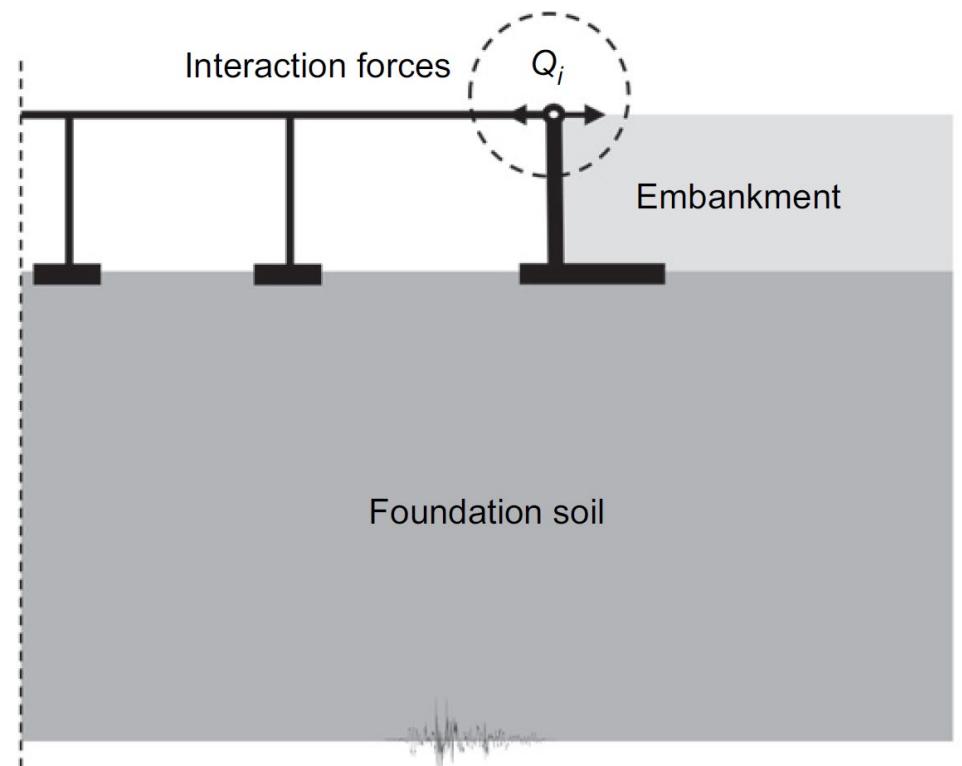
validation (entire database)



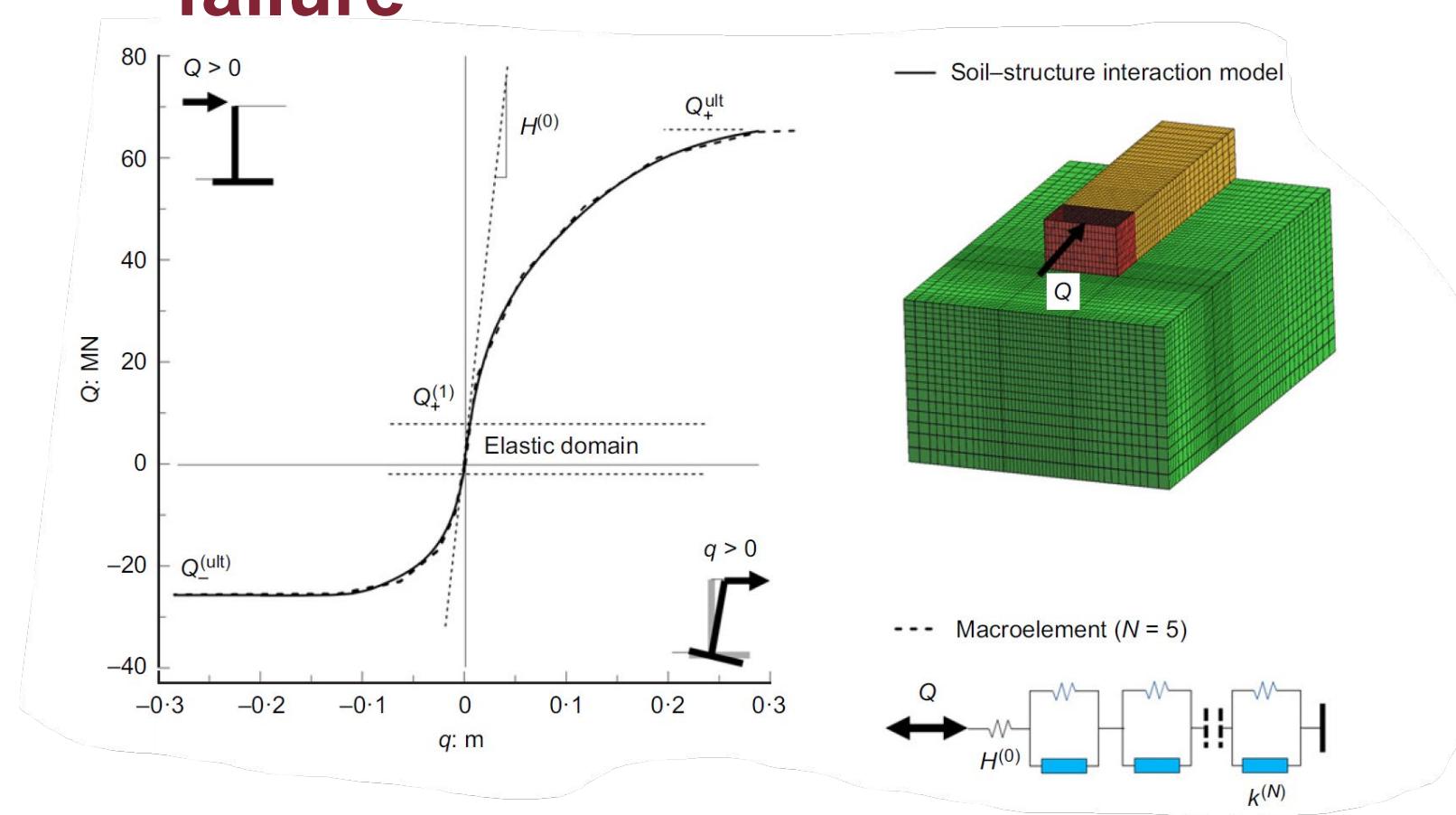
validation (code spectra)



more complex model, similar ingredients

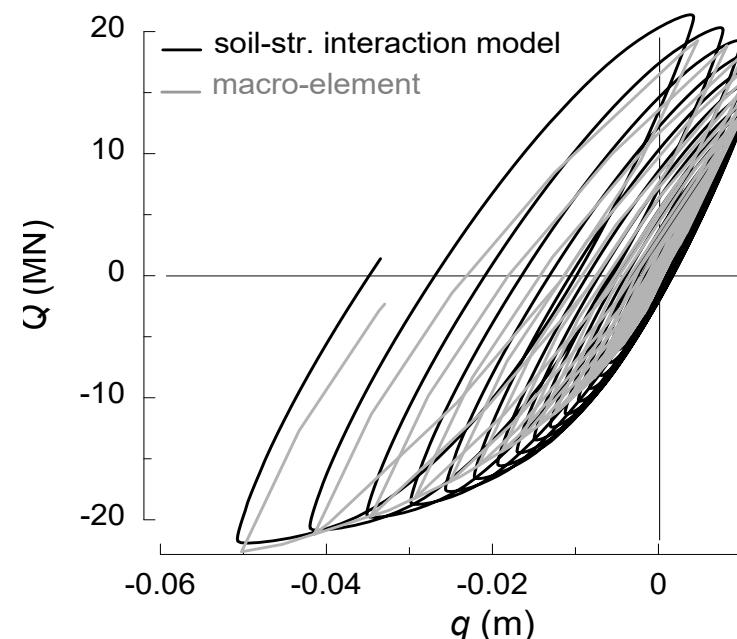
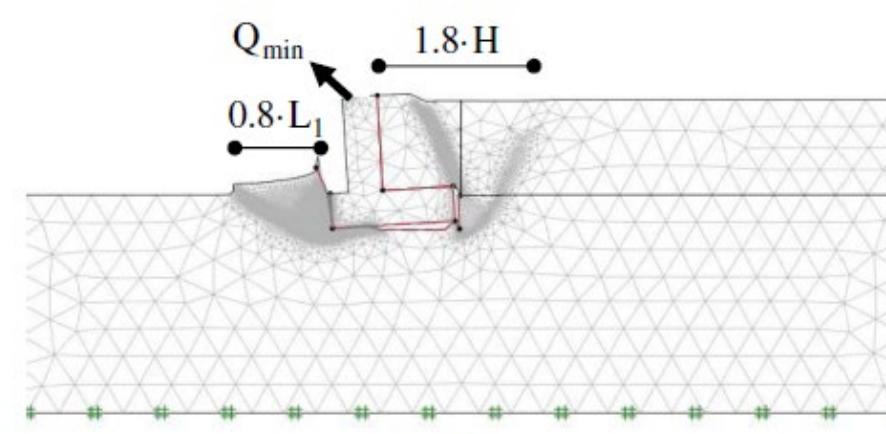
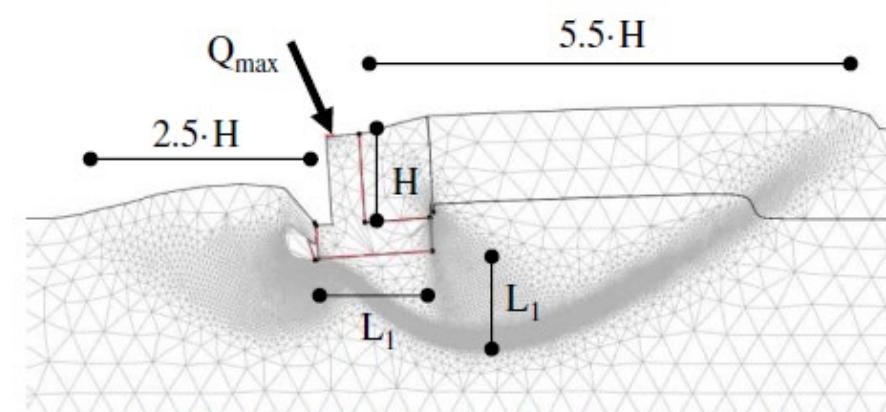
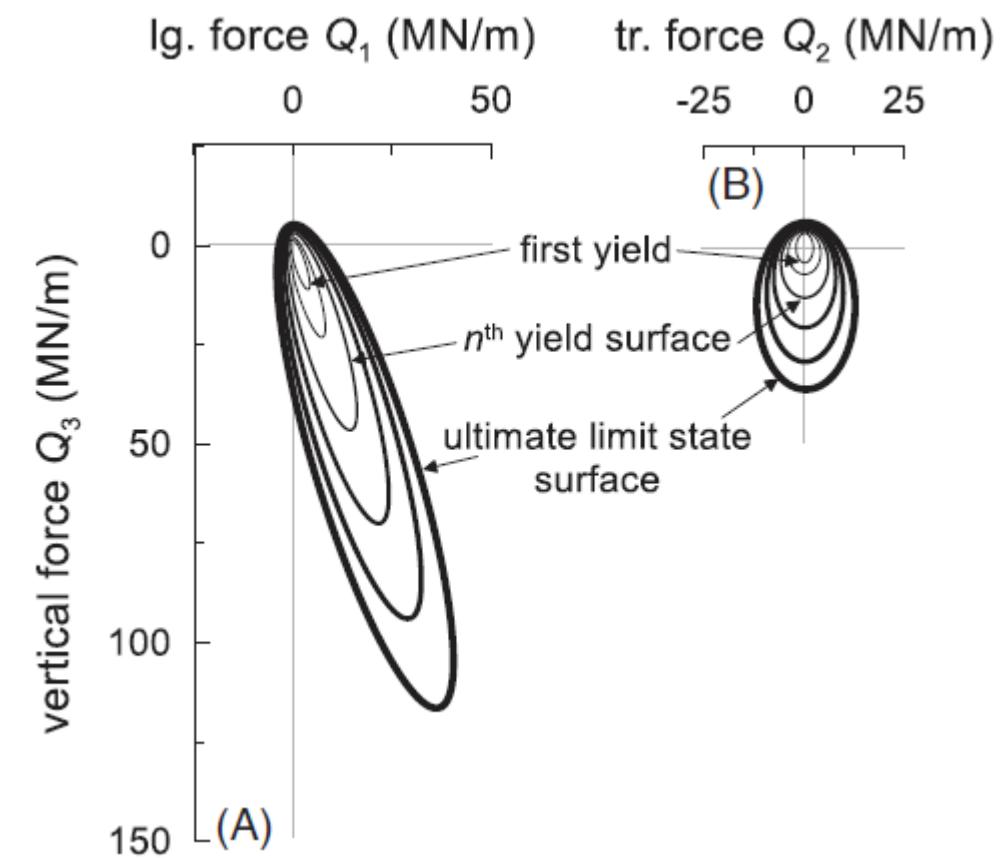


transition to failure

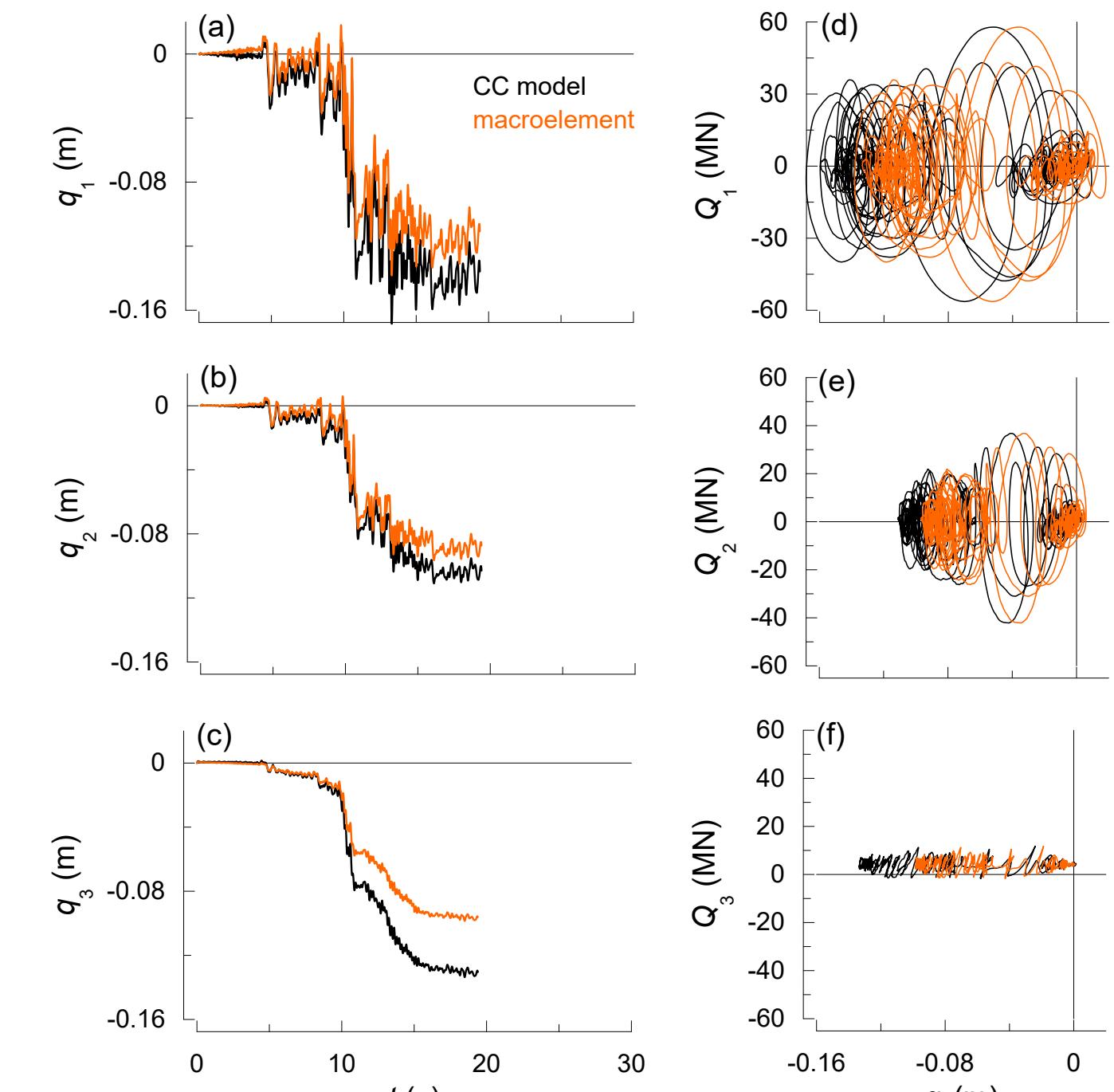


cyclic response

capacity



validation



Callisto, Gorini, Whittle
(2020-2023)

conclusions

- soil strength relevant to the actual behaviour of retaining structures
- pre-failure response essential, but related to capacity
- system capacity essential to develop macro-models for prediction of displacement and internal forces
- ability to predict capacity
 - we owe to the *Essai*

