# Slope stability with partial safety factor method

## Stabilité des pentes à l'aide de la méthode de sécurité partielle

Länsivaara T., Poutanen T.
*Tampere University of Technology, Tampere , Finland*

ABSTRACT: In most European countries Eurocode 7 (EN 1997) has been taken into use in geotechnical design. Although there are several design approaches in EN 1997, slope stability is commonly addressed by applying partial safety factors to material properties and variable loads. The outcome of such procedure has been evaluated with emphasis on uncertainty and consequences of failure. An attempt to improve the EN-1997 partial safety method by the introduction of consequence classes into stability analysis is then presented. Therein the partial safety factors for soil strength will be connected into consequence classes, i.e. the consequences of possible failure. Eurocode 1990 defines the minimum reliability index values for different Reliability Classes, which are then associated with the consequences classes. The result by the alternative method is then compared with results from the original approach.

RÉSUMÉ : Dans la plupart des pays européens, l' Eurocode 7 (EN 1997) a été mis en service dans la conception géotechnique. Bien qu'il existe plusieurs approches de conception de la norme EN 1997, la stabilité des pentes est généralement abordée en appliquant des facteurs de sécurité partiels pour les propriétés des matériaux et des charges variables. Le résultat d'une telle procédure a été évaluée en mettant l'accent sur l'incertitude et les conséquences de l'échec. Une tentative pour améliorer la méthode de sécurité EN-1997 partielle par l'introduction de classes de conséquence dans l'analyse de la stabilité est ensuite présenté. D'apès celle-ci, les facteurs partiels de sécurité pour la résistance du sol seront connecté en classes de conséquence, c'est à dire les conséquences d'un échec possible. L'Eurocode 1990 définit les valeurs minimales indices de fiabilité pour les différentes classes de fiabilité, qui sont ensuite associées à des classes de conséquences. Le résultat par la méthode alternative est finalement comparé avec les résultats de l'approche originale.

KEYWORDS: slope stability, partial safety method, reliability, consequence class

## 1 INTRODUCTION

In most European countries, Eurocodes have been taken into use as the main design standard. Eurocodes are based on the principles of limit states design in conjunction with the partial factor of safety method. EN 1990 describes the principles and requirements for safety, serviceability and durability of structures for all materials, whilst EN 1997 sets the rules for geotechnical design. For ultimate limit states design, Eurocode requires the verification of various ultimate limit states by applying partial safety factors to actions or the effect of actions, material properties or resistances. For geotechnical design, it has so far been impossible to find a single way of combining factors between actions, ground properties and resistances, and thus three different design approaches (DA) are permitted in EN1997. However, for slope stability there seems to be a rather large consensus on how safety should be applied. Most countries have chosen either design approach 1 (DA1) or 3 (DA3) for slope stability. DA1 consists of two combinations of sets of partial factors, of which combination 2 is the one relevant for slope stability and analogous to DA3. As there is such a large agreement on applying safety for slope stability it could be considered that this part of the geotechnical design is well formulated in the Eurocodes. The intention of this article is to critically review the applied partial factor of safety method for the slope stability with respect to reliability and consequences of failure.

## 2 SHORT OVERVIEW ON THE EUROCODES

### 2.1 *Safety in Eurocodes*

According to EN-1990 the partial safety factors should account for the possible unfavourable deviation of the property from its characteristic value and the uncertainties in the model used in calculations. The consequences of the ultimate limit states are further considered based on three consequences classes. Therein the consequence of failure is accounted by multiplying the factors for actions by a separate factor depending on the consequence. In principle the partial factors of safety can be determined in two ways. One is the conventional method where the factors are calibrated to past experience. The other is to use probabilistic methods and calibrate the factors against a target reliability index value.

For the most common design situations, corresponding to the reliability class RC2, the recommended reliability index β for a 50 years reference period is 3.8. This corresponds to nominal probability for ultimate limit states of approximately 1/15 000. Whilst some of the partial factors for actions have been determined based on probabilistic methods, it is the understanding of the authors that material and resistance factors for geotechnical design has mainly been determined based on calibration to old codes.

### 2.2 *Slope stability in EN-1997 according to DA1 and DA3*

As previously discussed, most European countries have chosen either DA1 combination 2 or DA3 to be used in slope stability.

In practice this means, that safety is placed on material properties (strength) and on actions. The recommended values for partial factors on soils strength are $\gamma_{\phi'} = \gamma_{c'} = 1.25$ for effective stress analysis and $\gamma_{cu} = 1.4$ for total stress analysis. The largest action on slope stability comes often from the soil weight itself. It is often considered difficult to factor soil weight properly and thus permanent loads are, also in the Eurocodes, left unfactored. Actions from variable loads like e.g. traffic load are on the other hand factored. Accordingly, the recommended values for permanent actions in EN-1997 is $\gamma_G = 1.0$ and for variable actions $\gamma_Q = 1.3$.

According to the principles of EN1997 the overall stability is checked by requiring that the design value of the effects of actions $E_d$ driving instability is less than the design value of the resistances $R_d$, i.e. $E_d \leq R_d$. However, the common methods for slope stability don't usually provide these values, but rather their ratio as an overall factor of safety. Thus an over-design factor ODF is introduced (Frank et al.), and the requirement for overall stability for DA 1 combination 2 and DA 3 is written as the factor of safety calculated using design values equal to ODF $\geq 1$.

In short the recommended values mean that if only effective stress parameters are used for all soil layers, and there is no variable load, the total factor of the safety requirement is $F = \gamma_{\phi'} = \gamma_{c'} = 1.25$. Similarly, if only undrained shear strength is used and there is no variable load, the total factor of safety requirement is $F = \gamma_{cu} = 1.4$.

Traditionally, slope stability analysis is most commonly done applying the total safety factor approach. There is a lot of experienced based data on total safety factors and many engineers feel that it is easy to relate to a single safety factor. As discussed by Leroueil et al. (1990) based on the observations by Bourges et al (1969) a reduction of the total safety factor below a certain limit, increases the settlements due to increasing horizontal movements in the soil. The use of such empirical knowledge supports the continuous use of a total safety approach for slope stability.

The application of partial factors of safety in the Eurocodes is indirectly implying that safety is placed were uncertainty is found accounting also for the consequences of failure. It is the intention to evaluate how true this implication is in relation to slope stability and to consider what could be done to improve it.

## 3 RELIABILITY BASED PARTIAL SAFETY FACTORS

### 3.1 *Introduction*

In the following material partial safety factors are calculated based on reliability theory. Firstly the theoretical bases and the assumptions made in the calculations will be presented. Thereafter the calculations will be done corresponding to the present system in the Eurocodes placing safety on both the variable action and on material properties. An alternative calculation will then be presented where both the uncertainty related to loads and material properties will be placed on the material partial safety factor. In both the influence of a general uncertainty will also be studied.

In the alternative approach, the influence of the consequence classes into stability analysis will also be considered. As for now, the reliability differentiation in Eurocodes is done by applying a multiplication factor $K_{FI}$ to unfavourable loads. The recommended values for the factor are 0.9, 1.0 and 1.1 corresponding to Reliability classes RC1, RC2 and RC3. For slope stability problems the effect of external actions on the stability varies from zero to rather substantial. It seems thus rather random to apply safety related to reliability and consequence of failure on such basis. On the other hand it is also uncertain should the factor be applied only to variable loads or also for permanent loads. In the latter case the problem on how to treat ground weight arises again. However, in EN 1990 it is also stated that, quote" Reliability differentiation may also be applied through the partial factors on resistance $\gamma_M$. ". The material partial factors for the alternative approach will thus be calculated for different target reliability index values corresponding to the different reliability classes.

### 3.2 *Theoretical bases*

Firstly, the design point, the target reliability, the uncertainty, load distributions and the basic parameters must be set. The design point is set at unity and the target reliability feasible in the reliability calculation is chosen according to EN 1990. The permanent load distribution is assumed to be normal, the coefficient of variation equal to 0.1, the cumulative distribution is $FG(x, \mu_G, \sigma_G)$ and the density distribution is $fG(x, \mu_G, \sigma_G)$. For variable load a normal distribution is also used, although Gumbel distribution might also be considered. The distributions are $FQN(x, \mu_{QN}, \sigma_{QN})$, $fQN(x, \mu_{QN}, \sigma_{QN})$, 0.98 fractile is set at the design point according to one-year load. The coefficient of variation used for the variable load is 0.4 as in EN 1990.

The material property distribution is assumed lognormal, the cumulative distribution is $FM(x, \mu_M, \sigma_M)$ and the density distribution is $fM(x, \mu_M, \sigma_M)$, the characteristic value is a 5 % fractile value which is set at the design point.

When the cumulative distribution of the load is $FL(x, \mu_L, \sigma_L)$, density distribution of the material property $fM(x, \mu_M, \sigma_M)$, the load safety factor is $\gamma_L$ and the material safety factor $\gamma_M$, the formula for the failure probability $P_f$ calculation is

$$1-\int_0^\infty FL(x,\mu_L,\sigma_L)\cdot fM(x,\mu_M\cdot\gamma_L\cdot\gamma_M,\sigma_M\cdot\gamma_L\cdot\gamma_M)\,dx=P_f \quad (1)$$

When two loads $F_1(x, \mu_1, \sigma_1)$, $f_1(x, \mu_1, \sigma_1)$ and $F_2(x, \mu_2, \sigma_2)$, $f_2(x, \mu_2, \sigma_2)$ with items $x_{1,i}$ and $x_{2,i}$ in fractile i are combined dependently in proportion $\alpha$ and $1-\alpha$, $\alpha$ is the proportion of the load 1 in the total load, to obtain item $x_{1,2,i}$ of the combination load in fractile i, is calculated by adding up the partial items:

$$\mu_{1,2,i} = \mu_{1,i}\cdot\alpha + \mu_{2,i}\cdot(1-\alpha) \quad (2)$$

The graphs of the used distributions are presented in Figure 1.

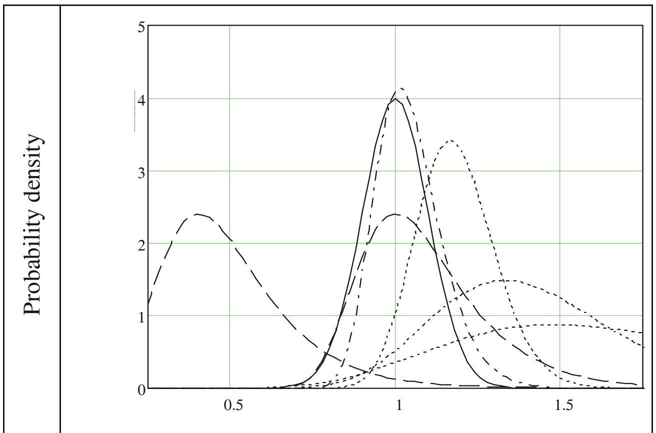


Figure 1. Distributions set at the design point. Solid line: permanent load; dashed line: variable load one-year location, dash-dotted line: variable load in 50-year location; dotted lines: material properties. The uncertainty distribution is equal to the permanent load distribution but it is located at the origin.

### 3.3 *Calculated material factors for DA3*

Herein the calculations are done to resemble the present partial safety factor approach DA3 in the Eurocodes. The loads are combined dependently and the partial safety factors for the loads are $\gamma_G$ = 1 and $\gamma_Q$ = 1.3. The load distribution for both permanent and variable loads is assumed to be normal. The variable load distribution is set at 50-year loads and the target reliability corresponds to RC2 of the Eurocodes, i.e. $\beta_{50}$ = 3.8.

The calculations are done for material variation of $V_M$ = 0.1, $V_M$ = 0.2 and $V_M$ = 0.3. In addition a general uncertainty following a normal distribution and 0.1 deviation is added into a parallel calculation. The results of the calculations are shown in Figure 2 as the function of the load ratio (the proportion of the variable load in the total load, %).

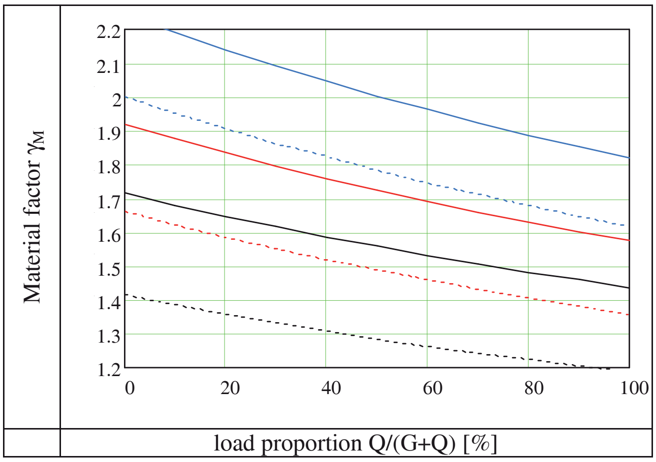


Figure 2. Material factors of the Eurocodes corresponding to $\gamma_G$ = 1, $\gamma_G$ = 1.3 and $\beta_{50}$ = 3.8 as function of load ratio. The lowest, black lines denote to $V_M$ = 0.1; read lines, middle to $V_M$ = 0.2 and blue lines, highest to $V_M$ = 0.3. The dotted lines correspond to values without uncertainty, $V_U$ = 0 and the solid lines with uncertainty, $V_U$ = 0.1.

The results presented in Figure 1 are of course dependent on the assumptions made. It is though notable, that the material factors are not constant while they depend on the load ratio. For an independent load combination the material factors would also be highly non-linear. The application of a constant material factor as in EN 1997-1 does not thus result in a constant reliability index. Considering effective stress analysis, the variation of the parameter is according to data gathered by Abramson et al. (2002) in the range of 2-21%. On average one could use the value of 10 % corresponding to the black line in Figure 2. The recommended partial safety factor for friction in EN 1997-1 is 1.25, which corresponds to a load ratio of 60% in Figure 2. Stability problems are often less load driven while in some cases the external load has no significant effect on safety. For such cases DA3 would according to Figure 2 clearly overestimate the safety.

### 3.4 *Calculated material factors for alternative approach*

Next an alternative approach will be presented where all uncertainty is placed on the material partial safety factor, i.e. $\gamma_G = \gamma_Q$ = 1.0. In addition the material factors are calculated for three different reliability index values, corresponding to the three reliability classes in the Eurocodes. The target $\beta$ values are $\beta_{50}$ = 4.3 (RC3), $\beta_{50}$ = 3.8 (RC2) and $\beta_{50}$ = 3.2 (RC1).

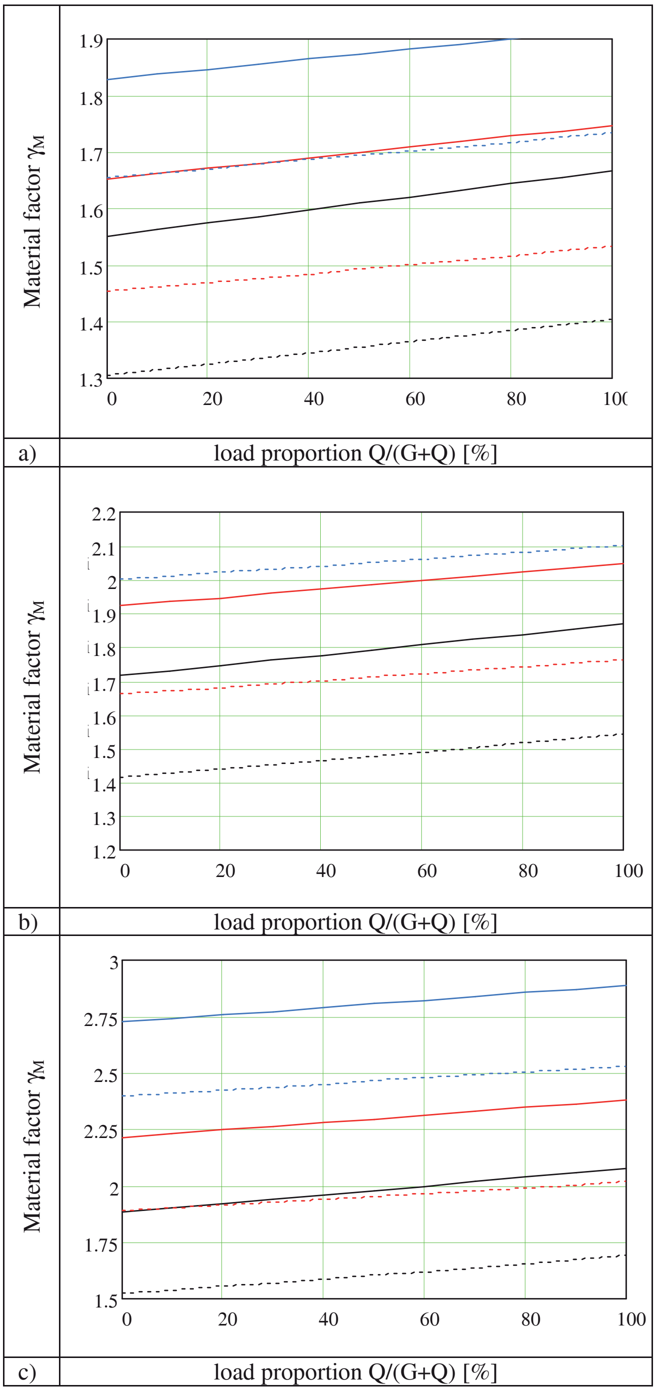


Figure 3. Material factors corresponding to $\gamma_G = \gamma_Q$ = 1.0 and a) $\beta_{50}$ = 4.3 (RC3), b) $\beta_{50}$ = 3.8 (RC2) and c) $\beta_{50}$ = 3.2 (RC1) as function of load ratio. The lowest, black lines denote to $V_M$ = 0.1; read lines, middle to $V_M$ = 0.2 and blue lines, highest to $V_M$ = 0.3. The dotted lines correspond to values without uncertainty, $V_U$ = 0 and the solid lines with uncertainty, $V_U$ = 0.1.

As can be seen, the material safety factors are the same as in Figure 2 for equal reliability index when the load proportion is zero. When Figure 2 and Figure 3 b are compared we find out that an equal design outcome is obtained when the safety is set both in the action and in the resistance or in the resistance only (e.g. load proportion 60 %, $V_M$ = 0.1, Figure 2: $\gamma_M$ = .1.53, i.e. (0.4+0.6·1.3)·1.53 = 1.81 which is equal to the value of Figure 3 b).

## 4 EXAMPLE OF APPLICATION

In the following the Eurocode EN 1997 will be applied in an example and compared to the alternative method presented in the previous section. The intention is to outline some issues the authors consider as problematic in a simplistic way. The example is thus very simplified and does not as such represent a true case study.

Let us consider the situation given in Figure 4. The soil conditions are the following. A 1m thick embankment is laid upon a dry crust layer. The unit weight of the embankment material is $\gamma = 20 kN/m^3$ and the characteristic friction angle is $\phi = 38^o$. The dry crust layer is 1m thick and has a unit weight of $\gamma = 17 kN/m^3$ and the characteristic undrained shear strength is 30kPa. Under the dry crust there is a layer of soft clay with a unit weight of $\gamma = 16 kN/m^3$ and a characteristic undrained shear strength of 10kPa at top of layer increasing with 1.4kPa/m deapthwise. A 5m wide load of 40kPa is placed two meters from the crest of the embankment. The problem in question is much load driven. The total safety factor without any load is around 4 while the 40kPa load decreases it to 1.46 for a circular failure surface analysed by the Bishop method.

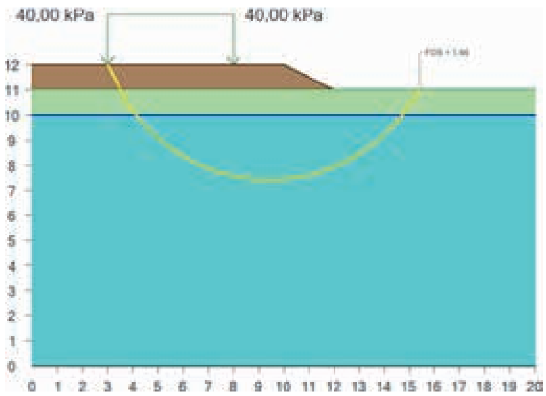


Figure 4. Geometry of the problem and calculated total factor of safety.

According to the recommended values to EN 1997 the partial safety coefficient for effective stress strength a parameters is $\gamma_{\phi'} = \gamma_{c'} = 1.25$ and for total stress analysis $\gamma_{cu} = 1.4$. Applying these yields for design values a friction angle of $\phi = 32^o$ and undrained strength values of 21.4kPa for the dry crust and 7.1kPa + 1kPa/m for the soft clay. If the load comes from a permanent load, the recommended partial factor is 1.0 while it is 1.3 for variable loads. So in case of variable loads the design value entering the calculation is 52kPa.

In case of a permanent load the resulting over dimensioning factor is ODF = 1.04 indicating that the situation is safe. For the variable load case the ODF reduces to ODF = 0.88. To have enough safety, the characteristic initial value of undrained shear strength of the clay should increase by some 30 % to 13kPa. This corresponds to a total safety factor of 1.69 for the situation. Is it then reasonable to require a higher safety for the variable load case? A general argument in favour of this is that there might be more uncertainty for the variable load than for the permanent one. This is however not necessarily true. A typical high variable load representative to embankment stability would be train load from a heavy freight train. However, railway tracks are classified and there is an upper load allowed for a certain track part. So the characteristic load is rather a maximum load. In such cases the partial factor for action should rather be calculated using a log-normal distribution than a normal distribution. Also, there will always be uncertainty also in the permanent load which is disregarded in EN 1997-1. It is perhaps also more important to consider the consequences of failure. A permanent load might come from a residential building. The consequence of failure might thus be very severe with lots of casualties. On the other hand if the variable load is due to a freight train carrying e.g. iron ore the consequences of failure in an uninhabited area are perhaps not that severe – at least the risk for loss of lives is minor. However, if a train with toxic material goes through an inhabitant area the consequences of failure are of course harsh.

The alternative approach presented in 3.4 allows for such considerations. It is emphasized that the results presented are aimed to give an example how safety could be applied. The assumed distributions, variations and target reliabilities needs of course careful consideration. However, if one assumes a variability of 0.1 for friction and 0.2 for the undrained shear strength one finds, that the recommended partial safety factors $\gamma_{\phi'} = \gamma_{c'} = 1.25$ and $\gamma_{cu} = 1.4$ in EN 1997-1 corresponds to the calculated one at approximately a load ratio of 80% assuming no additional uncertainty. While this is a very high load ratio its use is justified by the sake of comparison and the fact that the case is highly load driven. For the alternative approach corresponding safety factors for same $\beta$ would be $\gamma_{\phi'} = \gamma_{c'} = 1.52$ and $\gamma_{cu} = 1.73$. Applying this as a load factor on unity yields an ODF = 0.84. Now to have enough safety the undrained shear strength of the clay would need to increase to 13.3 kPa. This corresponds to on total safety factor of 1.71 i.e. close to the partial safety factor used for the undrained shear strength. Similarly the total safety requirement for a high consequence class (RC3) would be close to 2.0 and for a minor consequence class (RC1) approximately 1.5.

## 5 CONCLUSIONS

The partial safety factor approach in EN 1997-1 adopted in most European countries for slope stability is reviewed. The author's conclusions are that risk and consequence of failure are not necessarily properly accounted for. For situation with no variable loads the safety level applied in EN 1997-1 does not correspond to the implied reliability index, but is below that. Also the consequence of failure is not properly addressed, as the load factor in EN 1990 have a negliable affect to safety for some slope stability problems. An alternative approach is presented, where all uncertainty is placed on the material partial safety factor and the consequence of failure is accounted by calculating the material safety factors separately for different consequence classes with different target reliability index values.

## 6 REFERENCES

Abramson, L., Lee, T. S., Sharma, S. and Boyce, G. M. 2002. Slope stability and stabilization methods. John Wiley & Sons, Inc.

European committee for standardization, CEN. 2002. EN 1990 Eurocode: Basis of structural design. Brussels: European committee for standardization.

European committee for standardization, CEN. 2004. EN 1997-1 Eurocode 7: Geotechnical design. Part 1:General rules. Brussels: European committee for standardization.

Frank, R., Bauduin, C., Driscoll, M., Kavvadas, M., Krebs, Ovesen, N., Orr, T., Scuppener, B. 2004. Designer's guide to EN 1997-1 Eurocode 7: Geotechnical design-General rules. London: Thomas Telford Ltd.

Leroueil, S. Magnan, J.-P. and Tavenas, F. 1990. Embankments of soft clays. Ellis Horwood.Darcy H. 1856. *Les fontaines publiques de la ville de Dijon*. Dalmont, Paris.

Poutanen T., 2011. Calculation of partial safety factors, Applications of Statistics and Probability in Civil Engineering – Faber, Köhler & Nishijima (eds), Taylor & Francis Group, London