

# Computational limit analysis and extensions

**Kristian Krabbenhoff**  
*University of Liverpool, UK*  
*Optum Computational Engineering*  
Email: [kk@optumce.com](mailto:kk@optumce.com)

## ABSTRACT

The use of the finite element method for engineering analysis goes back to the 1950s. Since then, the method has undergone tremendous developments and is today a standard tool in many fields of engineering including structural, mechanical, and aerospace engineering. In geotechnical engineering, the method is used routinely for both stability and deformation analysis. That is, to assess limit loads (or factors of safety) and to determine deformations under working loads.

As an alternative to conventional finite element analysis, computational limit analysis (or finite element limit analysis), has long been recognized as a powerful means of assessing stability. Computational limit analysis has been developed in parallel to, and quite separately from, the finite element method and was originally viewed as having little or nothing to do with conventional finite elements. Indeed, it was always recognized that the scope of computational limit analysis – the assessment of stability – was much narrower than that of the finite element method.

However, in the past decade or so, it has been realized that the scope of the methods that underpin computational limit analysis is much broader than originally expected. In particular, it turns out that it is possible to extend the basic computational limit analysis framework to cover also deformation analysis. The result is a framework that embodies the best of both worlds: the flexibility and generality of conventional finite elements and the robustness and efficiency of computational limit analysis.

The presentation will cover the theoretical basis of the new framework and demonstrate its advantages in typical 2D and 3D geotechnical deformation and stability analyses. These include slope stability, deep excavations, embankment construction, and tunnelling.

Finally, while soil mechanics to a large extent has been developed with reference to 2D plane strain conditions, current practice increasingly requires 3D analyses. This has led to a number of instances in which well-established theory has required modification. Examples of these will be presented and discussed.