The Material Point Method: A promising computational tool in Geotechnics

La méthode du point matériel : un outil prometteur de calcul en géotechnique

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ABSTRACT: In recent years, the Material Point Method (MPM) has been applied to a number of geotechnical problems and has been extended to solve coupled flow-deformation problems. The dynamic formulation and the dual description of the media (lagrangian material points and an eulerian numerical mesh) provide the MPM the capabilities of handling problems involving large displacements and deformations. The paper presents four examples with the aim of highlighting the dynamic formulation and the capability of the method to analyze in a unified mathematical framework the static-dynamic transition of a slope failure.

RÉSUMÉ: Ces dernières années, la Méthode du Point Matériel (MPM) a été appliqué à un certain nombre de problèmes géotechniques et a été étendu à résoudre les problèmes couplés de flux-déformation. La formulation dynamique et la double description du média (des points matériels lagrangiens et du maillage de calcul eulérien) fournissent au MPM la capacité à résoudre des problèmes impliquant de grands déplacements et de grandes déformations. Le document présente quatre exemples afin d'illustrer la nature dynamique de la formulation et la capacité de la méthode pour analyser la transition statique-dynamique d'une rupture d'un talus avec un système de calcul unifié.

KEYWORDS: material point method, large deformations, slope failures, dynamics, consolidation.

1 INTRODUCTION

Problems involving large deformations such as the dynamic evolution of landslides or problems involving history-dependent constitutive models are of great interest in the geotechnical field.

The capability of the material point method (MPM) to deal with large displacements and its natural dynamic formulation make this technique an important numerical tool to tackle a number of geotechnical problems, such as landslides (Beuth *et al.*, 2008), run-outs (Andersen & Andersen, 2009) or installation of structures (pile driving and anchor pull-out (Beuth *et al*, 2007).

In this paper two different features of the method are illustrated in different examples. First, the dynamic nature of the formulation is emphasized in two cases. Afterwards, the capability of the method to analyze the static-dynamic transition of a slope failure in a unified mathematical framework is illustrated in two additional examples.

2 OUTLINE OF MPM FORMULATION

The MPM (Sulsky *et al.* 1995) represents the material as a collection of unconnected points so-called material points where the mass is concentrated (see Fig.1). Then, the density of the mixture can be expressed as

$$\rho(\mathbf{x},t) = \sum_{p=1}^{N_p} m_p \delta(\mathbf{x}-\mathbf{x}_p) \quad (1)$$

in which m_p and $\mathbf{x}_p$ are the mass and the position of the p^{th} material point, $\delta(\mathbf{x})$ is the Dirac delta function, and N_p is the total number of material points. An important assumption is that the mass assigned to each material point remains fixed during the calculation, thus assuring mass conservation. Other quantities such as velocities, strains and stresses, are also carried by the material points. Otherwise the governing equations are solved in a support numerical mesh, which covers the full domain of the problem. The standard shape functions provide the relationship between the material points and the nodes of any point of the domain.

The MPM formulation for a mechanical problem was presented by Sulsky *et al.* (1995). They discretized the equation of dynamic momentum balance. MPM has also been extended to solve coupled hydro-mechanical problems in granular media based on the well known equations described in Zienkiewicz & Shiomi (1984) and Verruijt (2010). Two different formulations have been applied, the most common one based on the solid velocity-liquid pressure formulation (Zabala & Alonso, 2011), and a solid velocity-liquid velocity formulation (Jassim *et al.* 2012).

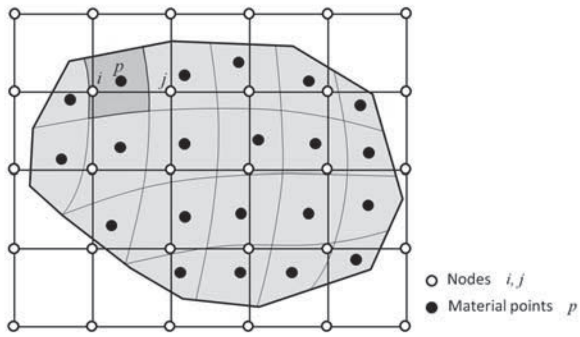

Figure 1. Discretization in material points and a finite element mesh used in MPM.

3 EXAMPLES OF DYNAMIC FORMULATION

3.1 *Wave propagation in a blasting problem*

The dynamic formulation of the MPM is useful to study some geotechnical problems such as the determination of stress and deformation in the vicinity of a blasting.

In the case analyzed the detonation is applied in a fractured granite rock mass, which is covered by a more superficial layer of sand 5m thick. Both materials have been modeled using a Mohr-Coulomb constitutive law (see parameters in Table 1) which has been obtained from a linear approximation of the rock failure Hoek-Brown criterion at a mean stress of 2MPa.

The problem is three-dimensional and, taking into account two planes of symmetry, a cubic geometry is modeled (see Fig. 2). The blasting is simulated by means of a horizontal pressure acting on a borehole 8 m long on one of the edges as shown in Figure 2. This pressure is applied as a triangular ramp loading during 0.034 seconds, with a maximum of 2.5 GPa at 0.017 seconds. The mesh was made denser in the vicinity of the borehole.

Table 1. General characteristics of the granite rock and the sand.

Material parameter	*granite*	*sand*
Specific weight γ (kN/m^3)	25	23
Young modulus E (MPa)	10000	100
Poisson ratio ν	0.33	0.33
Cohesion *c* (kPa)	600	50
Frictional angle φ (º)	42	35

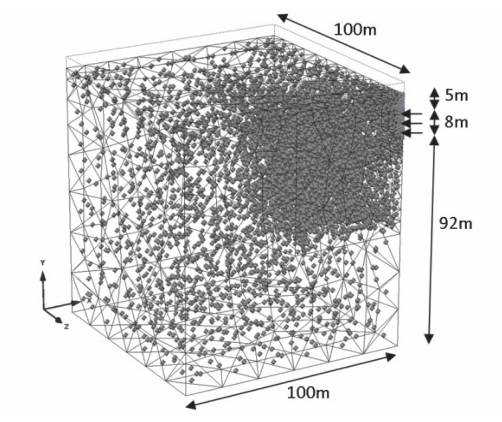

Figure 2. Simulation scheme, dimensions and initial discretization of the blasting problem.

The rapid application of the load generates a wave which extends in all directions throughout the domain. The speed of the wave propagation depends on the Young modulus and the specific weight of each material: 250m/s in the upper sand and 2500m/s in the granitic rock.

The evolution of the calculated displacement field is presented in the Figure 3. The maximum displacements are concentrated in the area of the blasting and they are of the order of 10^{-4}m. The larger the affected area the lower is the displacement amplitude of the wave front.

Figure 4 presents the stress paths for three points (P1, P2 and P3) located at a depth of 20m and at distances of 10, 20 and 50m respectively from the origin of the blast. The Hoek-Brown rock failure criterion with the corresponding parameters of the granite is also represented in the figure. Only P1 reaches the Hoek-Brown failure criterion.

3.2 *Oedometric consolidation*

Consider the consolidation of a soil defined in Table 2. The sample is a 1m long column, in which traction of 1 kPa was applied and maintained at the upper boundary. The bottom is impervious.

The aim of this example is to show the difference between the (static) Terzaghi analytical expression, and the dynamic solution, calculated via the MPM code. Moreover, the effects of damping in the dynamic solution has been analyzed.

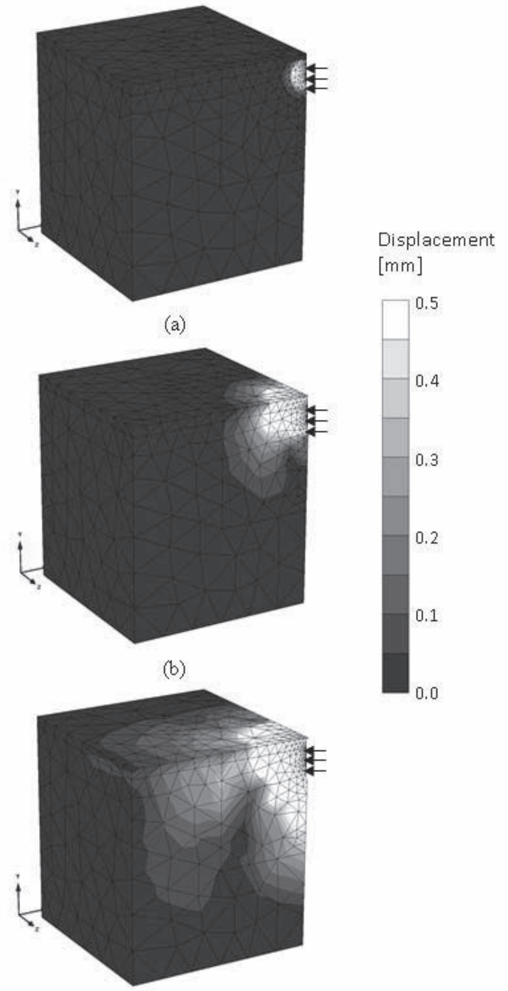

Figure 3. Displacements produced by the pressure wave propagation at different times after the blasting: (a) 0,01s; (b) 0,03s; (c) 0,06s.

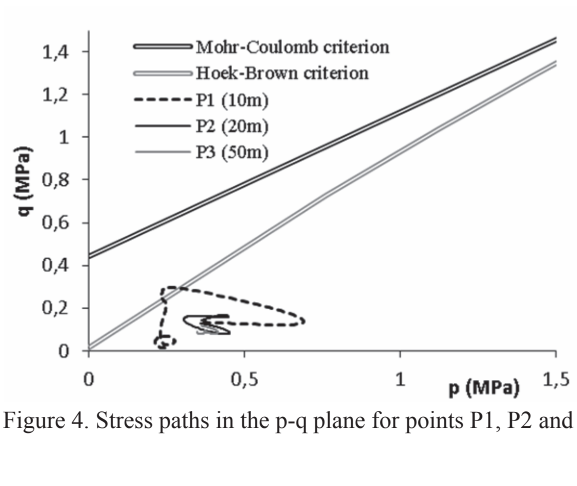

Figure 4. Stress paths in the p-q plane for points P1, P2 and P3.

Two different simulations of the problem were made, the first one is a purely dynamic one and the second one is affected by an extra damping at the bottom, which was imposed with the aim of reaching earlier the static solution.

Table 2. General characteristics of the tested soil.

Material parameter	
Dry unit weight γ (kN/m³)	23
Young modulus E (MPa)	10
Intrinsic permeability *k* (m²)	10^{-10}
Porosity *n*	0.3
Water viscosity *μ* (kg/m·s)	10^{-3}
Water bulk modulus K (MPa)	300

Figure 5 shows the evolution of the pore pressure along depth at different times for both simulations. Figure 6 provides the evolution of the pore pressure of the material point located at the bottom of the sample. The numerical solution is naturally damped in any case because of the coupling term of the hydro-mechanical formulation, which is explained by water flow in soil pores (at t_2 both MPM numerical solutions fit the static solution). However, the implementation of viscous boundaries (extra damping) is essential to damp the solution as quick as possible if the aim is to capture the quasi-static equilibrium. At t_1 (Fig. 5 and 6) the MPM solution with extra damping almost adjusts the static solution while the MPM solution with fixed boundary on the bottom still has a strong dynamic behavior.

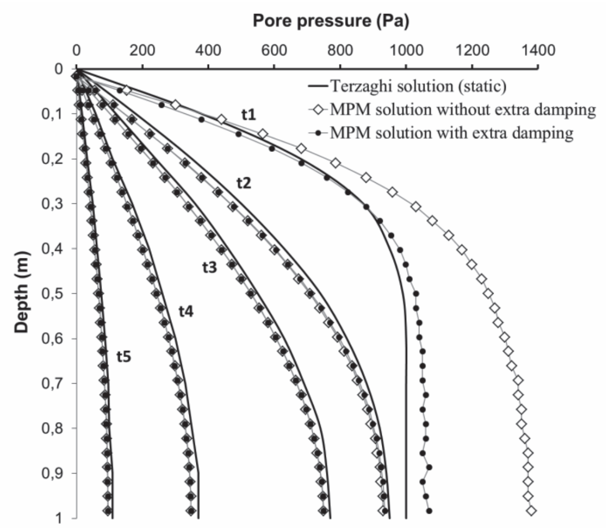

Figure 5. Comparison of analytical and MPM solutions (with and without extra damping on the bottom) for one-dimensional consolidation at different times (t_1=0.02s, t_2=0.1s, t_3=0.2s, t_4=0.5s, t_5=1s).

4 SLOPE FAILURES

4.1 *Simple case*

Two plane strain theoretical cases are presented below. Both simulations have been solved using a purely mechanical formulation and they concern slope failures with the same initial geometry and boundary conditions (the lowest boundary of the model is fixed and horizontal displacements are restricted in the lateral boundaries.

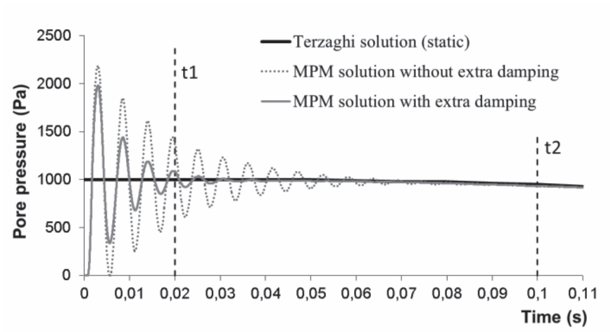

Figure 6. Evolution of the pore pressure for the deepest material point.

The constitutive model used in both cases is the Mohr-Coulomb criterion. The first case is characterized by a frictional material, while the second is a cohesive material. In order to initiate the failure of the slopes the strength parameters were suddenly decreased. In the first simulation, the friction angle has been reduced from 42º to 28º whereas the undrained strength was reduced from 100kPa to 10kPa. Other common material parameters are given in Table 3.

Table 3. Material parameters for the simulation of slope stability cases

Material parameter	
Dry unit weight γ (kN/m³)	16
Young modulus E (MPa)	10
Poisson ratio ν	25

Figure 7a shows the initial particle distribution for both simulations, and figures 7b and 7c show the two final distributions after the failure. In both simulations large deformations occur but the typology of the movement is completely different. For the frictional material, a shallow failure is developed and the main part of the movement occurs during the first 50 seconds. On the contrary, the failure induced for the cohesive material is deeper and in this case the time elapsed to stabilize the slope is around 450 seconds.

This example shows the great importance of the strength parameters and their evolution in the geometry and formation of a failure. The method provides in a natural way the highly deformed geometry of the slope after failure.

4.2 *Aznalcóllar dam*

The Aznalcóllar dam failure was described in Alonso & Gens (2006). In a recent contribution, Zabala & Alonso (2011) described an MPM analysis of the dam using a strain softening constitutive model for the foundation soil.

A significant result of the analysis was an accurate prediction of the geometry of the failure surface. Also the first few meters of displacement after the instability where modeled.

A saturated porous media was considered and the hydro-mechanical interactions were formulated in MPM. The model was two-dimensional and a regular computational mesh was used. A non-associated strain softening Mohr-Coulomb constitutive law was implemented and calibrated for the clay foundation. Figure 8 shows the development of the failure surface preceding the final rupture. Figure 9 shows the deformation of the mesh. The position of material points provides a direct visual representation of the failure.

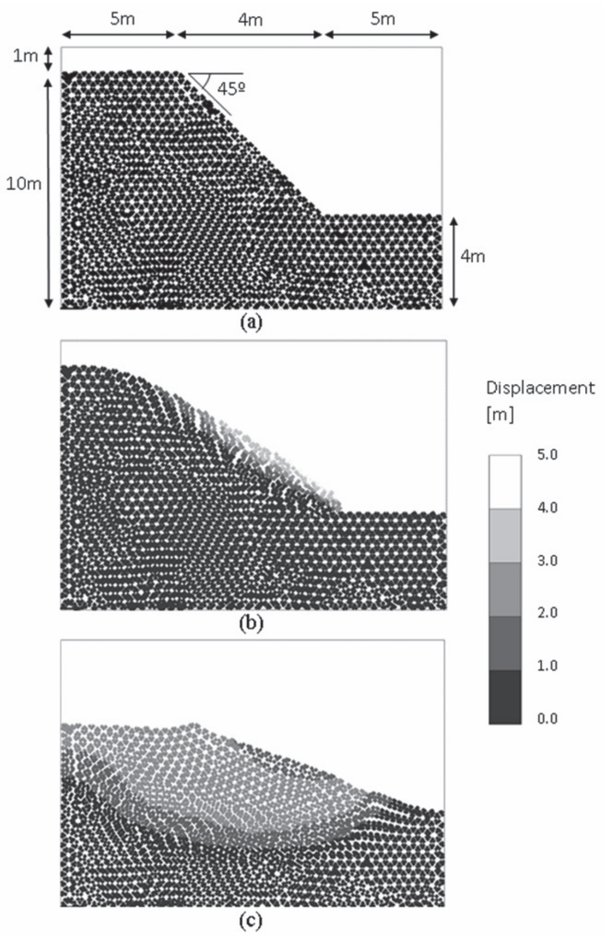

Figure 7. (a) Initial geometry and particle distribution for both cases; (b) Final displacements for the frictional material; (c) Final displacements for the cohesive material.

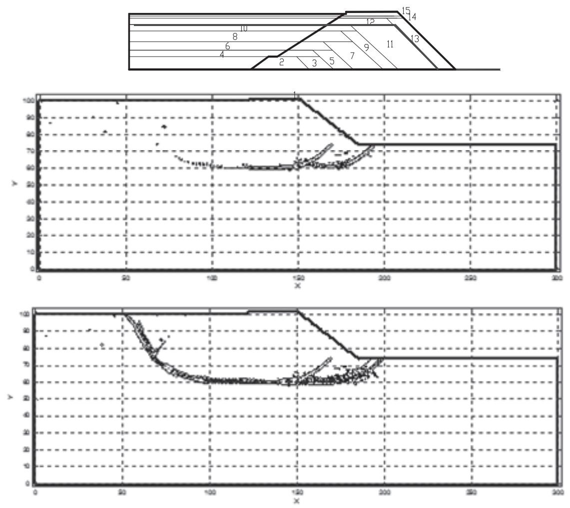

Figure 8. Construction sequence and development of contours of equal equivalent deviatoric plastic strain (from Zabala & Alonso, 2011).

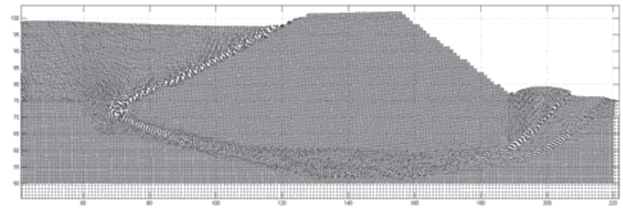

Figure 9. Model of Azalcóllar dam few seconds after the tailing's liquefaction. Particle displacements are not amplified. (Zabala & Alonso, 2011).

5 CONCLUSION

The Material Point Method is a potentially powerful tool in the geotechnical engineering because its capabilities to model dynamical problems and to integrate the analysis of failures and the subsequent large displacements. Four examples have been presented in this work, illustrating all this capacities in mechanical and coupled hydro-mechanical frameworks.

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