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COMITÉ FRANÇAIS DE MÉCANIQUE DES SOLS ET DE GÉOTECHNIQUE

ACADÉMIE DES SCIENCES INSTITUT DE FRANCE

ACADÉMIE DES TECHNOLOGIES

SIMSG ISSMGE

LES TRAVAUX PUBLICS FÉDÉRATION NATIONALE

Charles-Augustin COULOMB - A geotechnical tribute

Paris, september 25 & 26, 2023

1773
Coulomb's memoir on Soil Mechanics

Jean Salençon

BUREAU VERITAS Shaping a World of Trust · Geos · SoilCloud · SOLETANCHE BACHY · spie batignolles fondations · SSI 2U · terrasol setec

Jean LEHUEROU KERISEL (1908-2005)

ДВУХСОТЛЕТИЕ МЕМУАРА 1773 г.
ШАРЛЯ ОГЮСТЕНА КУЛОНА

Проф. Ж. Керизель. Париж, Франция

BICENTENAIRE DE L,ESSAI DE 1773
CHARLES AUGUSTIN COULOMB

J.Kerisel – Professeur Honoraire. Paris, France

BICENTENARY OF THE 1773 PAPER
OF CHARLES AUGUSTIN COULOMB

J.Kerisel, Prof. Emeritus, Paris, France.

VIII
1973
МОСКВА

1773
Coulomb's Memoir on Soil Mechanics…

Jean Salençon

Académie des sciences
Académie des technologies, Paris
Senior Fellow HKIAS, City University, Hong Kong

250th anniversary of its presentation
to the French Academy of Sciences.
A most popular landmark text
in civil and construction engineering.

1773
Coulomb's Memoir on Soil Mechanics…

Charles-Augustin Coulomb
(1736 – 1806)

Member of the French *Académie des sciences*
« Section de mécanique » (1795)

"This Memoir, written some years ago, was only intended for my own personal use in the many works I am in charge of. If I now dare to present it to the Academy, it is because it always kindly welcomes the weakest attempt, when it has utility as its object."

1773
Coulomb's Memoir
on Soil Mechanics…

Charles-Augustin Coulomb
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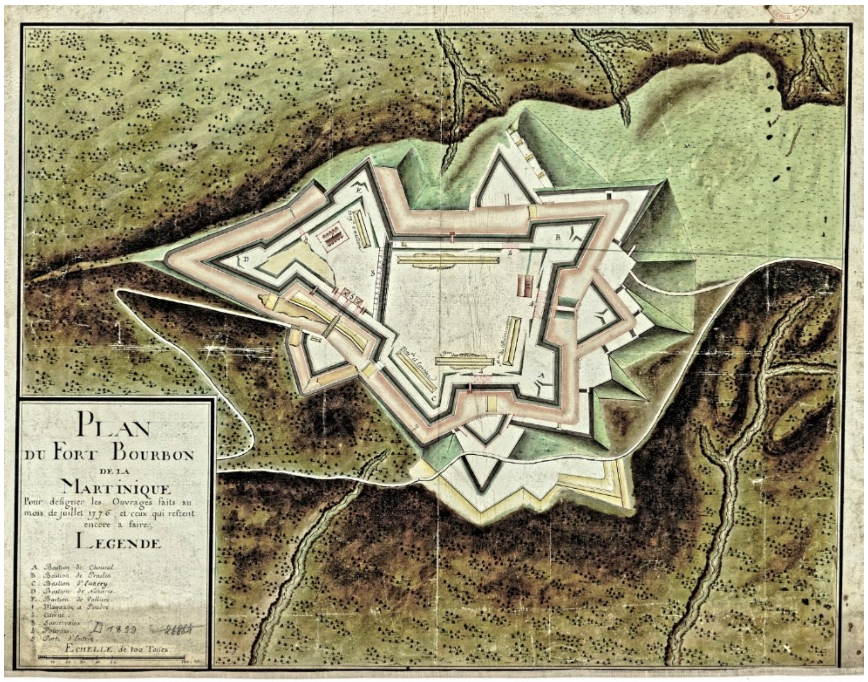

Fort Bourbon
As of 1776

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1773
Coulomb's Memoir on Soil Mechanics…

E S S A I

Sur une application des règles de Maximis & Minimis *à quelques Problèmes de Statique, relatifs à l'Architecture.*

Par M. COULOMB, Ingénieur du Roi.

N O T E

On an application of the Rules of* Maxima & Minima *to some Statics Problems, relevant to Architecture.

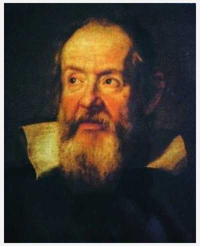
Par M. COULOMB, Ingénieur du Roi.

- Coulomb's memoir scientific temporality
- Friction & Cohesion in Coulomb's Memoir
- Coulomb's Approach to Active and Passive Thrusts
- Beyond Coulomb's Analysis
- A Theory of Yield Design
- Yield Design or Plastic Limit Analysis?
- Conclusion

Coulomb's Memoir
Scientific Temporality

Coulomb's Memoir

Scientific Temporality

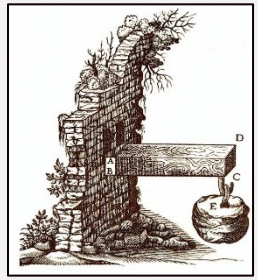


Galileo Galilei
(1564 – 1642)

DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuove scienze
Attenenti alla
MECANICA & i MOVIMENTI LOCALI;
del Signor
GALILEO GALILEI LINCEO,
Filosofo e Matematico primario del Sereniſſimo
Grand Duca di Toſcana.
Con una Appendice del centro di gravità d'alcuni Solidi.

IN LEIDA,
Appreſſo gli Elſevirii. M. D. C. XXXVIII.

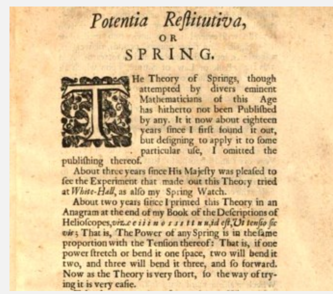
(1638)



Discorsi

Elasticity: Hooke's Law

Robert Hooke
(1635 - 1703)



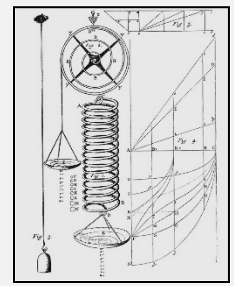
Potentia Reſtitutiva,
OR
SPRING.

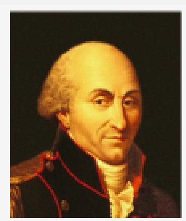
The Theory of Springs, though attempted by divers eminent Mathematicians of this Age has hitherto not been Publiſhed by any. It it now about eighteen years ſince I firſt found it out, but deſigning to apply it to ſome particular uſe, I omitted the publiſhing thereof.

About three years ſince His Majeſty was pleaſed to ſee the Experiment that made out this Theory tried at *White-Hall*, as alſo my Spring Watch.

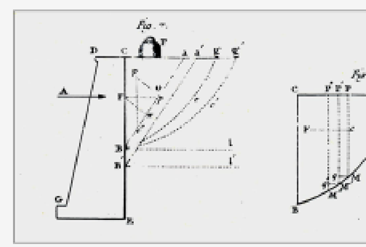
About two years ſince I printed this Theory in an Anagram at the end of my Book of the Deſcriptions of Helioſcopes, *viz. c e i i i n o s s s t t u u, id eſt, Ut tenſio ſic vis*; That is, The Power of any Spring is in the ſame proportion with the Tenſion thereof: That is, if one power ſtretch or bend it one ſpace, two will bend it two, and three will bend it three, and ſo forward. Now as the Theory is very ſhort, ſo the way of trying it is very eaſie.

(1678)





Charles-Augustin Coulomb
(1736 -1806)

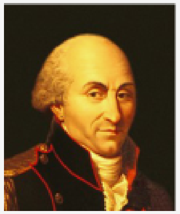


(1773)

Coulomb's Memoir

Scientific Temporality

Stability Analysis



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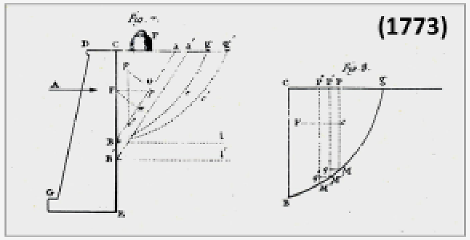
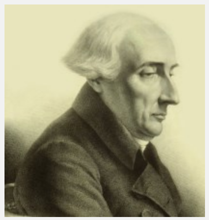


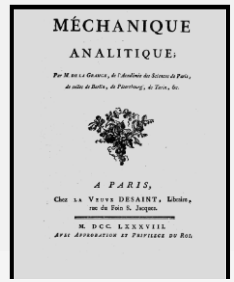
(1773)

Principle of virtual velocities

Resisting forces



J-L Lagrange
(1736 – 1813)

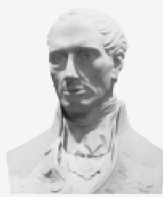


MÉCHANIQUE
ANALITIQUE.
A PARIS,
M. DCC. LXXXVIII.

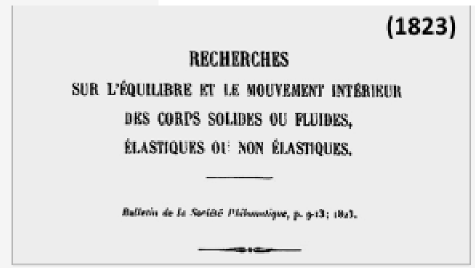
1788

Stress vector

Stress tensor



Augustin Louis Cauchy
(1789 -1



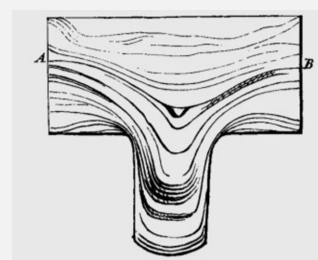
RECHERCHES
SUR L'ÉQUILIBRE ET LE MOUVEMENT INTÉRIEUR
DES CORPS SOLIDES OU FLUIDES,
ÉLASTIQUES OU NON ÉLASTIQUES.

Bulletin de la Société Philomatique, p. 9-13; 1823.

(1823)

Fluidity of solids

Henri-Édouard Tresca
(1814 – 1885)



(1864 - 1870)

COMITÉ FRANÇAIS DE MÉCANIQUE DES SOLS ET DE GÉOTECHNIQUE

Friction & Cohesion in Coulomb's Memoir

Coulomb's concept of Resistance

"Friction and cohesion are not active forces such as gravity that always fully exerts its effect, but only coercive forces; those two forces are assessed through their limits of resistance".

Du Frottement.

Le frottement & la cohéſion ne ſont point des forces actives comme la gravité, qui exerce toujours ſon effet en entier, mais ſeulement des forces coërcitives; l'on eſtime ces deux forces par les limites de leur réſiſtance. Lorſqu'on

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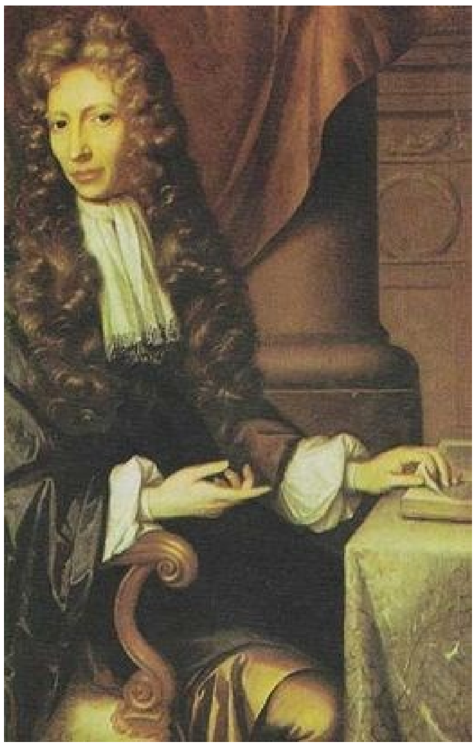
Friction

Coulomb refers to Amontons (1663-1705)

Member of the
Académie des sciences
(1699)



122 MEMOIRES DE L'ACADEMIE ROYALE

Par ces experiences on peut remarquer, en paſſant, que c'eſt une erreur de croire, que les frotemens dans les machines augmentent ou diminuent à proportion que les parties qui frottent, ont plus ou moins d'étenduë, & que la rouë, par exemple, d'un moulin tourne d'autant plus facilement, que ſes tourrillons ont moins de longueur, ce qui d'ailleurs eſt une mauvaiſe conſtruction, à cauſe qu'ils mangent incontinent les boëtes dans quoi ils tournent. Mais que ces frottemens augmentent ou diminuent à proportion des fardeaux qui ſont mûs, & de la raiſon de la longueur des leviers qui ſervent à les mouvoir à la longueur de ceux ſur leſquels ils s'appuïent.

Amontons G., 1699

Moyen de substituer commodément l'action du feu, à la force des hommes et des chevaux pour mouvoir les Machines. *Mémoires de l'Académie Royale des sciences, année* 1699, pp. 112-126.

Friction

Coulomb refers to Amontons

(1663-1705)

Member of the
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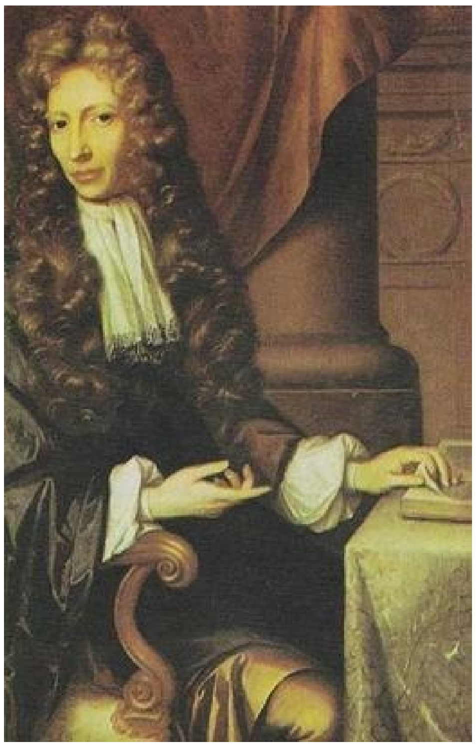


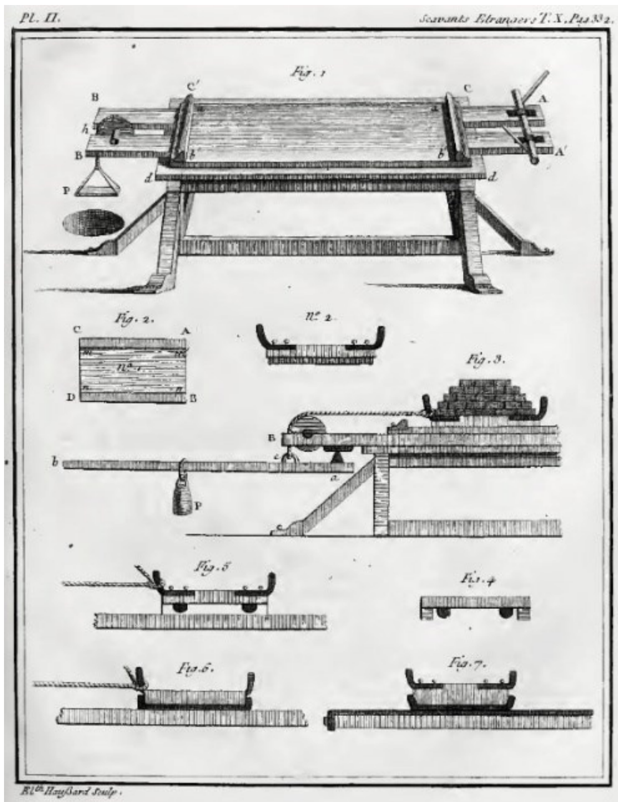
**"The resistance due to friction
is proportional to the pressure exerted"**

P coſ. x; & comme le frottement eſt proportionnel à la preſſion, il ſera égal à $\frac{P \text{coſ.} x}{n}$, n étant une quantité conſtante;

**friction is proportional
to the pressure exerted**

$$1/n = \tan\phi$$

Coulomb's own researches on friction would be presented 8 years later



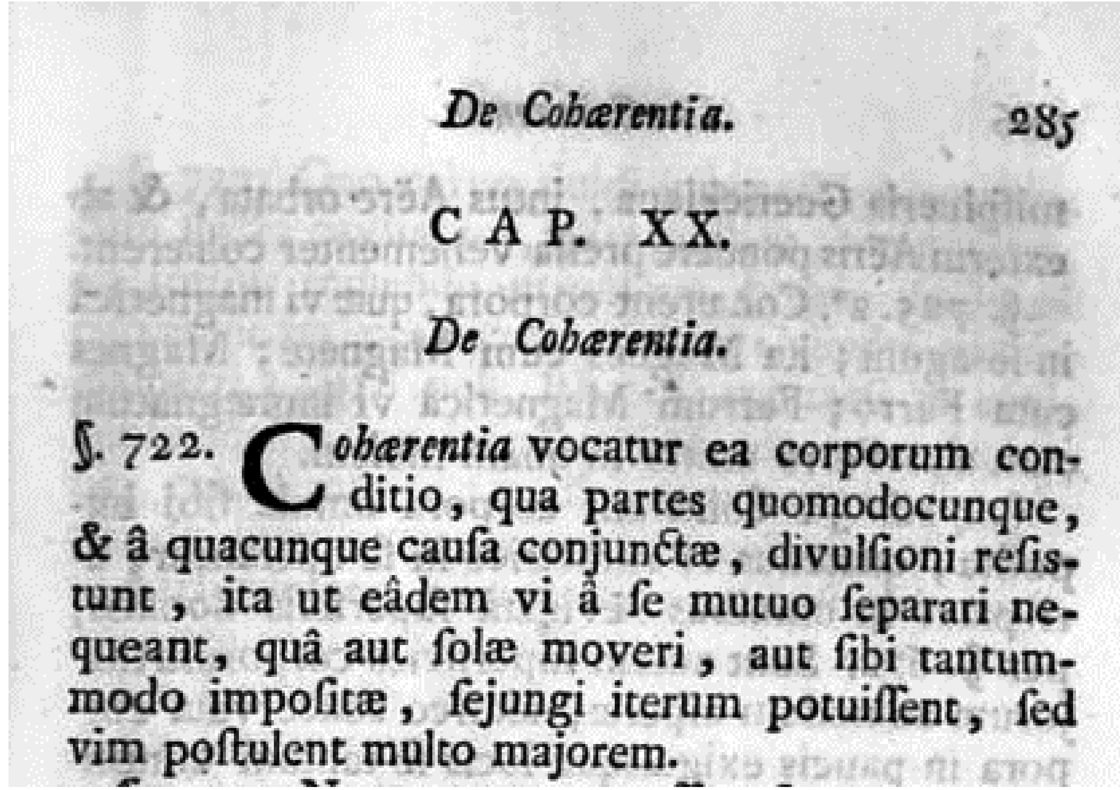
Coulomb C.-A, 1781. Théorie des machines simples en égard au frottement de leurs parties, et à la roideur de leurs cordages. *Recueil des savants étrangers de l'Académie royale des sciences*, 10, 161-332.

Cohesion

Coulomb refers to

van Musschenbroek (1692-1761)

Corresponding Member of
the *Académie des sciences*
(1734)



De Cohærentia. 285

CAP. XX.

De Cohærentia.

§. 722. *Cohærentia* vocatur ea corporum conditio, qua partes quomodocunque, & â quacunque cauſa conjunctæ, divulſioni reſiſtunt, ita ut eâdem vi â ſe mutuo ſeparari nequeant, quâ aut ſolæ moveri, aut ſibi tantummodo impoſitæ, ſejungi iterum potuiſſent, ſed vim poſtulent multo majorem.

van Musschenbroek P., 1748

Institutiones Physicae conscriptae in usus academicos. Lugduni Batavorum apud Samuelem Luchtmans et filium, Leyden, 1748.

Cohesion

Coulomb refers to

van Musschenbroek (1692-1761)

Corresponding Member of
the *Académie des sciences*
(1734)

"Cohesion is measured by the resistance that solid bodies oppose to the direct disunity of their parts. Total cohesion is proportional … to the fracture surface. "

It recalls Galileo's statement

> "this consideration dealt with the resistance which all solids offer to fracture and depended upon a certain cement which held the parts glued together so that they would yield and separate only under considerable pull [*potente attrazzione*]."

Galileo Galilei, 1638. *Dialogues Concerning Two New Sciences.* Crew, H. & Salvio, A. transl., Dover, New York, 1954.

20

Cohesion

+

Friction

Two independent additive contributions to material resistance

van Musschenbroek

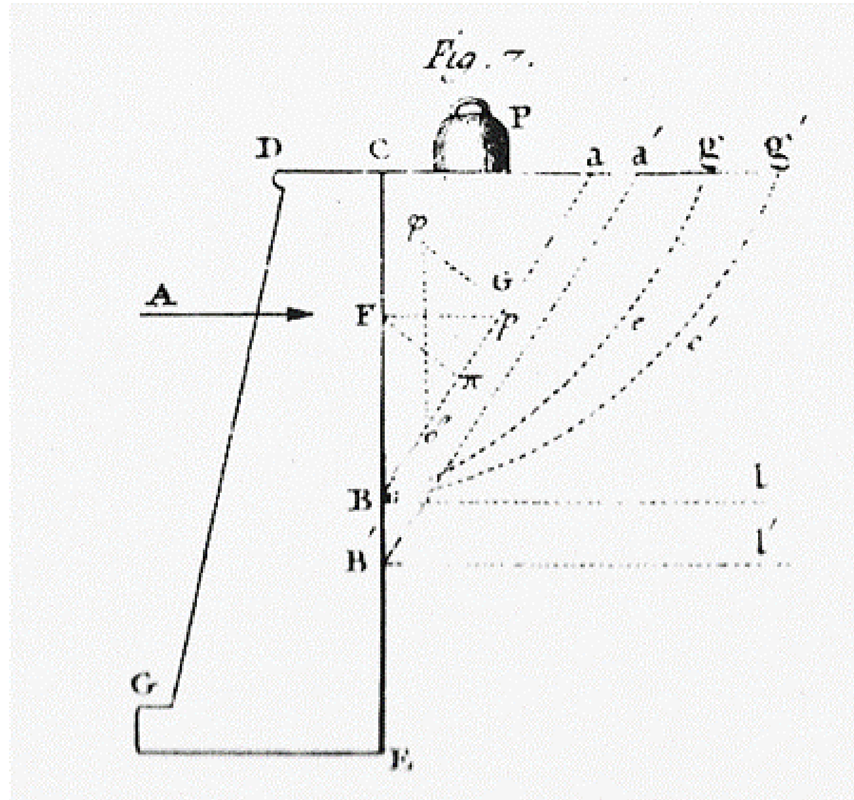
+

Amontons

COMITÉ FRANÇAIS DE MÉCANIQUE DES SOLS ET DE GÉOTECHNIQUE

Coulomb's Approach to Active and Passive Thrusts

ACTIVE and PASSIVE THRUSTS

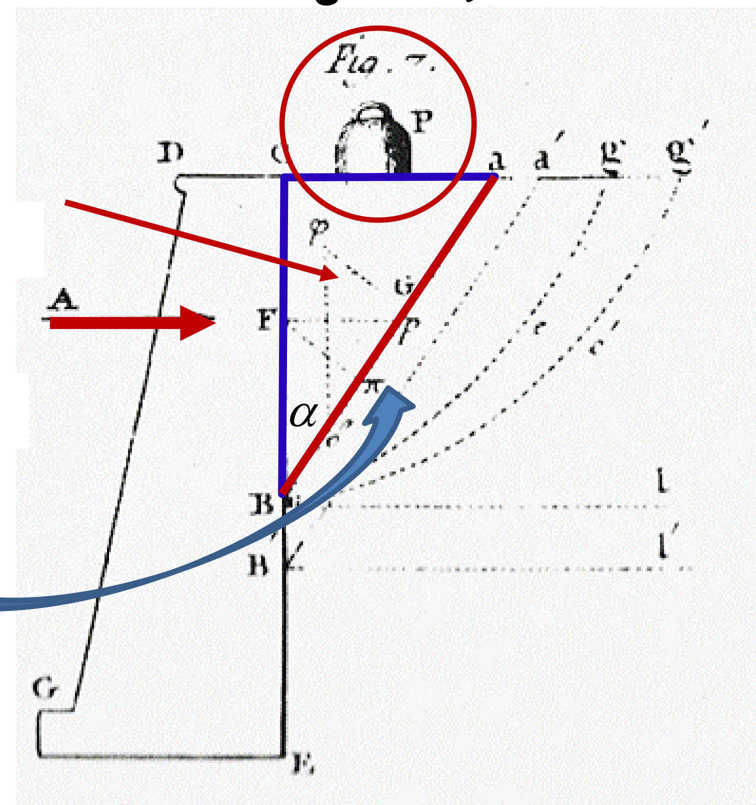


ACTIVE and PASSIVE THRUSTS

a thought experiment

A SOLID TRIANGLE

Resistance

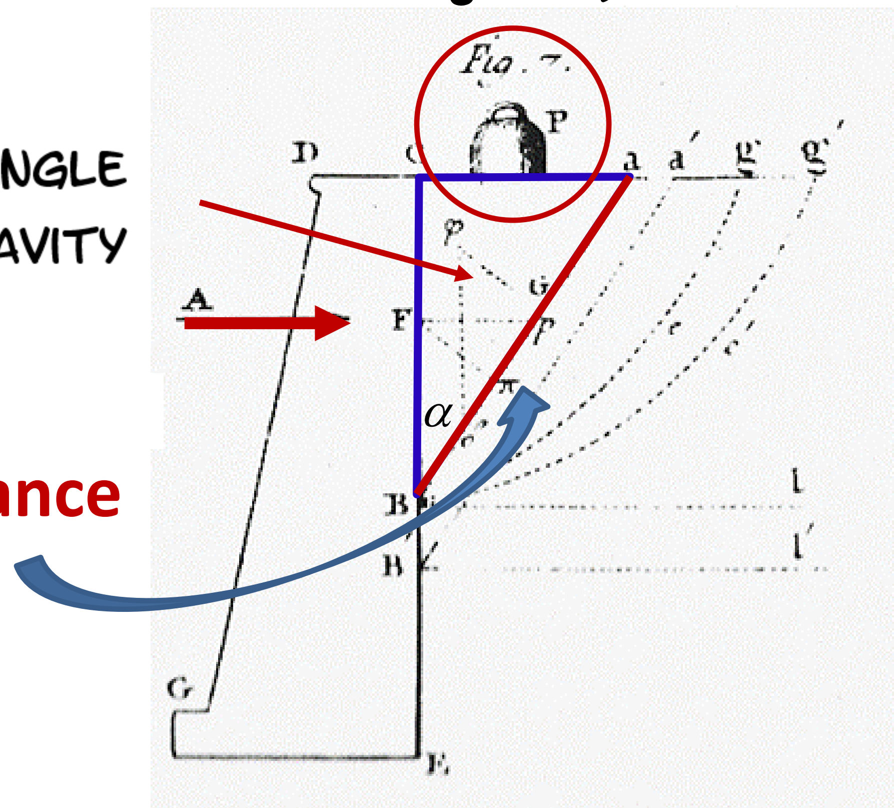


ACTIVE and PASSIVE THRUSTS

a thought experiment

A SOLID TRIANGLE
SUBJECTED TO GRAVITY

Resistance

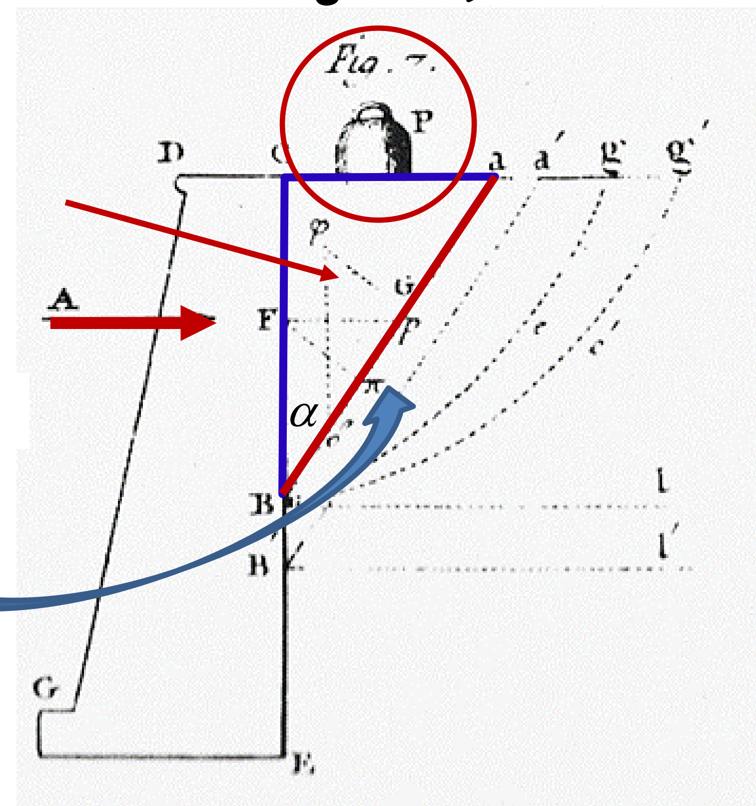


ACTIVE and PASSIVE THRUSTS

a thought experiment

A SOLID TRIANGLE
SUBJECTED TO GRAVITY
SUSTAINED BY FORCE A

Resistance

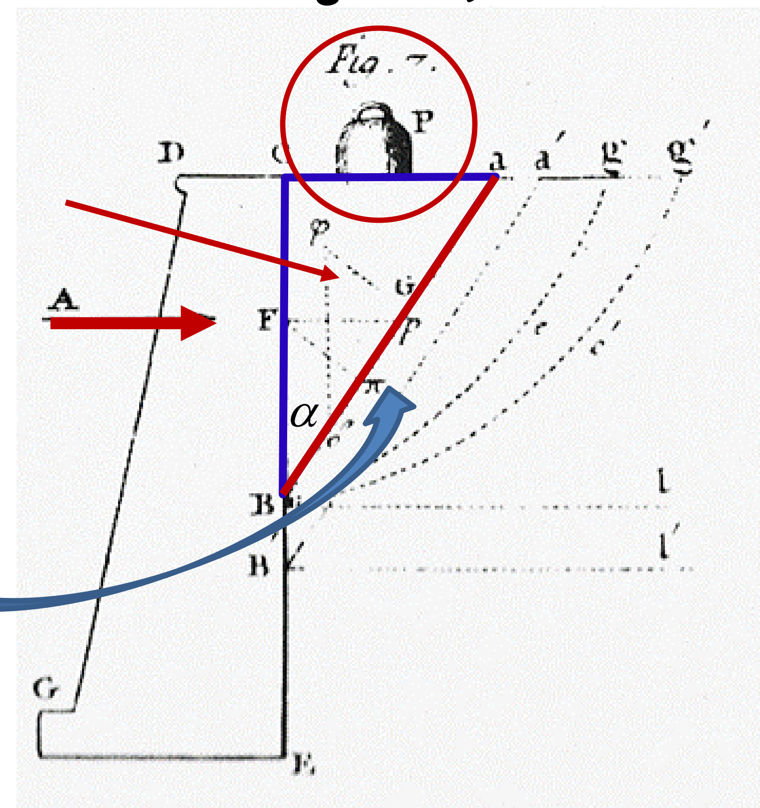


ACTIVE and PASSIVE THRUSTS

a thought experiment

A SOLID TRIANGLE
SUBJECTED TO GRAVITY
SUSTAINED BY FORCE A
AND RETAINED ALONG Ba

Resistance



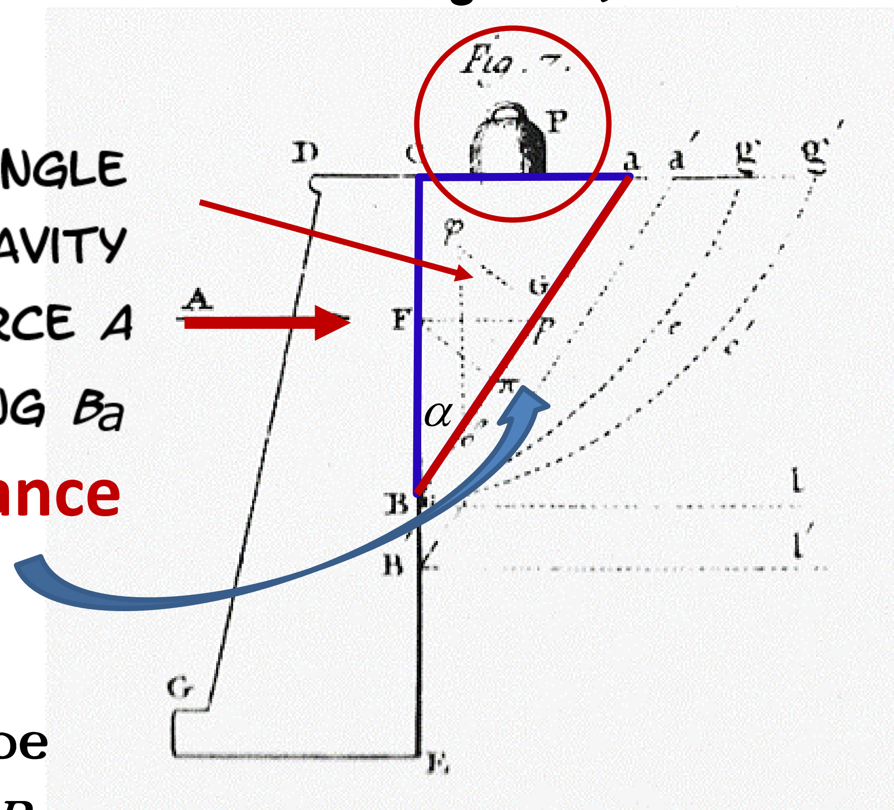
ACTIVE and PASSIVE THRUSTS

a thought experiment

Equilibrium

A SOLID TRIANGLE
SUBJECTED TO GRAVITY
SUSTAINED BY FORCE A
AND RETAINED ALONG Ba

Resistance



Assess the value of force A that can be exerted at point F, perpendicular to CB, without causing the triangle to move.

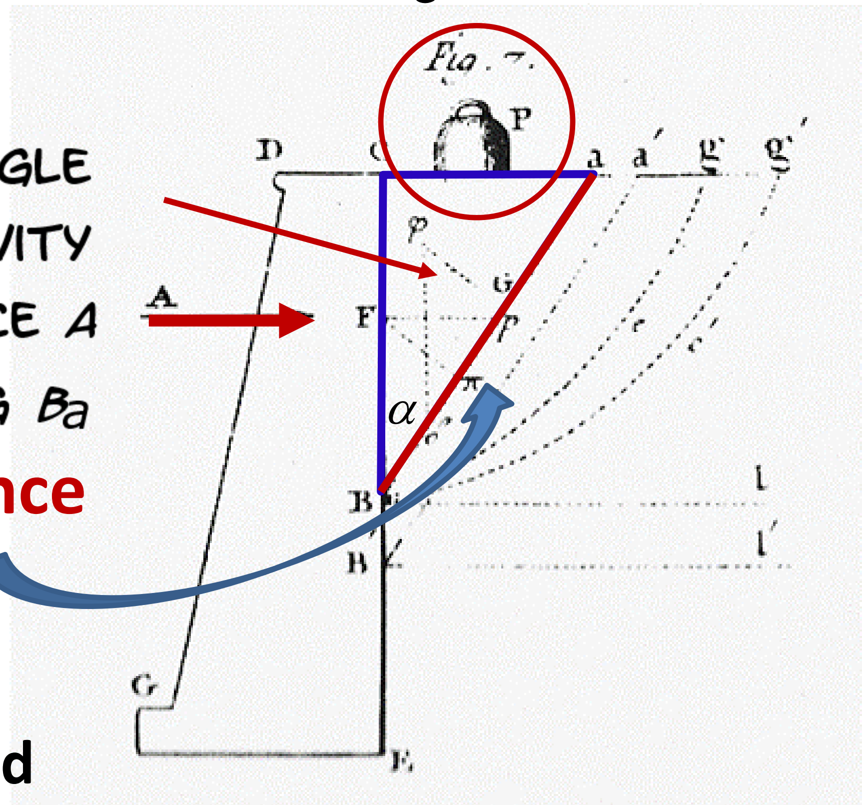
ACTIVE and PASSIVE THRUSTS

a thought experiment

Equilibrium

A SOLID TRIANGLE
SUBJECTED TO GRAVITY
SUSTAINED BY FORCE A
AND RETAINED ALONG Ba

Resistance

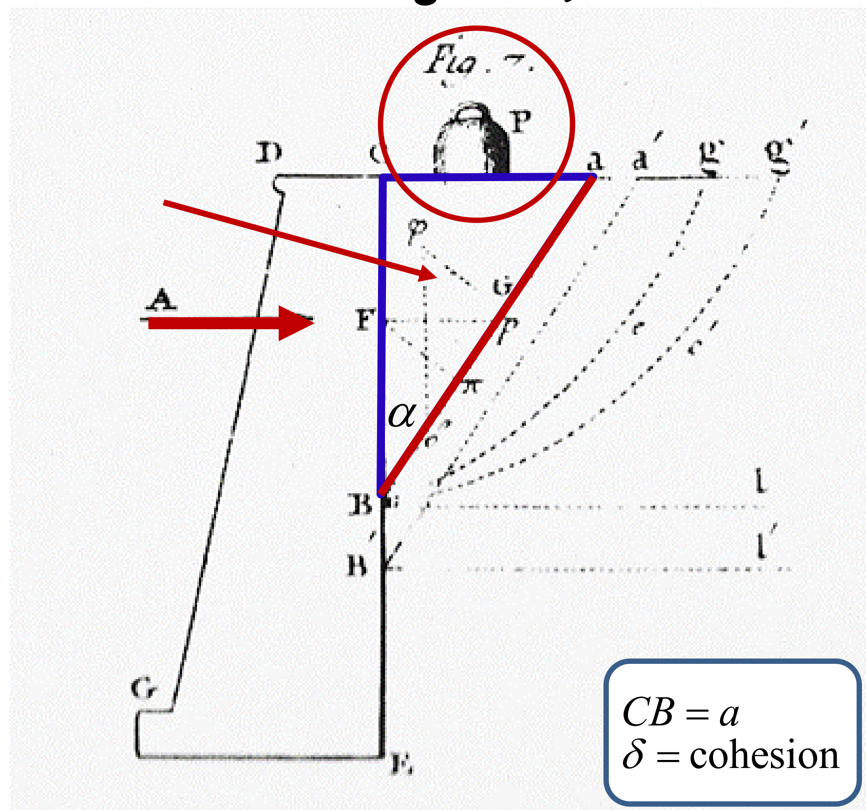


Force *A* prevents solid *Cba*
from "flowing" downwards

Resisting force shall be directed
upwards along *Ba*

ACTIVE THRUST

a thought experiment



Compatibility
Equilibrium
Resistance

Force *A* prevents solid *Cba* from "flowing" downwards

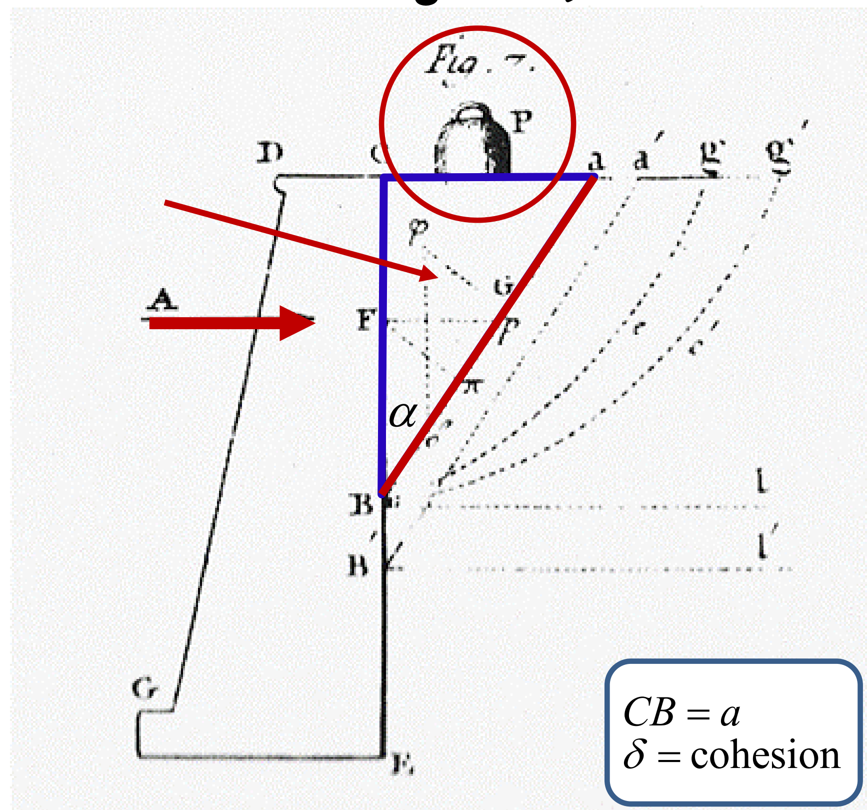
Resisting force fully mobilized upwards along *Ba*

$$A(\alpha) = \frac{1}{\tan\alpha + \tan\phi}\left(\gamma \frac{a^2}{2\tan\alpha}(1 - \tan\alpha\tan\phi) - \delta\frac{a}{\cos^2\alpha}\right)$$

ACTIVE THRUST

a thought experiment

Compatibility
Equilibrium
Resistance



Force *A* prevents solid *Cba* from "flowing" downwards

Resisting force fully mobilized upwards along *Ba*

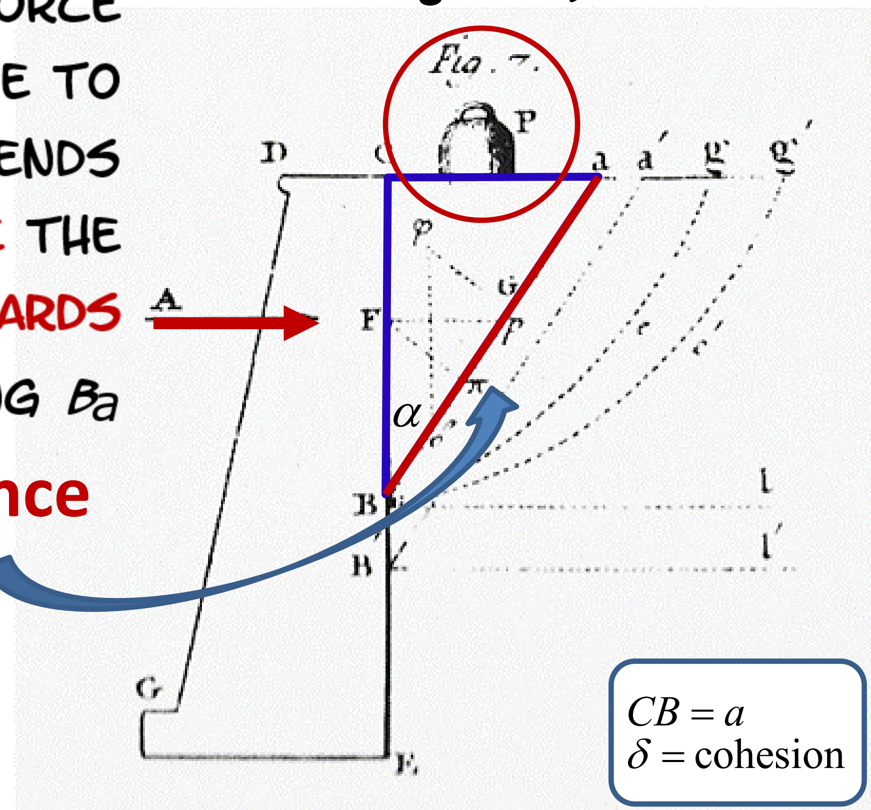
$$A \geq A(\alpha) = \frac{1}{\tan\alpha + \tan\phi}\left(\gamma \frac{a^2}{2\tan\alpha}(1 - \tan\alpha \tan\phi) - \delta \frac{a}{\cos^2\alpha}\right)$$

PASSIVE THRUST

a thought experiment

IF THE FORCE IN F IS MADE TO INCREASE AND TENDS TO IMPULSE THE TRIANGLE UPWARDS ALONG Ba

Resistance



Equilibrium

Resistance

Resisting force shall be directed downwards along *aB*

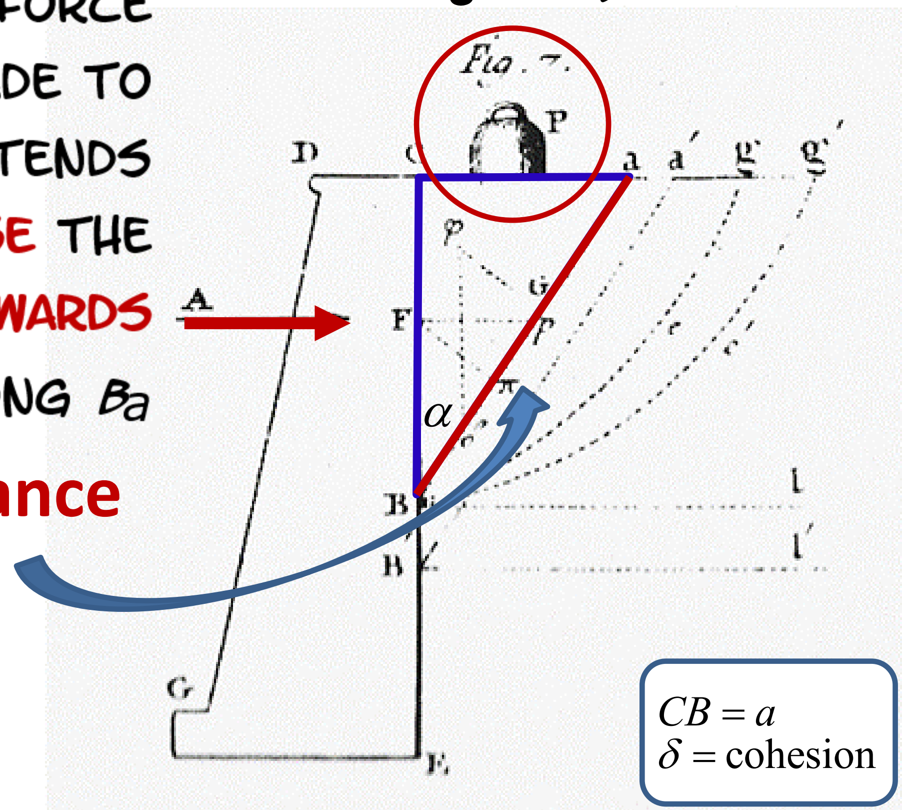
PASSIVE THRUST

Compatibility
Equilibrium
Resistance

IF THE FORCE IN *F* IS MADE TO INCREASE AND TENDS TO **IMPULSE** THE TRIANGLE **UPWARDS** ALONG *Ba*

Resistance

a thought experiment



Resisting force fully mobilized downwards along *aB*

$$A'(\alpha) = \frac{1}{\tan\alpha + \tan\phi}\left(\gamma \frac{a^2}{2\tan\alpha}(1 + \tan\alpha \tan\phi) + \delta \frac{a}{\cos^2\alpha}\right)$$

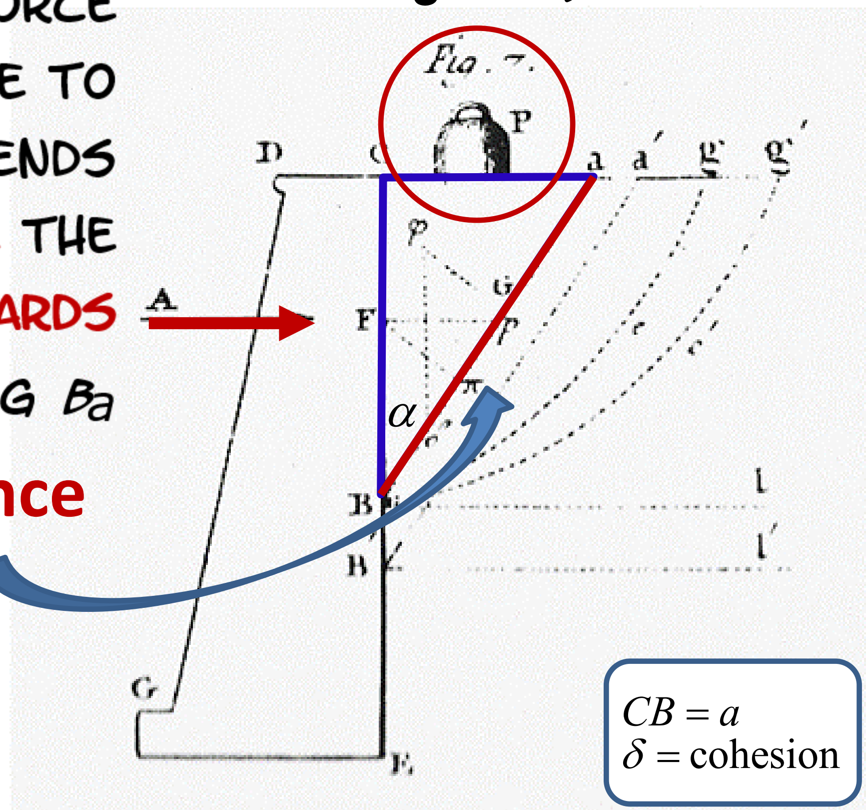
PASSIVE THRUST

Compatibility
Equilibrium
Resistance

IF THE FORCE IN F IS MADE TO INCREASE AND TENDS TO IMPULSE THE TRIANGLE UPWARDS ALONG Ba

Resistance

a thought experiment



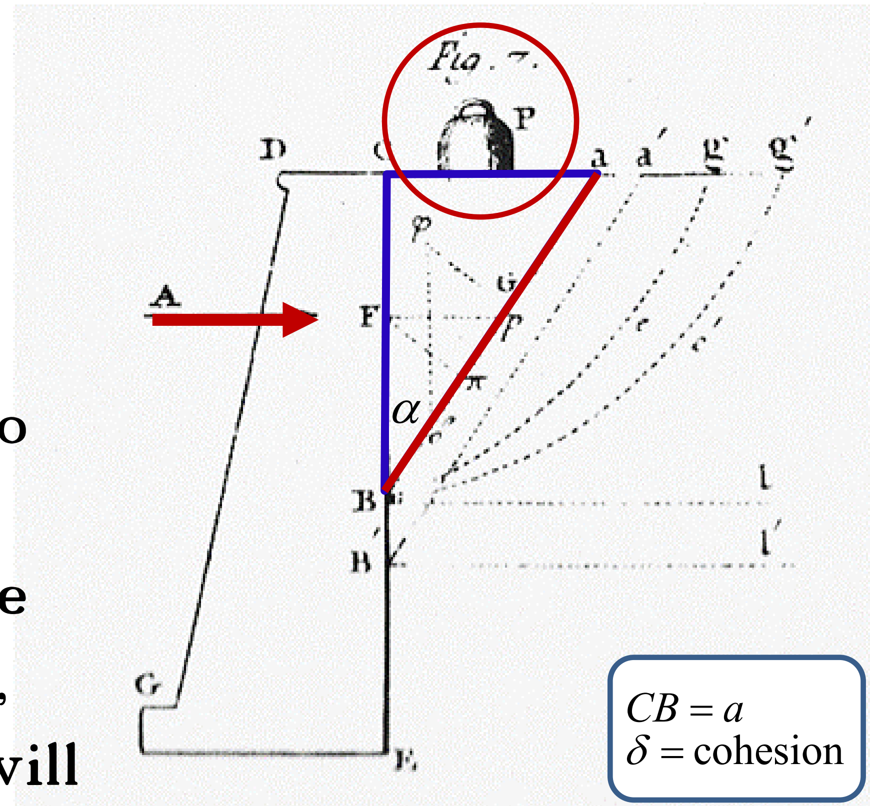
Resisting force fully mobilized downwards along *aB*

$$A \leq A'(\alpha) = \frac{1}{\tan\alpha + \tan\phi}\left(\gamma \frac{a^2}{2\tan\alpha}(1+\tan\alpha\tan\phi) + \delta\frac{a}{\cos^2\alpha}\right)$$

ACTIVE and PASSIVE THRUSTS

Compatibility
Equilibrium
Resistance

"It has just been proven that, when cohesion and friction both contribute to maintaining the triangle at rest, the limits to the value of a force that can be exerted at point *F*, perpendicular to *CB*, without causing the triangle to move, will lie between $A(\alpha)$ and $A'(\alpha)$."



ACTIVE and PASSIVE THRUSTS

Compatibility
Equilibrium
Resistance

N O T E

On an application of the Rules of Maxima & Minima *to some Statics Problems, relevant to Architecture.*

Par M. COULOMB, Ingénieur du Roi.

"It has just been proven that, when cohesion and friction both contribute to maintaining the triangle at rest, the limits to the value of a force that can be exerted at point F, perpendicular to CB, without causing the triangle to move, will lie between $A(\alpha)$ and $A'(\alpha)$."

ACTIVE and PASSIVE THRUSTS

"The limits of the horizontal force that can be exerted in F without setting the fluid in motion, will lie between the limits A and A', where A will be a maximum and A' a minimum,"

First appearance of the concept of estimates as a solution to a problem

ACTIVE THRUST

$$A(\alpha) = \frac{1}{\tan\alpha + \tan\phi}\left(\gamma \frac{a^2}{2\tan\alpha}(1 - \tan\alpha\tan\phi) - \delta\frac{a}{\cos^2\alpha}\right)$$

Maximized with respect to α

$$\alpha = \pi/4 - \phi/2$$

Coulomb

**A lower bound estimate of
the maximum pressure exerted on the wall**

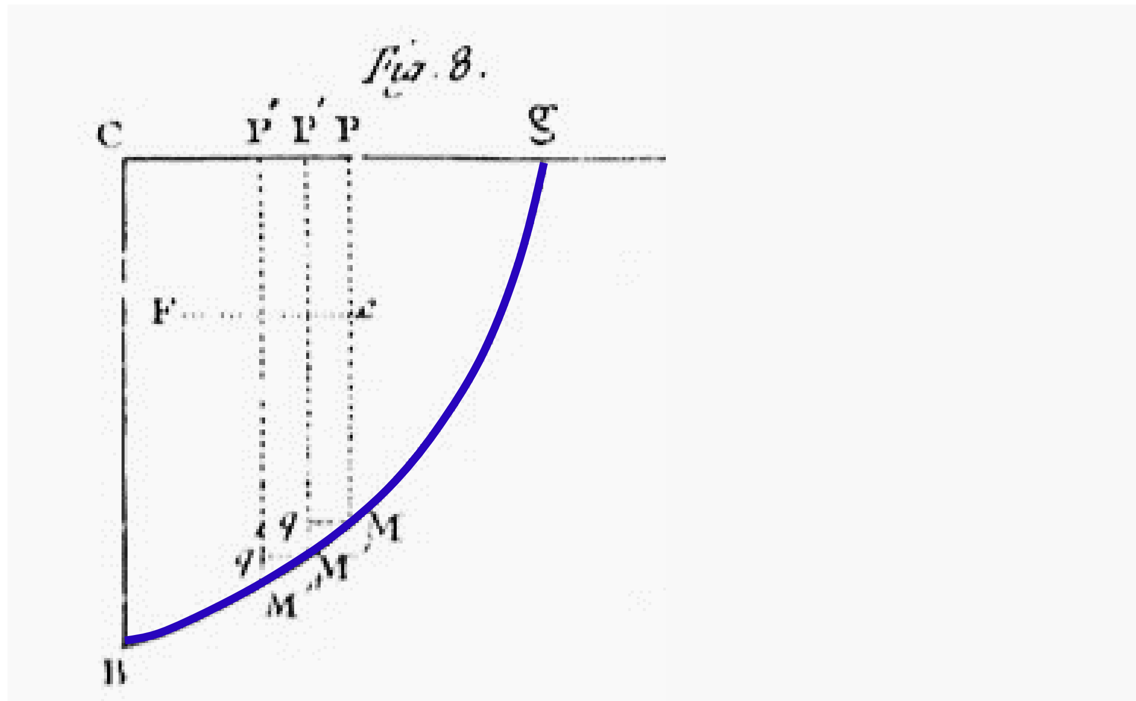
Non Triangular Wedges

"Up to now, we have assumed that the surface that produces the maximum pressure was triangular; the simplicity of the results derived from this assumption, their easy practical applicability and the desire to be useful and accessible to the Artists, are the reasons that decided for that choice; **but if we would want to determine the curve that produces the maximum pressure in an exact way, here is, I think, the way it could be done.**"

Non Triangular Wedges

Compatibility
Equilibrium
Resistance

A "slice-method"



Non Triangular Wedges

Compatibility
Equilibrium
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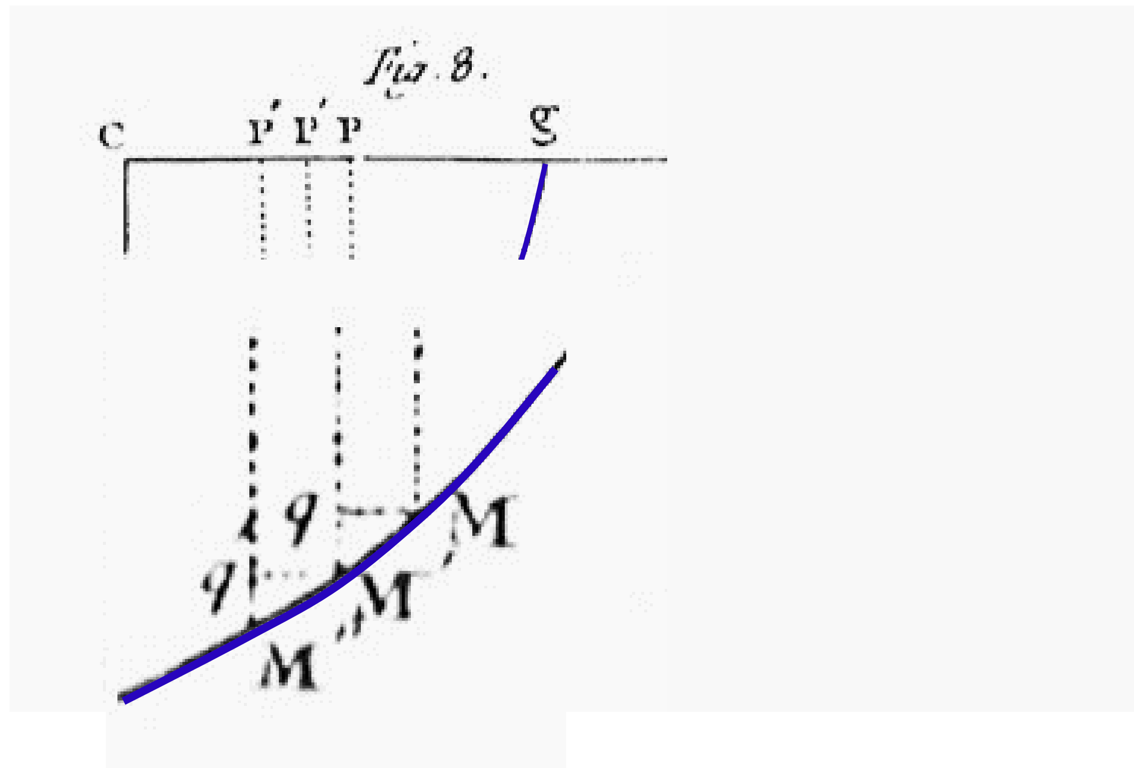
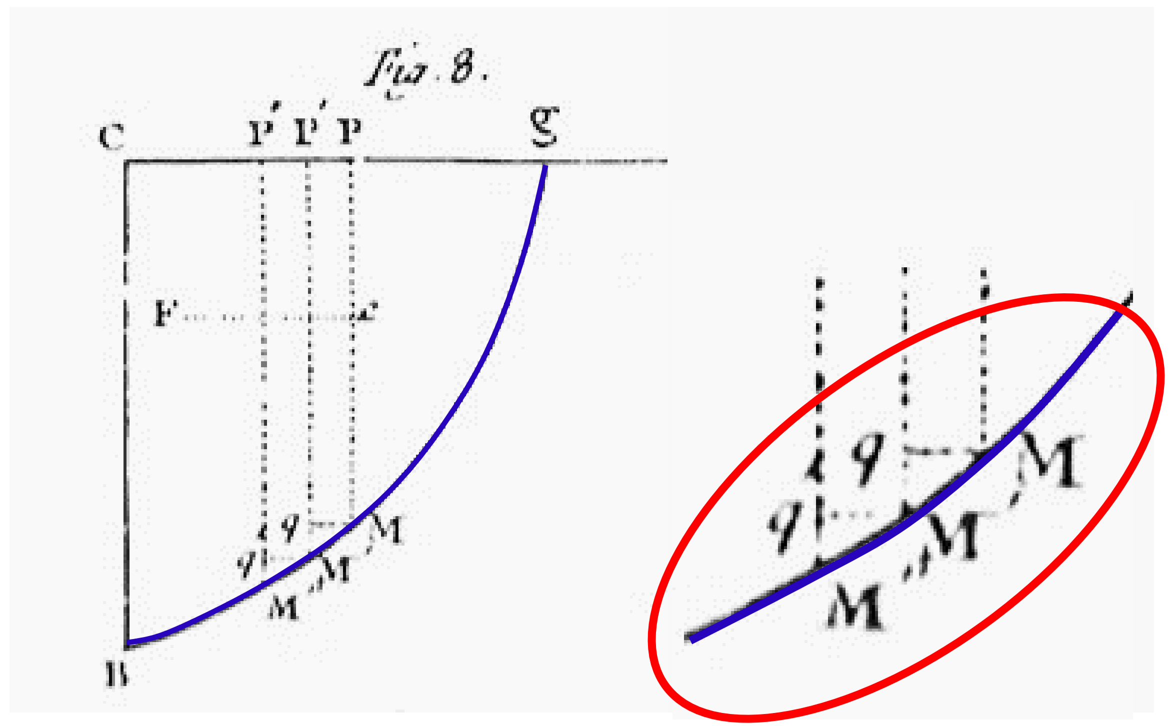
A "slice-method"



Non Triangular Wedges

Compatibility
Equilibrium
Resistance

A "slice-method"



PASSIVE THRUST

$$A'(\alpha) = \frac{1}{\tan\alpha + \tan\phi}\left(\gamma \frac{a^2}{2\tan\alpha}(1 + \tan\alpha \tan\phi) + \delta \frac{a}{\cos^2\alpha}\right)$$

Minimized with respect to α

$$\alpha = \pi/4 + \phi/2$$

$$\Downarrow$$

An upper bound estimate of
the resistance opposed by the soil mass to upward impulse

Beyond Coulomb's Analysis

Governing principles

1° Make a clear distinction between the imposed loads exerted on the structure under concern, and the resistances of the constituent materials that can be mobilized under the restrictions imposed by the limits of resistance.

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No energy concept involved

Following Coulomb's track

e.g.,

Fellenius slip-circle method (no complementary assumption for $\phi = 0$)

Rendulic log-spiral method (no complementary assumption)

Taylor's ϕ-circle **method**

Slice methods

. . .

often call for complementary assumptions about resisting forces …, to make the problem statically determinate

A kinematical or energy balance viewpoint

Governing idea

Checking a global equilibrium equation, is equivalent to writing that

the work by the "driving" forces exerted on the structure

can be balanced by

the work by resisting forces in the dual rigid body motion, under the constraints imposed by the material limit of resistance.

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A Theory of Yield Design

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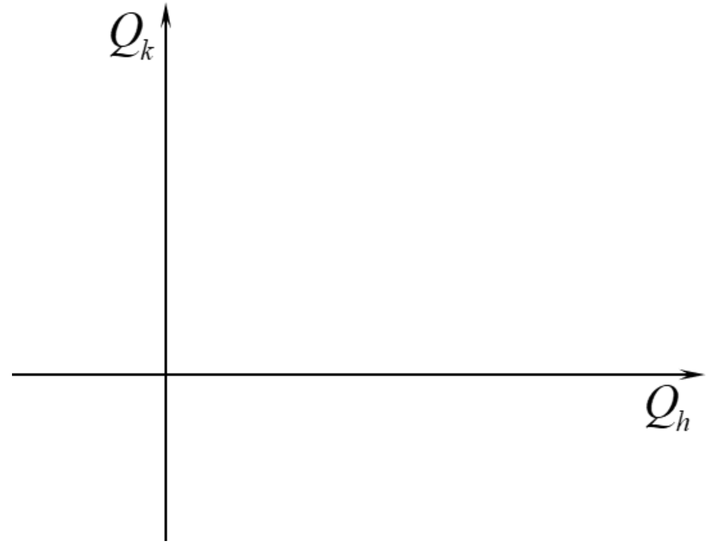
Domain of POTENTIAL STABILITY



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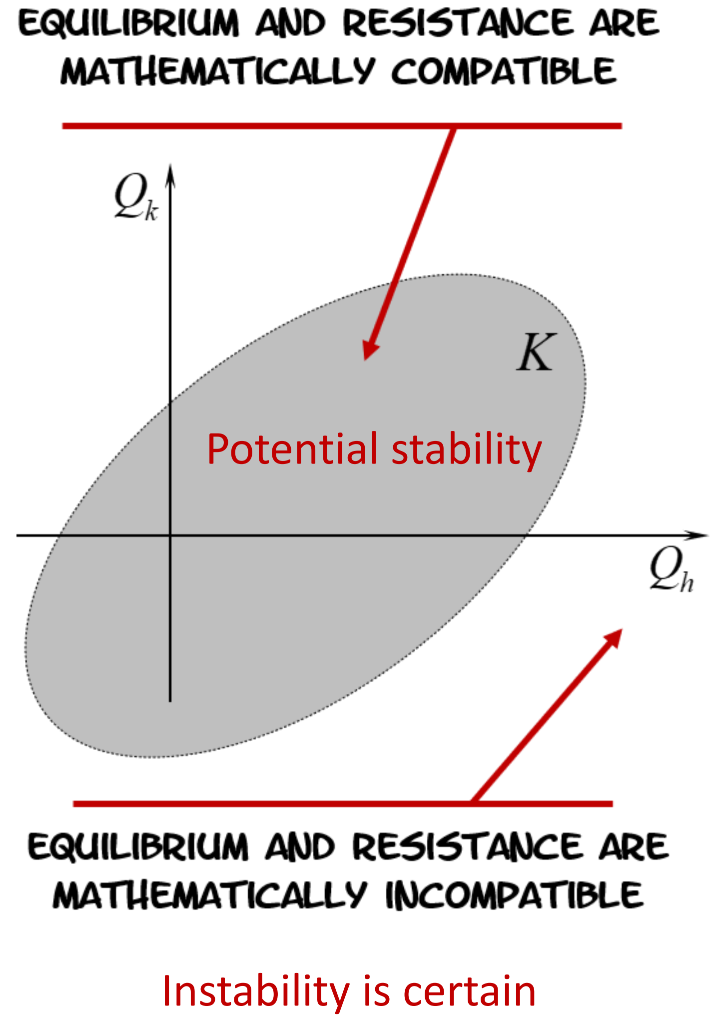

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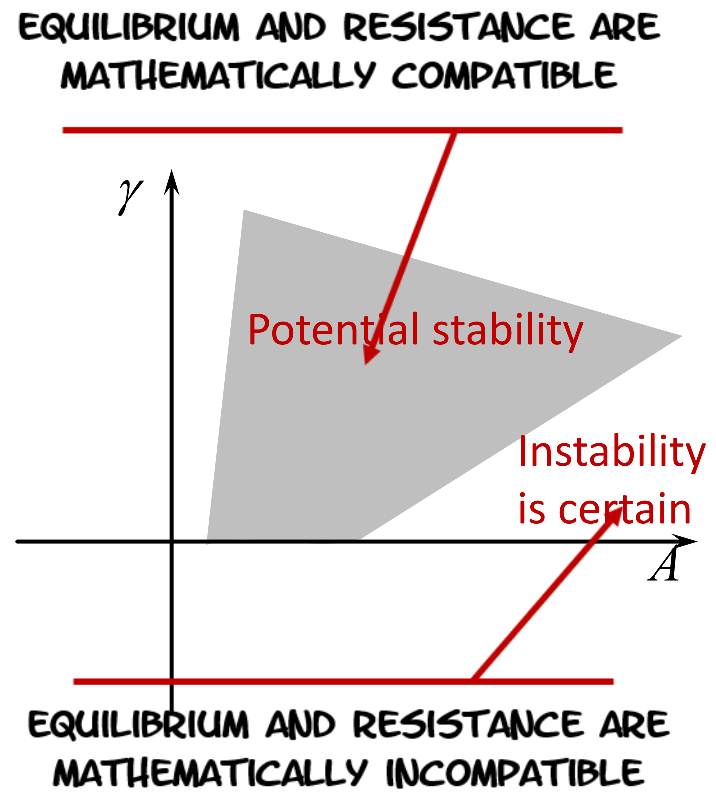

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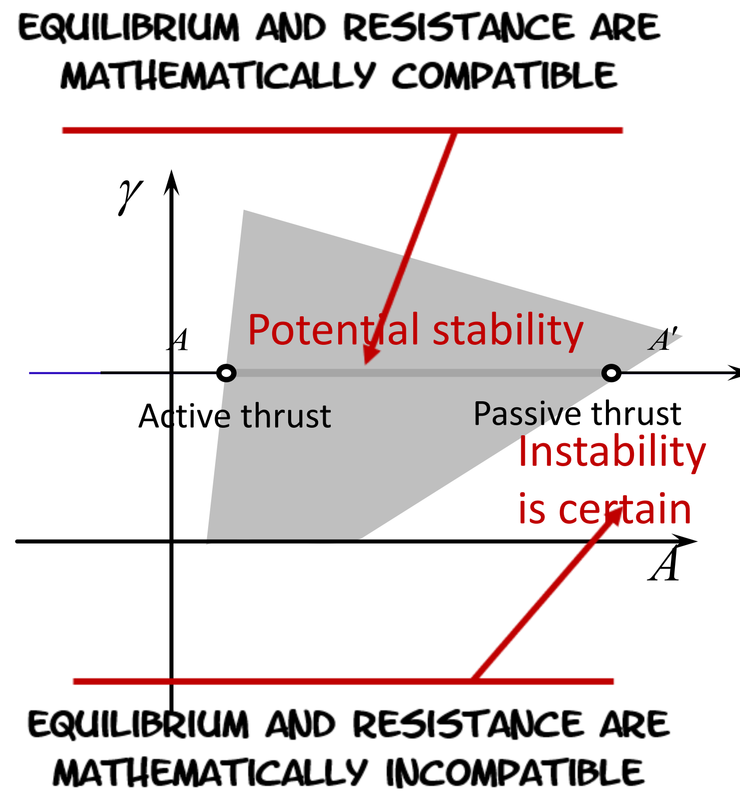

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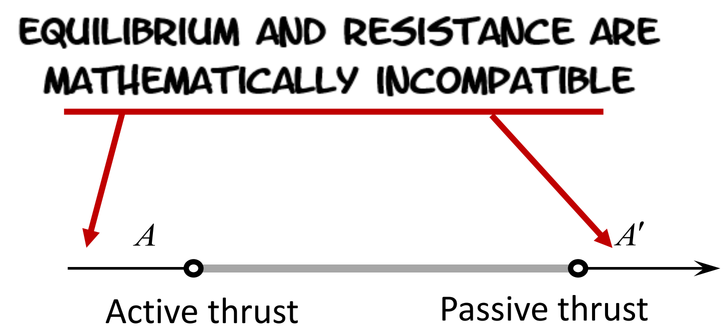

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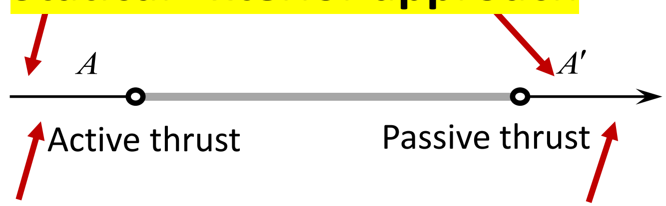
Domain of POTENTIAL STABILITY

EQUILIBRIUM AND RESISTANCE ARE MATHEMATICALLY INCOMPATIBLE

Statical Exterior approach



Coulomb's
Lower bound estimate
of Maximum pressure

Upper bound estimate
of Passive thrust

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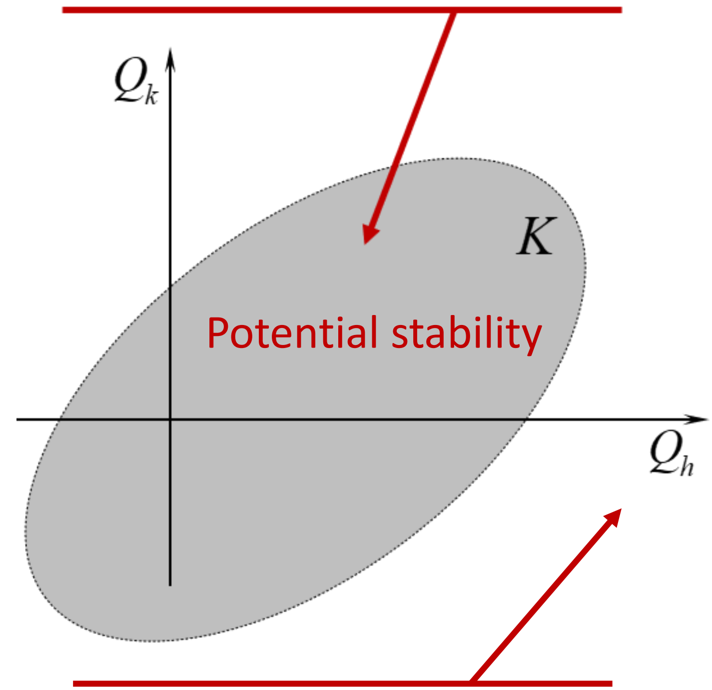
+

Principle of virtual work

EQUILIBRIUM AND RESISTANCE ARE MATHEMATICALLY COMPATIBLE



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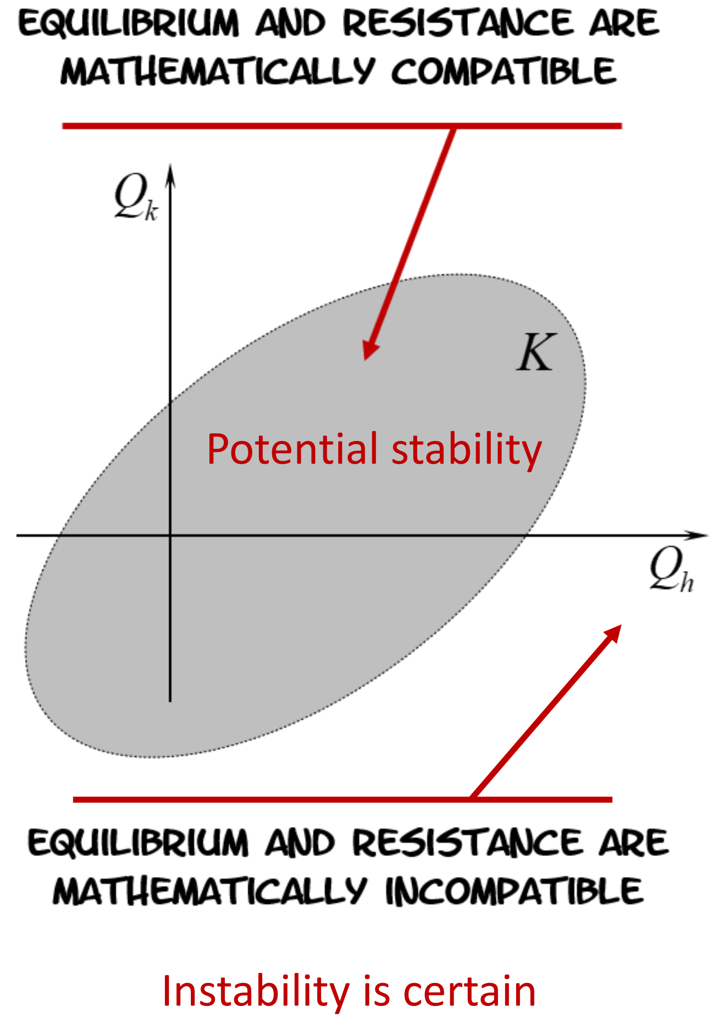

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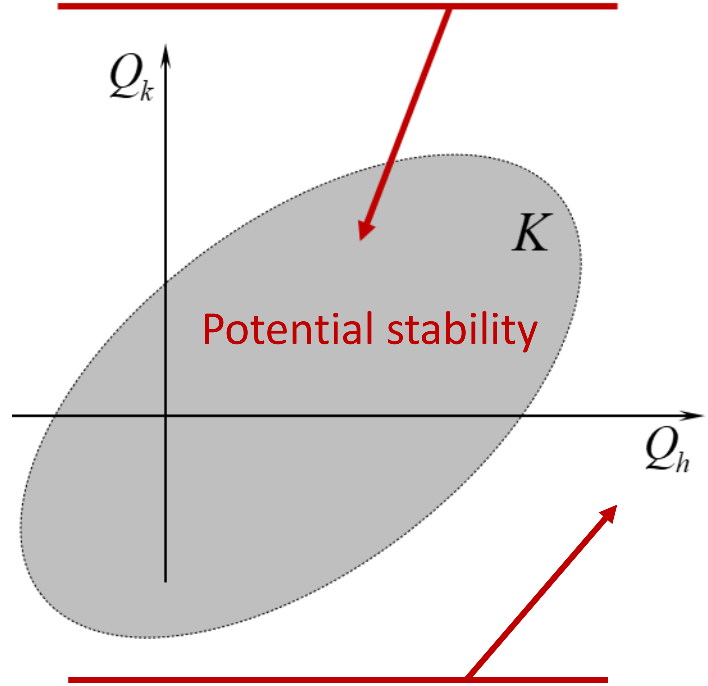

Given a potential collapse mechanism

Virtual work by external forces

>

MAXIMUM resisting work

EQUILIBRIUM AND RESISTANCE ARE MATHEMATICALLY INCOMPATIBLE

Instability is certain

Kinematical Exterior approach

Unambiguously mathematically defined
Does not call for any complementary assumptions

Given a potential collapse mechanism

Virtual work by external forces

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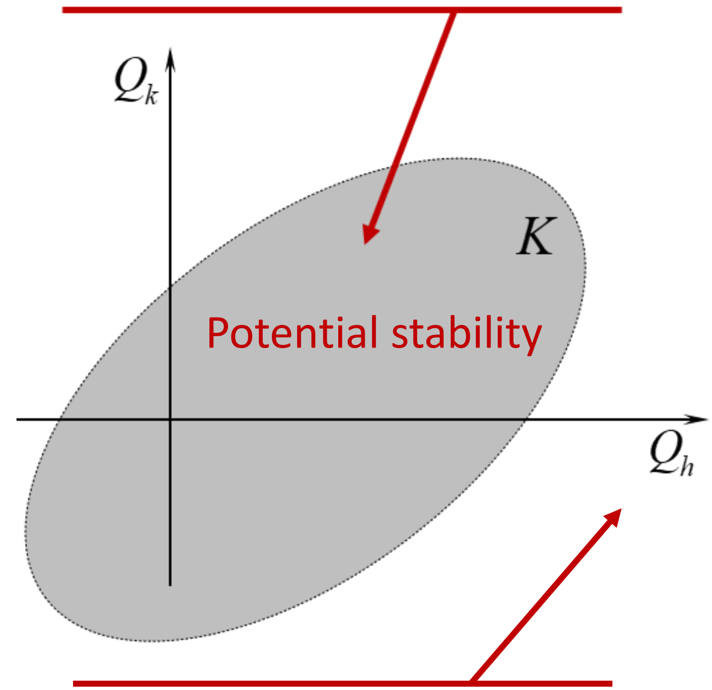

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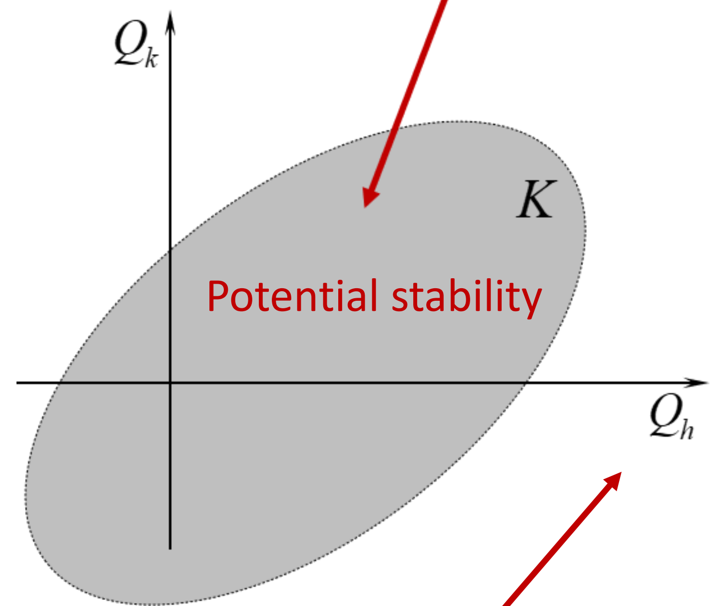
Instability is certain

Kinematical Exterior approach

EQUILIBRIUM AND RESISTANCE ARE MATHEMATICALLY COMPATIBLE

Mathematically derived from the material strength criterion

through a Normality rule



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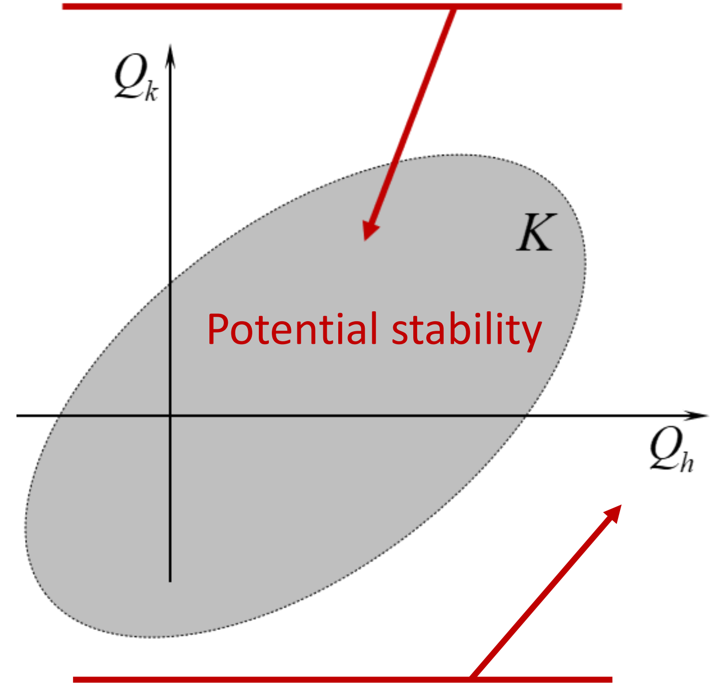

Mathematically derived from the material strength criterion

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Given a potential collapse mechanism

Virtual work by external forces

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MAXIMUM resisting work

Either finite or infinite, depending on the potential collapse mechanism

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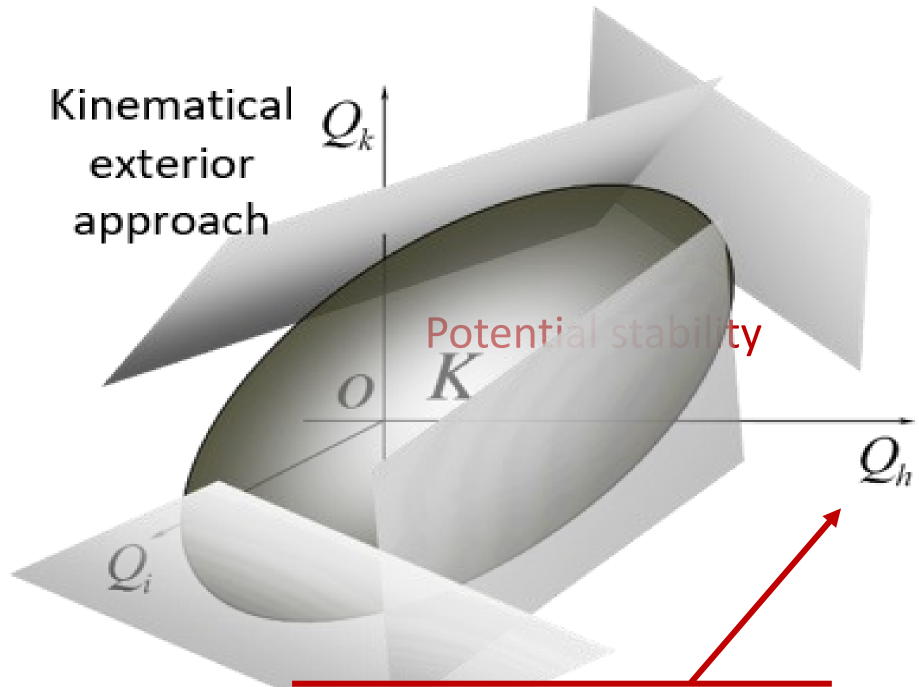

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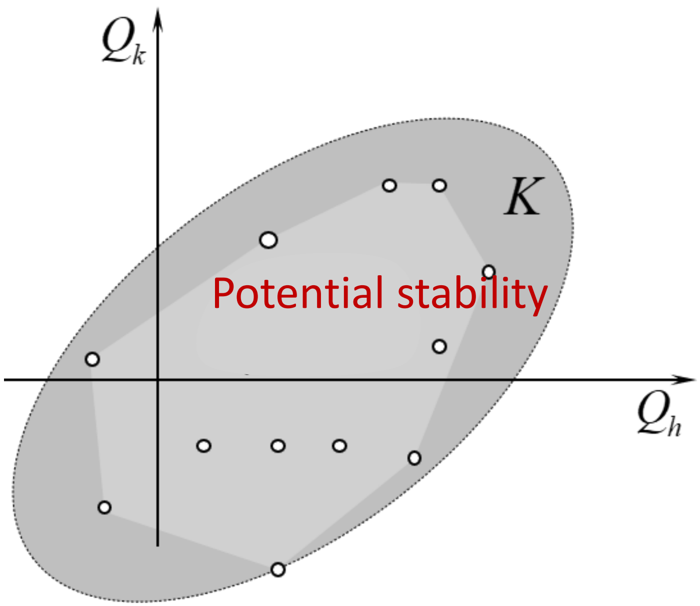
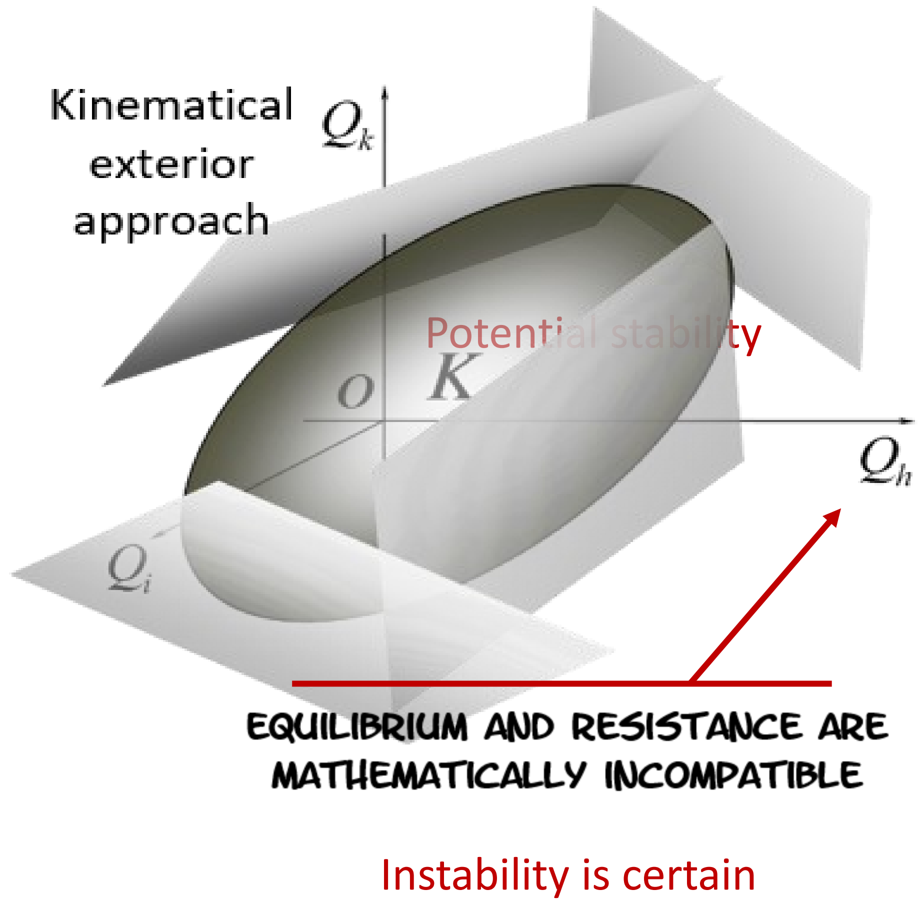
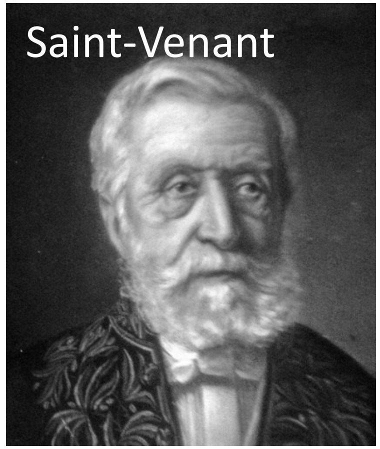


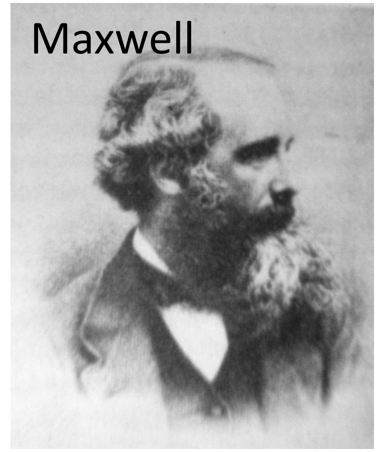
Statical Interior approach

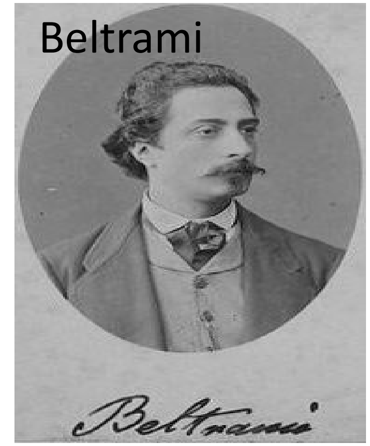


Kinematical Exterior approach

Yield Design
or
Plastic Limit Analysis ?

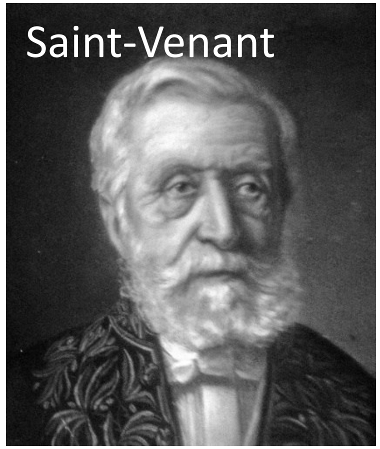


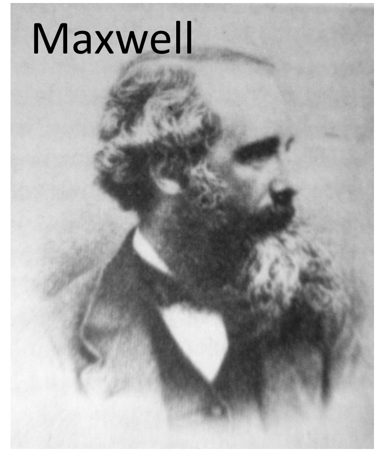


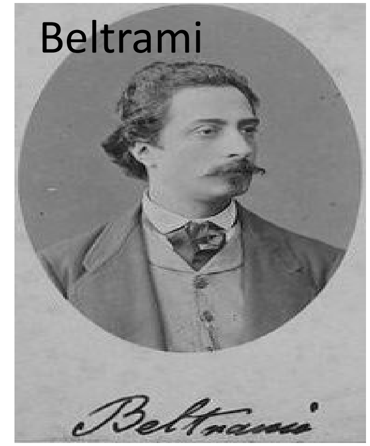


1864-72

Mathematical Theory of Plasticity



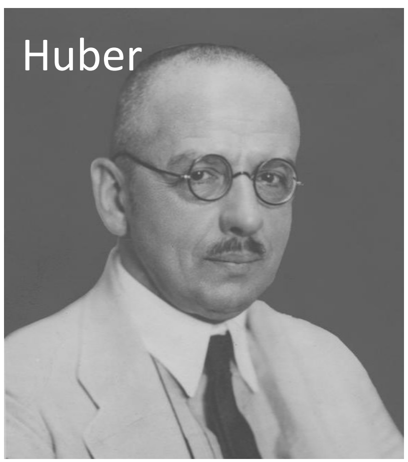


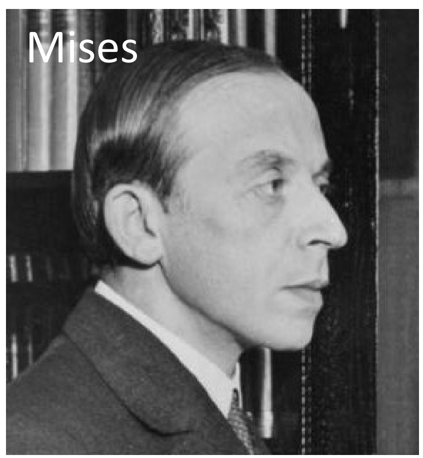


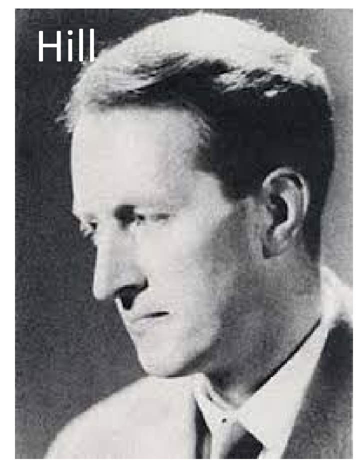
1864-72

Mathematical Theory of Plasticity

1948-50

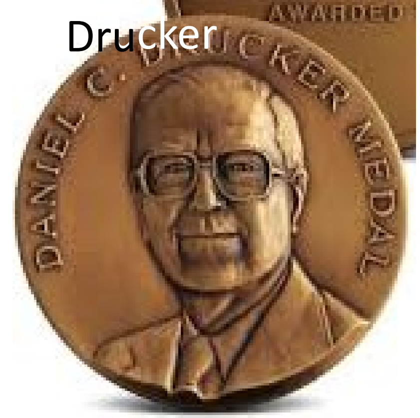


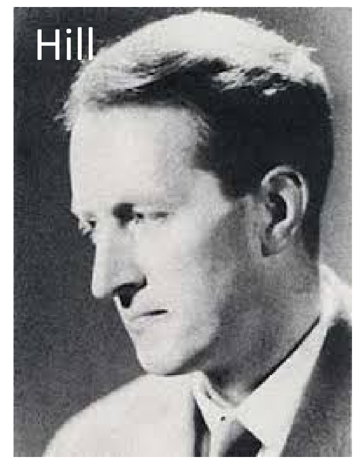




Yield Design
or
Plastic Limit Analysis ?

Mathematical Theory of Plasticity





Theory of Yield Design

Strength Criterion

Maximum Resisting Work
Mathematical result
(Convex analysis)

Normality Rule

Statical interior approach

Potential stability

Kinematical exterior approach

Mathematical Theory of Plasticity

Theory of Yield Design

Plastic Yield Criterion

Plastic potential
Von Mises

Normality Rule
Constitutive Equation
Plastic Flow Rule

Strength Criterion

Maximum Resisting Work
Mathematical result
(Convex analysis)

Normality Rule

Mathematical Theory of Plasticity

Theory of Yield Design

Plastic Yield Criterion

Principle
of Maximum Plastic Work
Hill

Normality Rule

Constitutive Equation

Plastic Flow Rule

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Normality Rule

Constitutive Equation

Plastic Flow Rule

Non-hardening materials

Existence theorem

Yield Design
or
Plastic Limit Analysis ?

Mathematical Theory of Plasticity

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Principle
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Hill

Normality Rule

Constitutive Equation

Plastic Flow Rule

Questionable !

Non-hardening materials

"Lower bound" theorem

Existence theorem

Limit loads

"Upper bound" theorem

Yield Design
or
Plastic Limit Analysis ?

Theory of Yield Design	Mathematical Theory of Plasticity
Statical interior approach	"Lower bound" theorem
Potential stability	Existence theorem
Extreme loads	Limit loads
Kinematical exterior approach	"Upper bound" theorem

No Constitutive Law

Normality as a
Plastic Constitutive Law

Yield Design
or
Plastic Limit Analysis ?

Theory of Yield Design	Plastic Limit Analysis
Statical interior approach	"Lower bound" theorem
Potential stability	Existence theorem
Extreme loads	Limit loads
Kinematical exterior approach	"Upper bound" theorem
No Constitutive Law	Normality as a Plastic Constitutive Law

Within its domain of relevance the theory of Yield Design does not call for the validity of the Normality Rule as a Constitutive Law

As a Conclusion

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Whatever its sophistication which may lead to remarkable theoretical results and whatever the discipline concerned, any mathematical model is only valid insofar as its underlying hypotheses are verified.

COMITÉ FRANÇAIS DE MÉCANIQUE DES SOLS ET DE GÉOTECHNIQUE

THANK YOU

for

Your Kind Attention