Seismic metamaterials

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joint with A. Colombi, B. Maling, O. Schnitzer (Imperial), D. Colquitt (Liverpool), P. Roux (ISTerre, Grenoble), S. Guenneau, Y. Achaoui, S. Enoch (Inst. Fresnel Marseille), T. Antonakakis (Multiwave AG), S. Brule (Menard), M. Clark, V. Ageeva (Nottingham), P. Sebbah, G. Levebvre (Inst Langevin) and others

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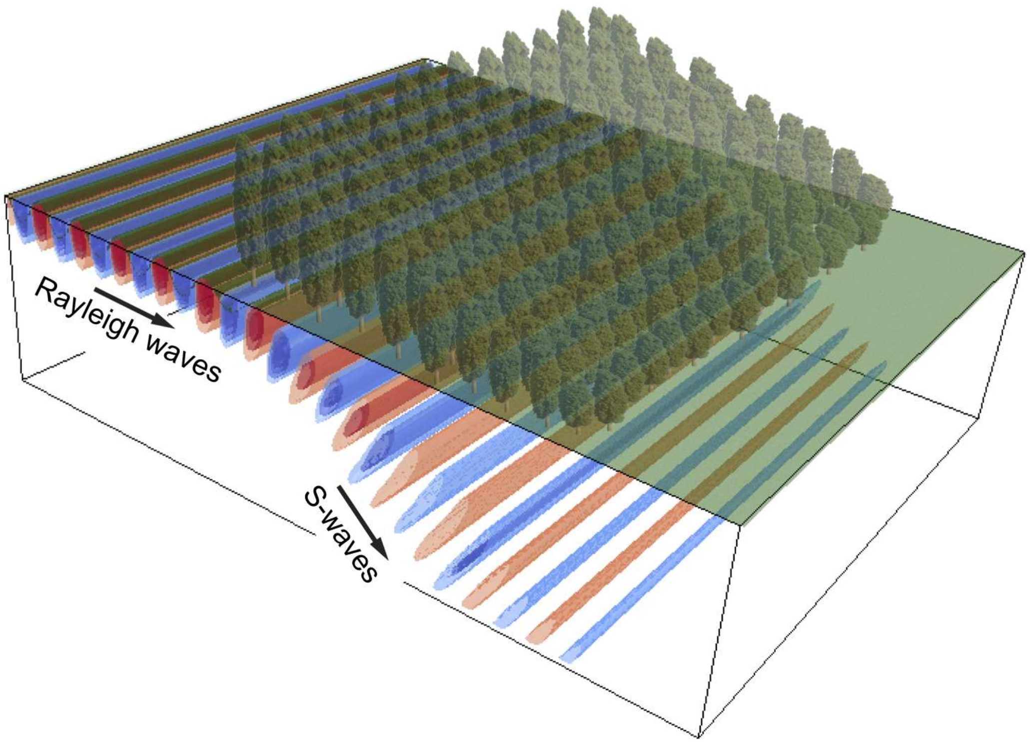
Challenges

- Very low frequency and long waves 0-10 Hz
- High material mismatch of soil, rock and building materials
- Soil mediation and seismic site effects
- Elasticity - full vector system, surface Rayleigh waves and P / S wave mode conversion and coupling at interfaces.
- Three routes/ concepts:
- : Seismic mode converters and the elastic rainbow.
- :Elastic surface wave lenses.
- :Elastic phononic crystals with ultra-broad stopbands.

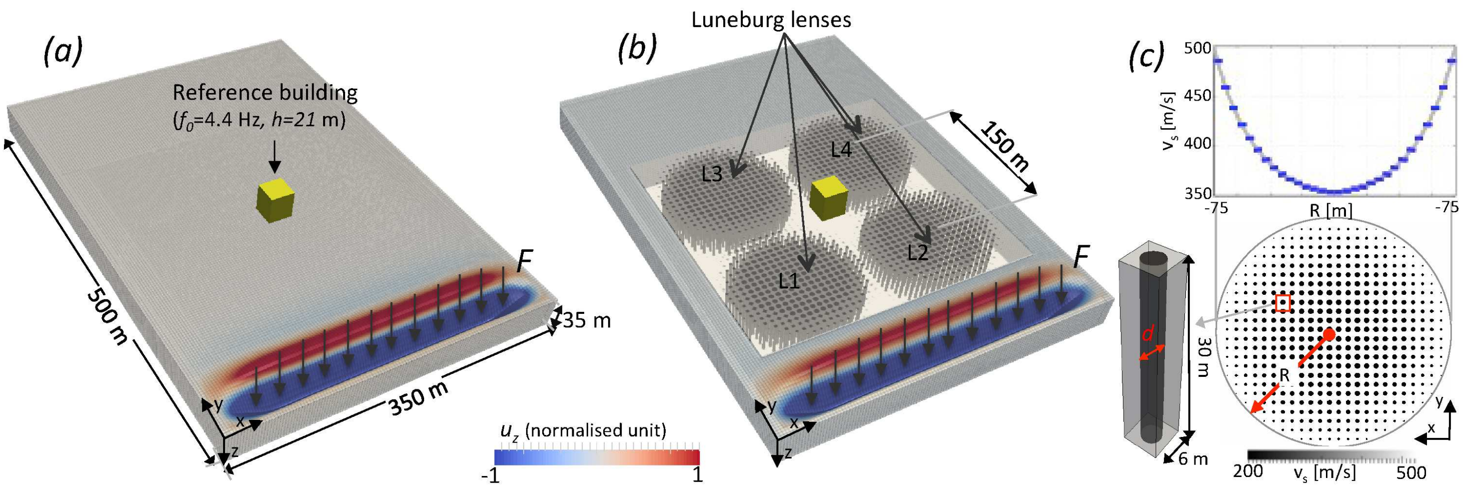
Seismic mode conversion

Can we divert waves somewhere “safe”? Mode conversion from surface to bulk waves, can this be achieved using sub-wavelength resonators?
A forest metamaterial !? See the recent experiments of Roux and collaborators
`https://metaforet.osug.fr/`



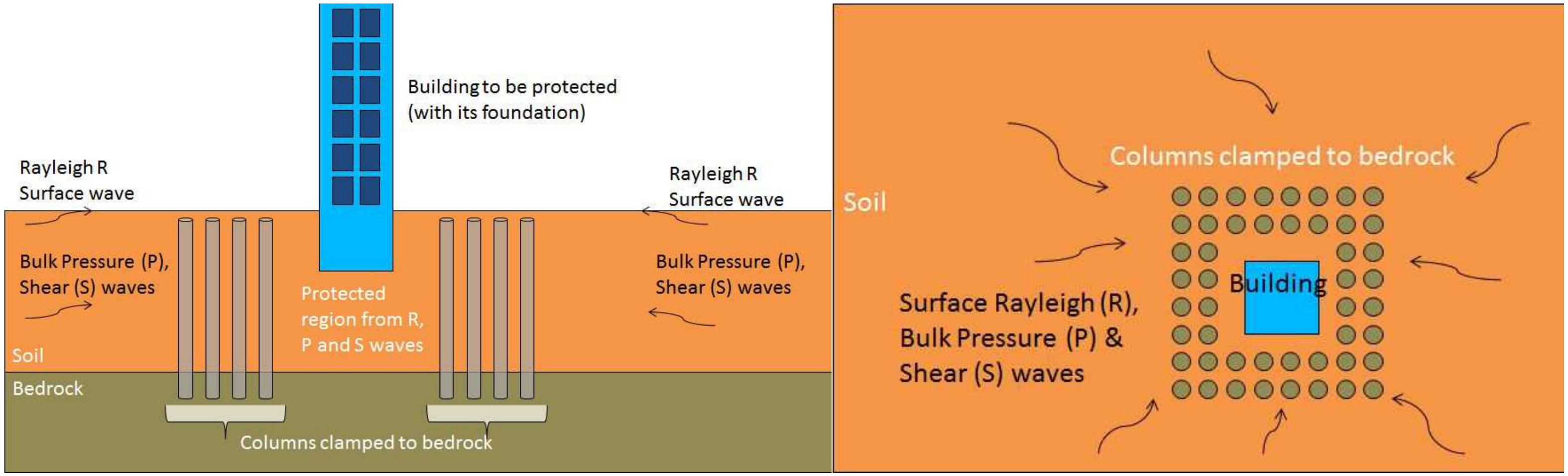
Seismic gradient index lenses

Can we steer surface waves around objects using gradient index lenses? Can this be achieved using sub-wavelength soil mediation?
A Luneburg lens arrangement similar to that in optics.

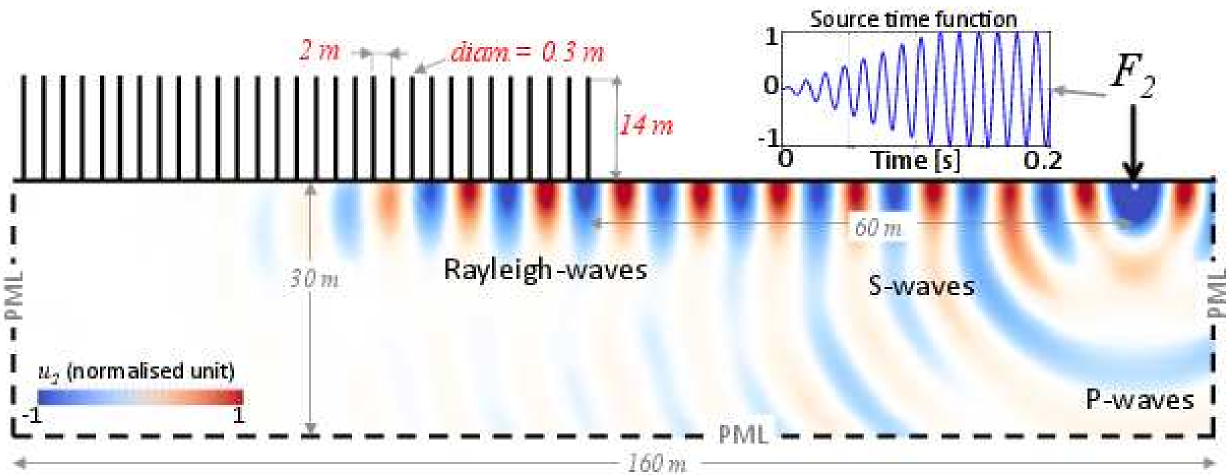


Seismic “shields”

Can we design structures that protect buildings (on soft soil overlaying bedrock) from seismic waves? Important to note that we need sub-wavelength structures that operate at $0 - 10$ Hz - V. hard to achieve.

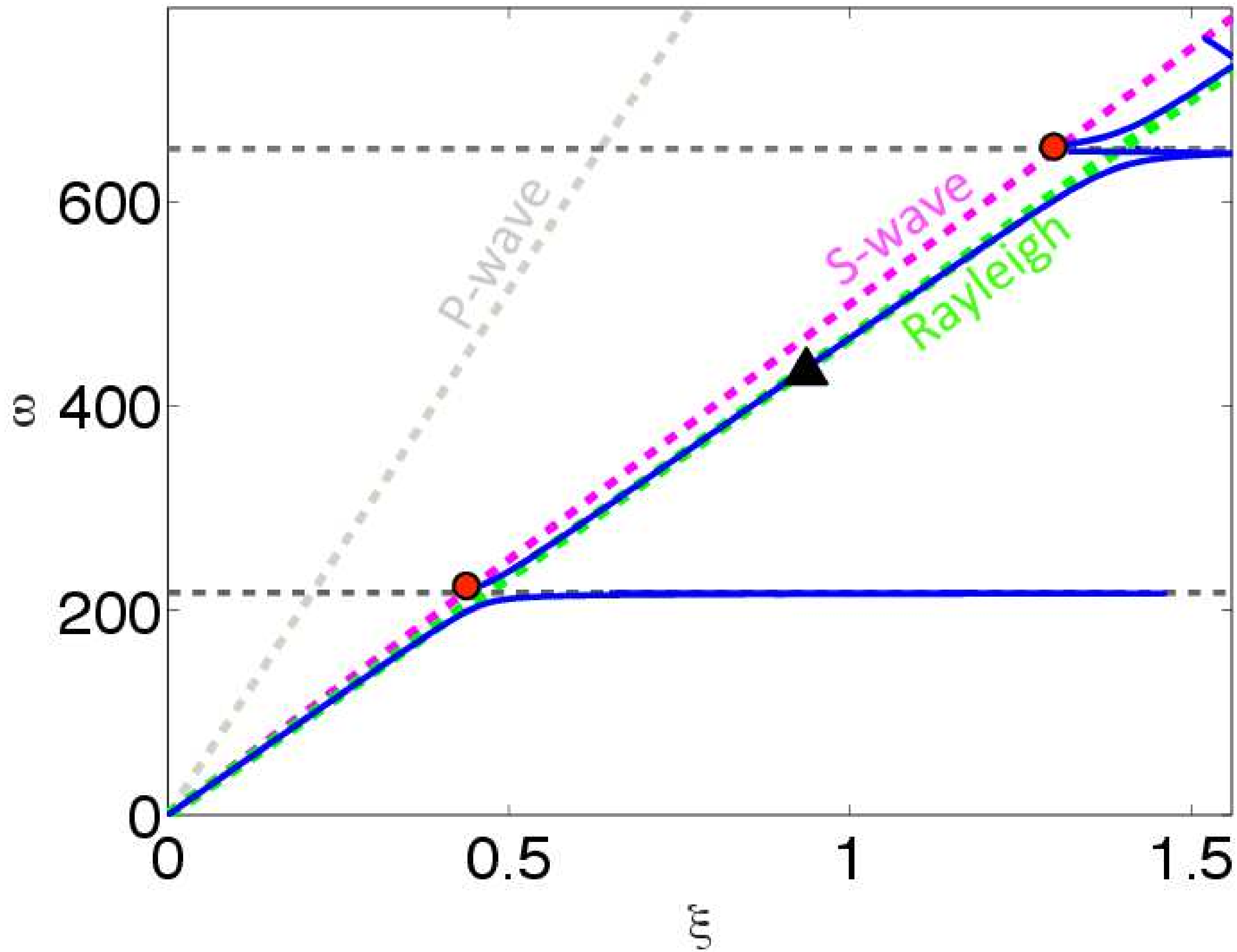


Surface sub-wavelength resonators

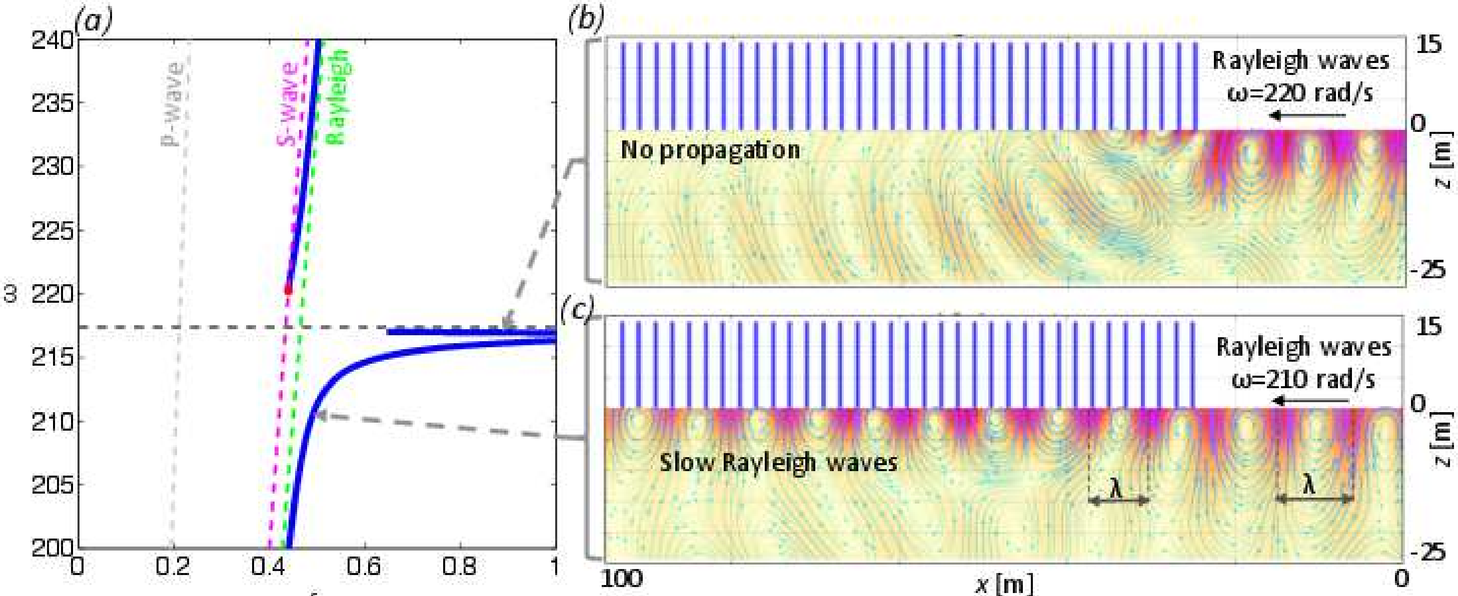


The 2D computational domain for the half-space also showing, in a red-blue colorscale, the vertical displacement u_2 (proportions are not to scale). The inset depicts the source time function generating the monochromatic Rayleigh wave.
This can be solved exactly for an infinite array using Fourier transforms and the Bloch problem gives the dispersion curves.
[D. J. Colquitt, A. Colombi, R. V. Craster, P. Roux, and S. R. L. Guenneau *J. Mech. Phys. Solids* Volume 99, 379-393, 2017]

Dispersion curves

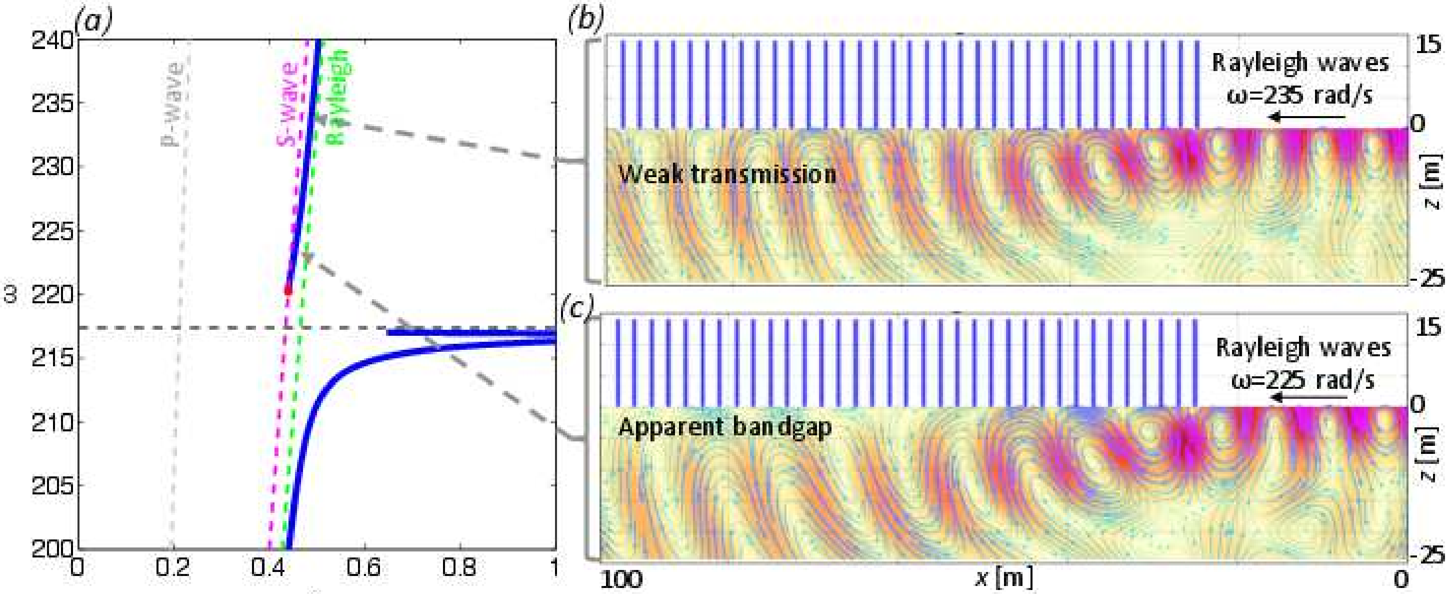


The dispersion curves for an infinite periodic array found using Bloch theory. Note the very narrow stop-band, the slow waves and hybridisation of Rayleigh wave.



(a) A zoom-in of dispersion curves, highlighting the first two branches and the first resonance. P, S and Rayleigh wave dispersion curves for the free half-space are also annotated in different colours. (b,c) Wavefield inside the half-space. (b) illustrates the stop band behaviour of the array for surface waves in the vicinity of a resonance, whilst (c) shows the usual pass band behaviour associated with the lower branch, λ represents the wavelength. Elastic streamlines are superimposed to the wavefield magnitude colorcode. The field in the resonators is not shown.

An animated version of this figure Start animation Start animation



The upper branch of the dispersion (blue) line. (b) illustrates the weak transmission of Rayleigh waves for frequency-wavenumber combinations just above the Rayleigh sound-line. (c) demonstrates the behaviour close to the intersection of the dispersion curves with the Rayleigh sound-line and responsible for the apparent bandgap.
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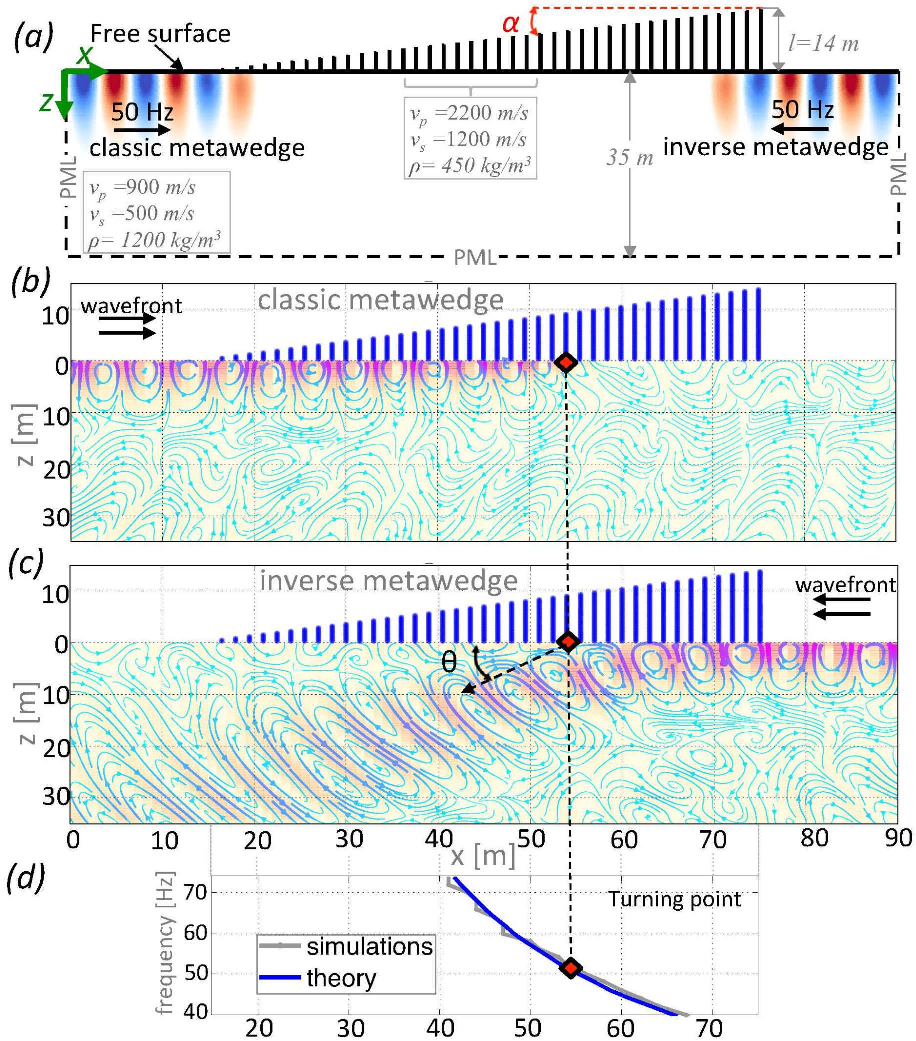
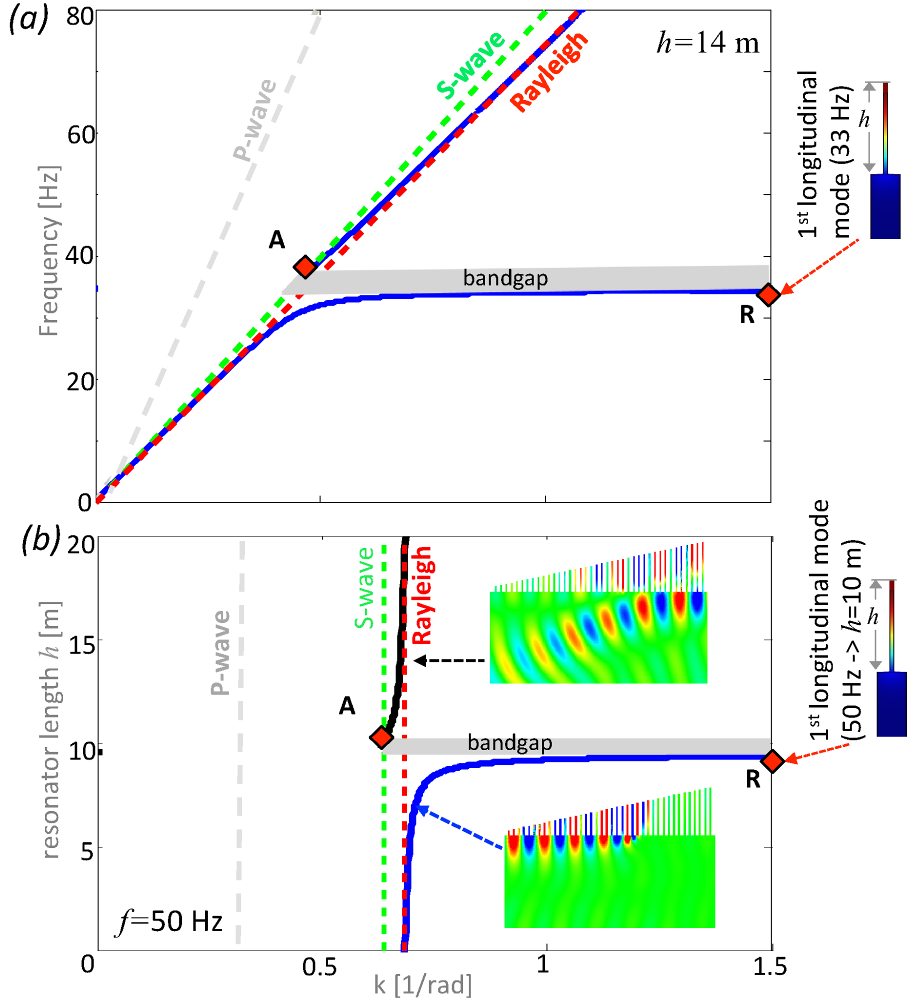
Surface wave mode converter - Metawedge

Start animation Here we design a graded surface array to convert Rayleigh waves into bulk waves (this is a genuine seismic metamaterial....) or to create a seismic filter.



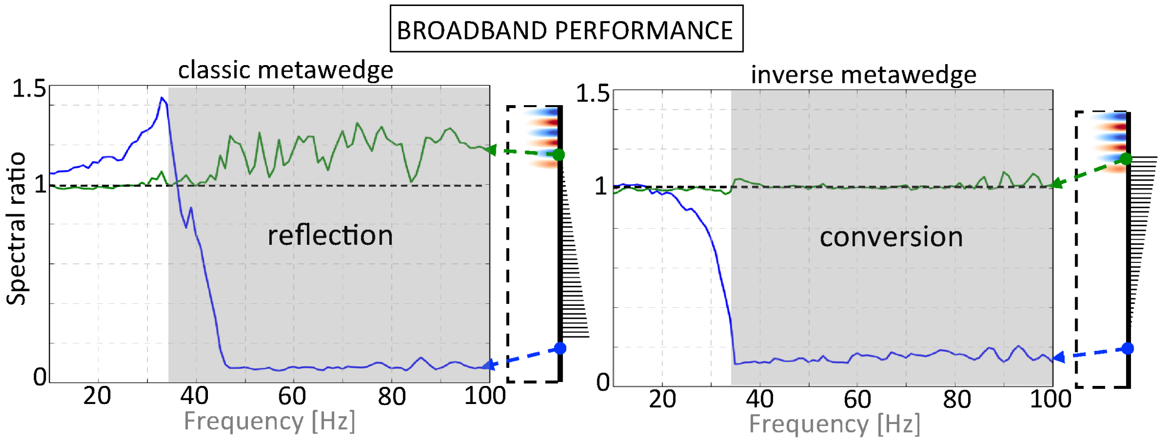
The dispersion curves and dependence upon resonator length



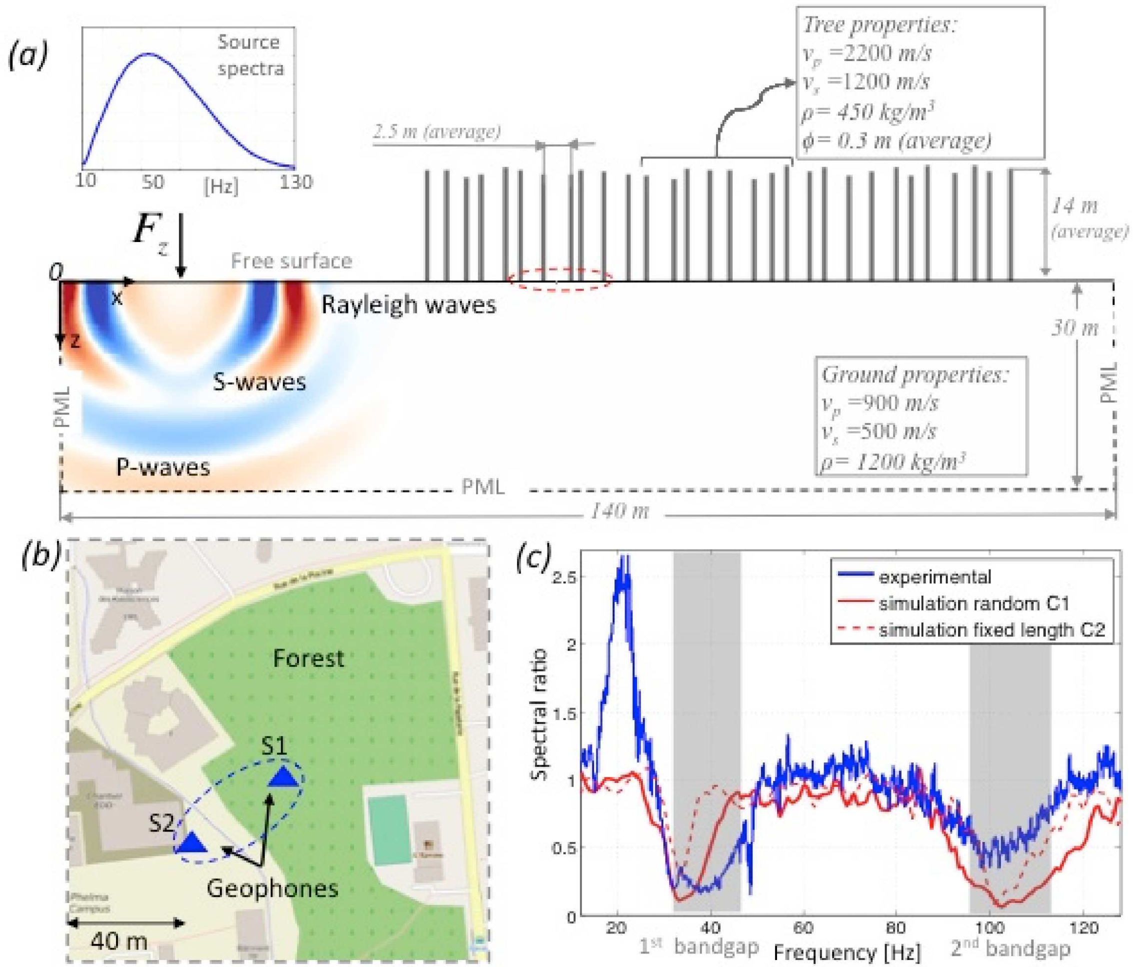
[A. Colombi, D. Colquitt, P. Roux, S. Guenneau, R. V. Craster *Scientific Reports*, **6**, 27717,2016]

Broadband performance

Absolutely key is to obtain broadband performance, and the metawedge does this over a very broad range of frequencies.

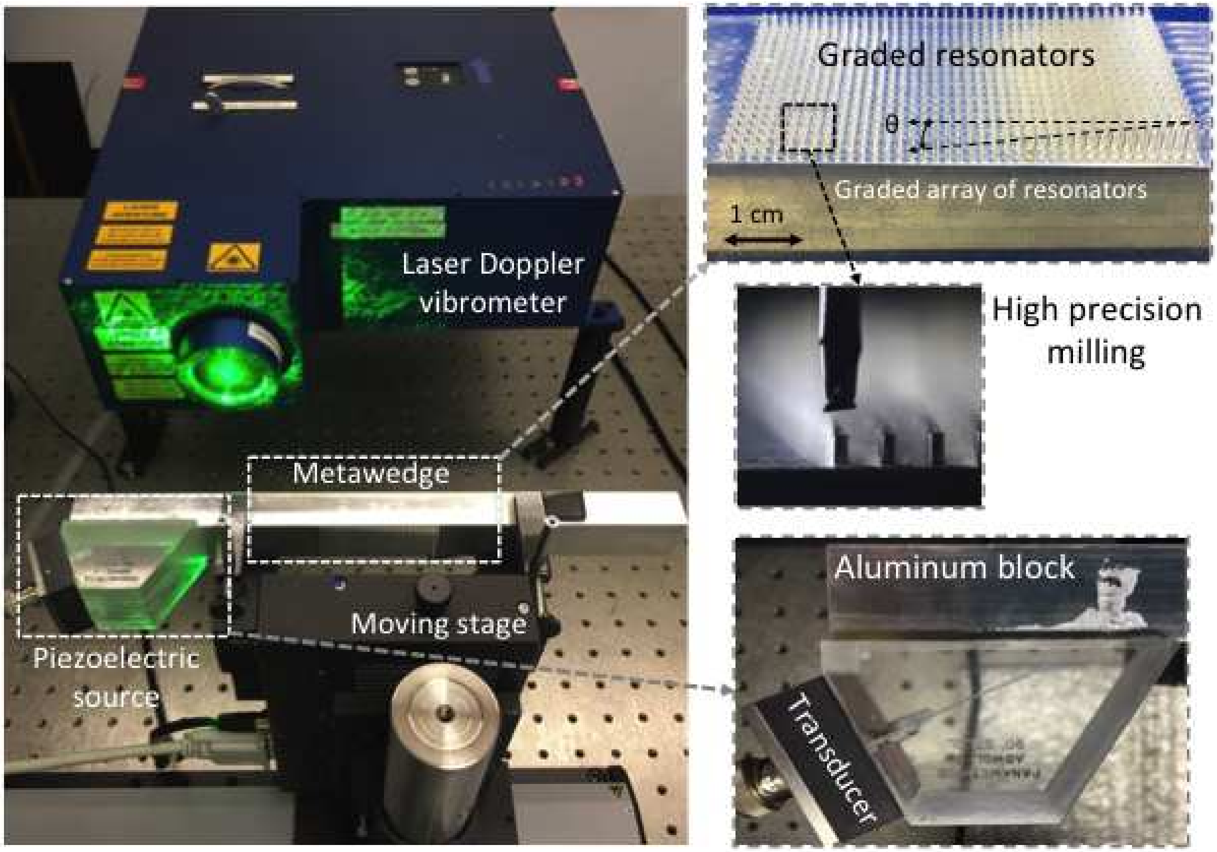


A natural metamaterial?



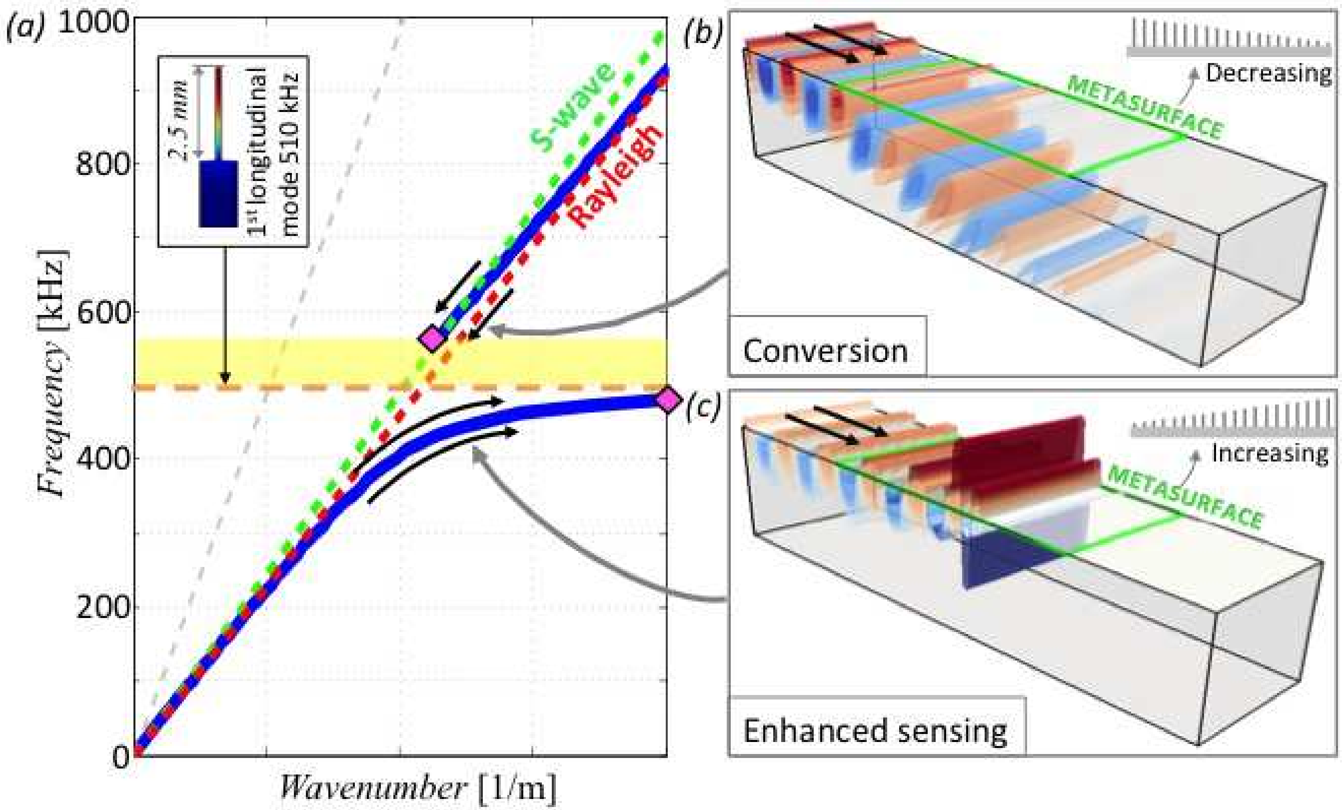
[A. Colombi, P. Roux, S. Guenneau, P. Gueguen and R. Craster “Forests as a natural seismic metamaterial: Rayleigh wave bandgaps induced by local resonances” *Scientific Reports*, **6**, 2016]

Ultrasonic experiments - scaled down in size

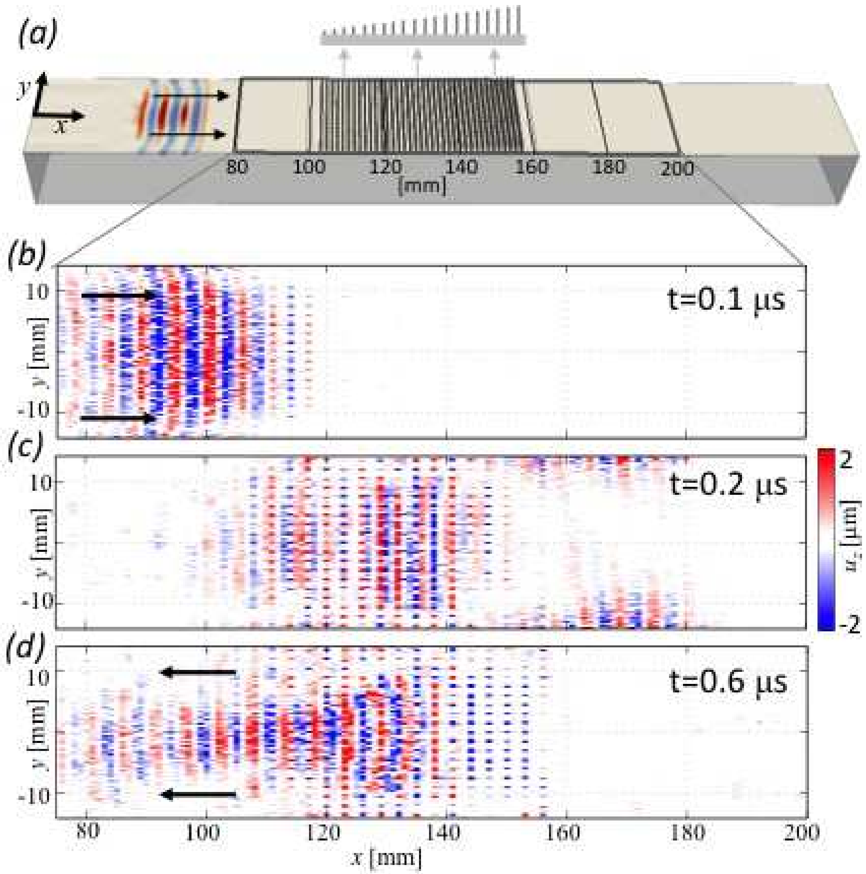


Experimental setup - laser doppler vibrometer scanning aluminium block created by micro-milling.

Scaled down in size but up in frequency

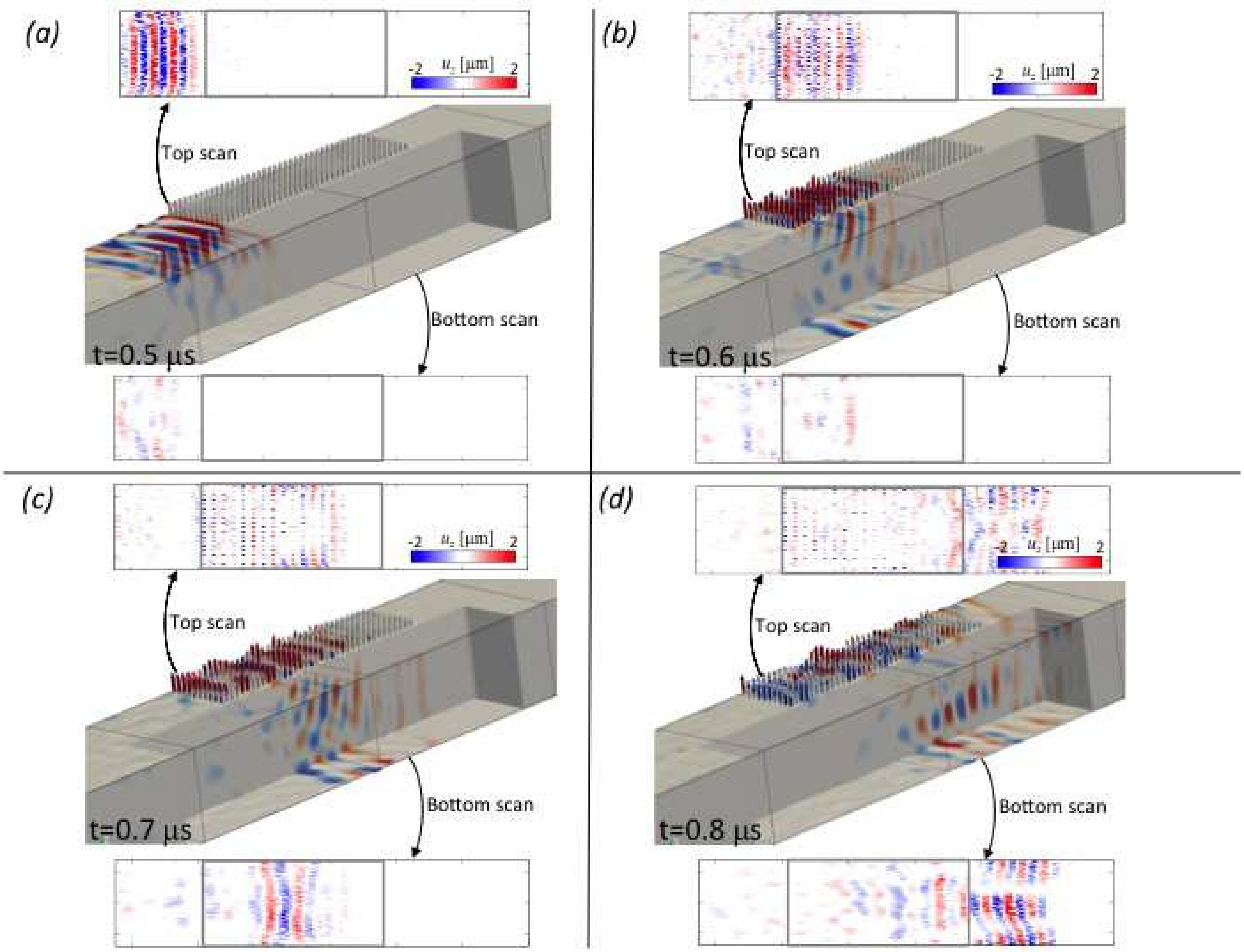


Elastic rainbow



Experimental results showing the reflection of the surface wave (filtered at 450-600 Hz)

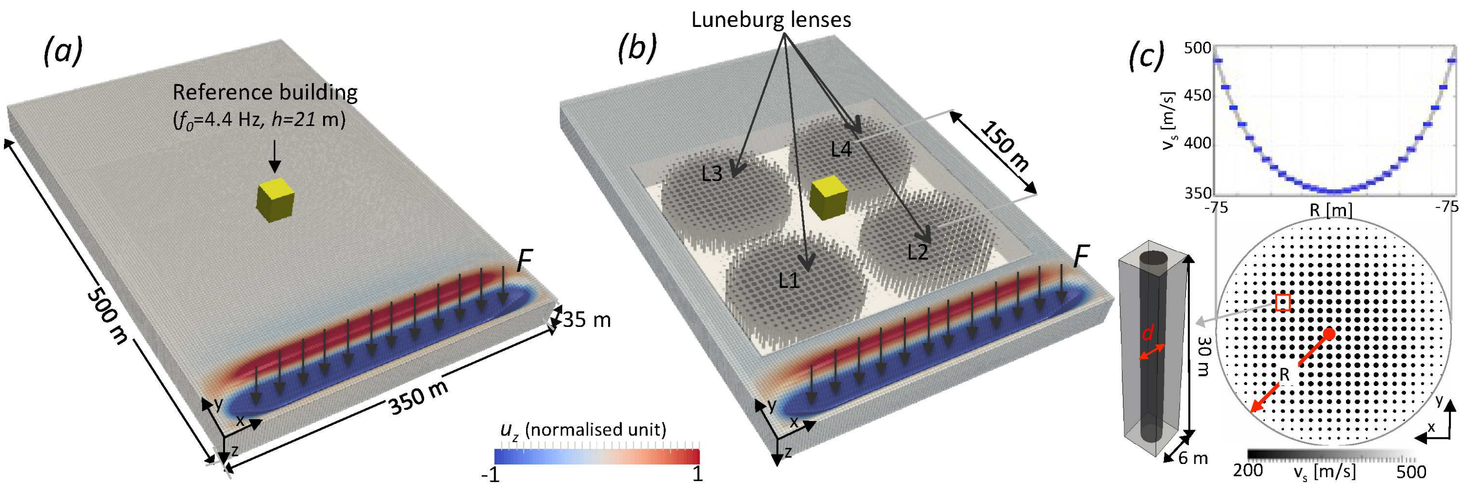
Mode conversion



Experiments and numerics

Designer elastic surfaces

Here we are trying to design elastic surfaces so that we can “steer” Rayleigh waves using ideas from metamaterials/transformation optics - Luneberg lens.

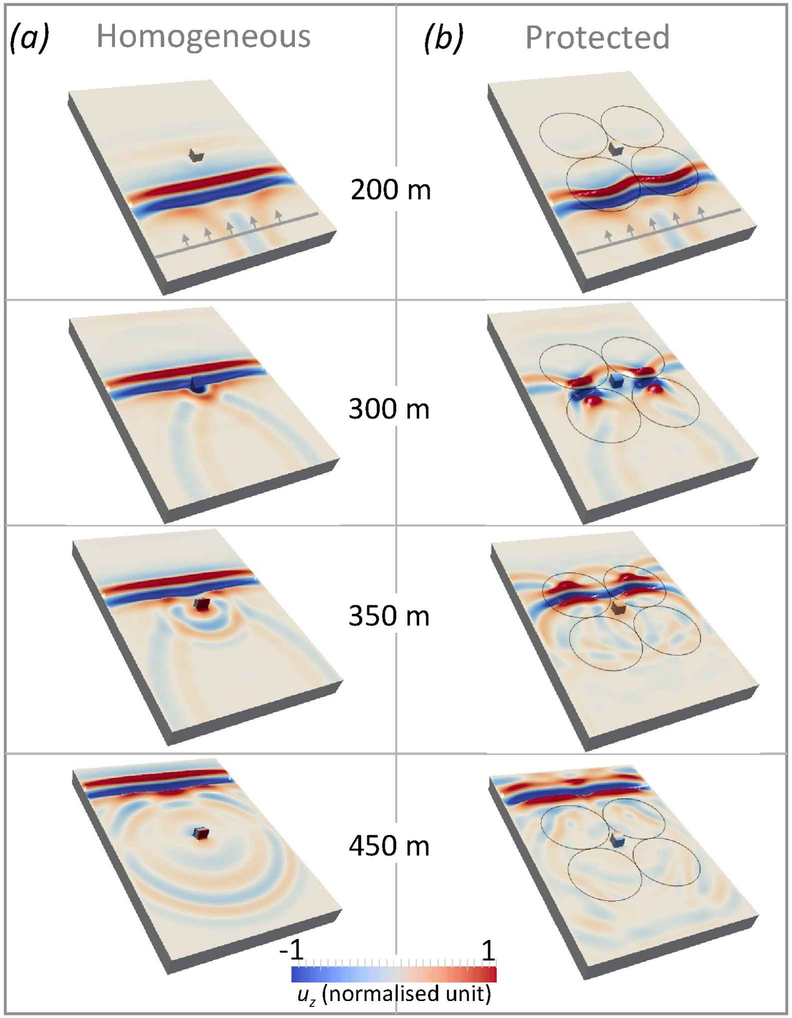


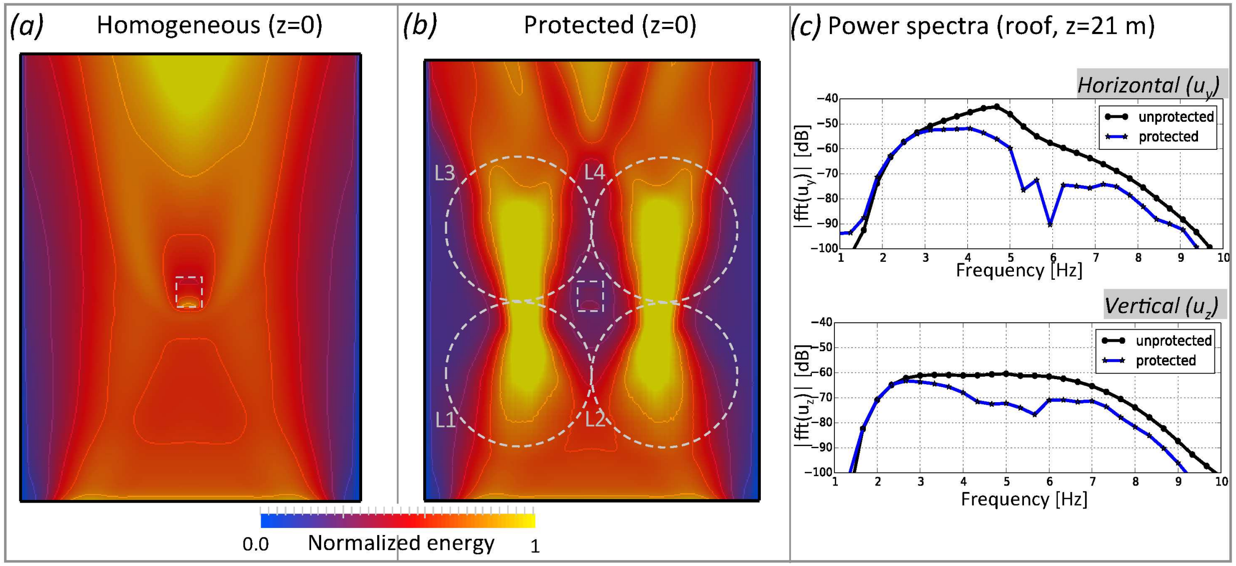
Start animation Start animation

Use the Maxwell-Garnett effective medium approach.

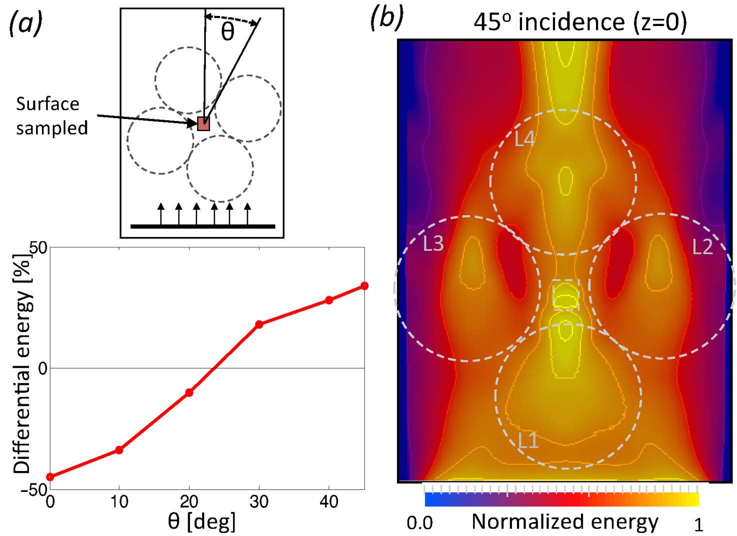
$$f = 1 - \frac{(v_{se}^2 - v_{s0}^2)(v_{s0}^2 + v_i^2)}{(v_{se}^2 + v_{s0}^2)(v_i^2 - v_{s0}^2)};$$

to tune the effective “refractive” index.





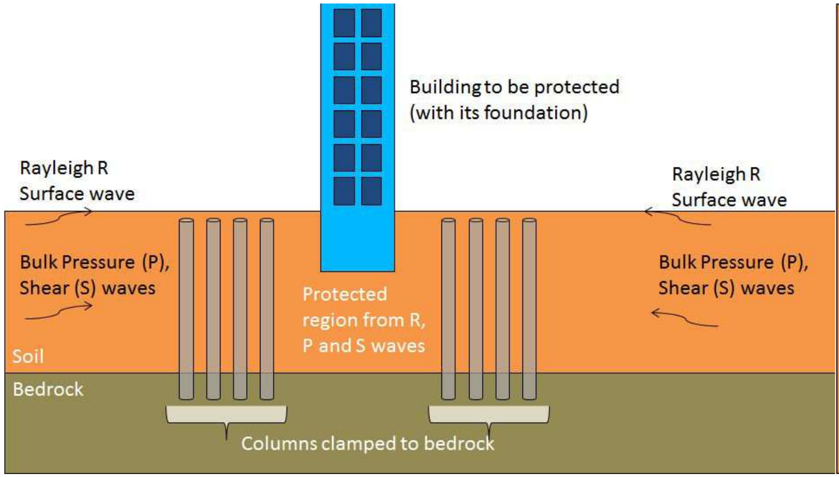
(a) Maps of the elastic energy distribution at the surface (z=0) for the homogenous case. Same as (b) but for the lenses case. (c) Spectral density of the rooftop motion of the building in dB. In the upper panel, the blue trace is calculated for the homogeneous case while the black is obtained when the lenses enclose the building. The lower panel is identical, but now for the vertical component.

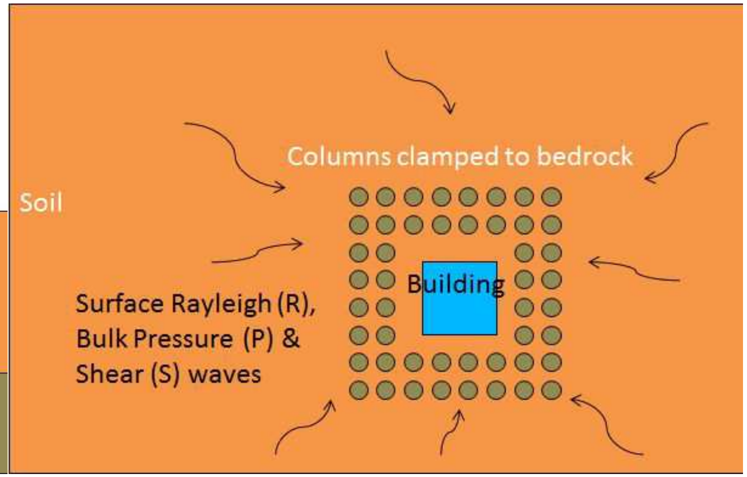


(a) Plot of the differential energy between reference and protected case *vs.* incidence angle of the wavefront with respect to the lenses. (b) Same as Fig. 3b but for a plane wave approaching the lenses with a 45° incidence angle.

Seismic "shields"

Can we design structures that protect buildings (on soft soil overlaying bedrock) from seismic waves? Important to note that we need sub-wavelength structures that operate at $0 - 10$ Hz - V. hard to achieve.





A phononic shield under construction (Brule - Menard)

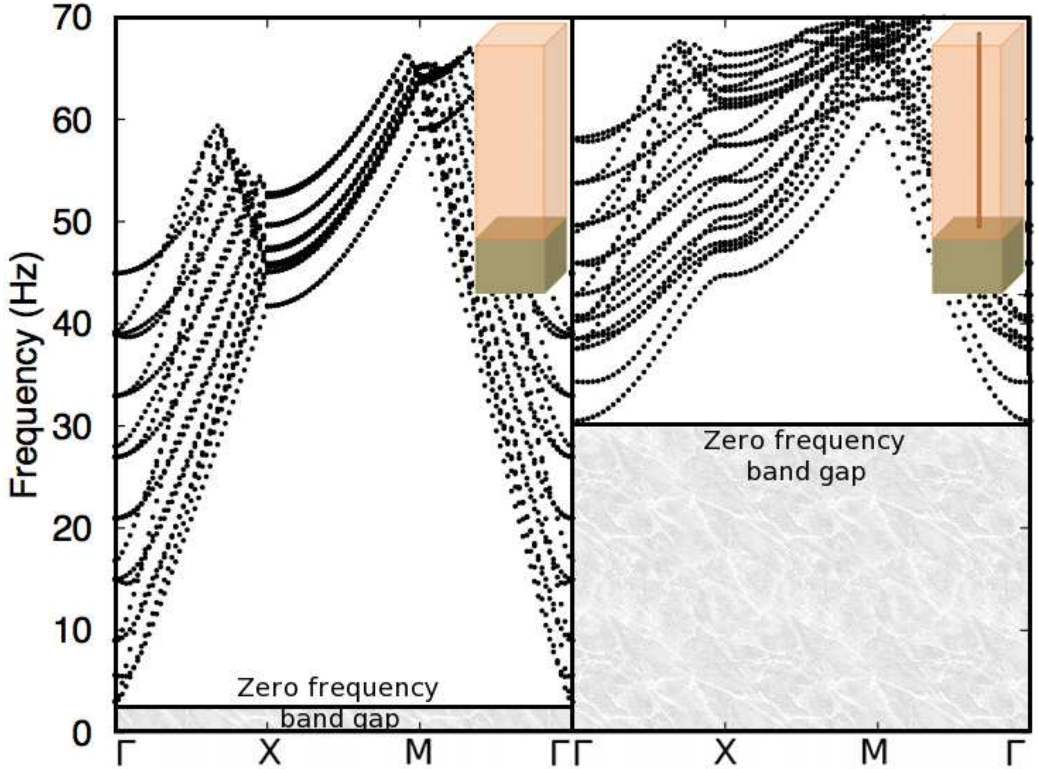
Fig. S1-2. Photograph of the seismic metamaterial experiment from the Ménard company.

Sensitive three components Velocimeters

Five meters deep 320 mm holes

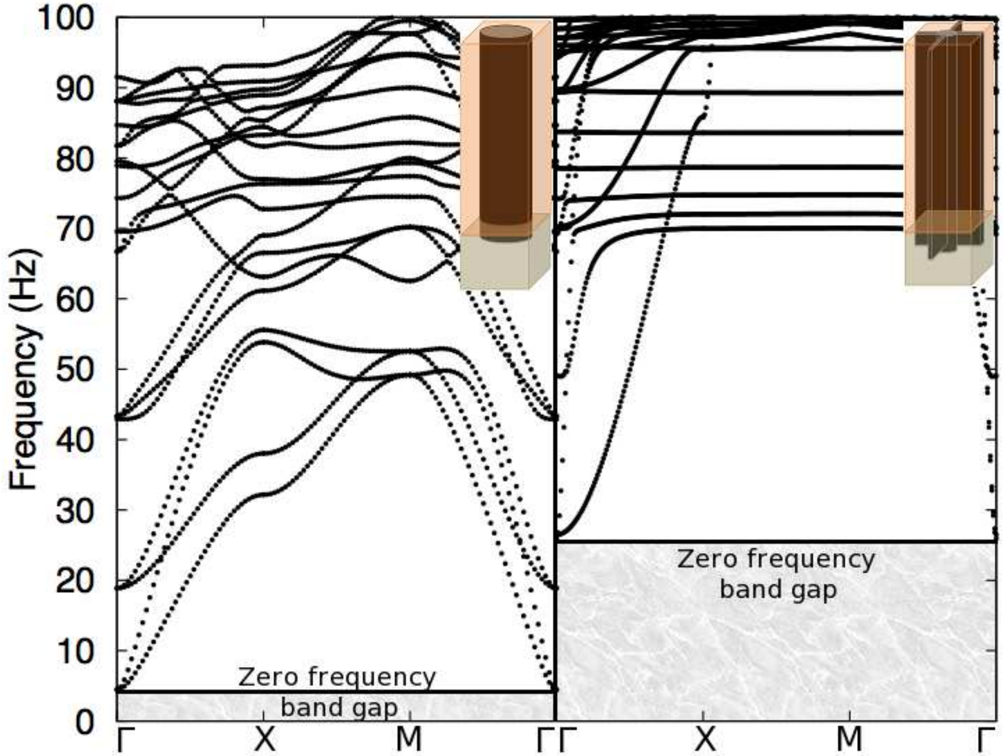
Source: - frequency: 50 Hz
- horizontal displacement : 14 mm

Idealised upper/lower bounds



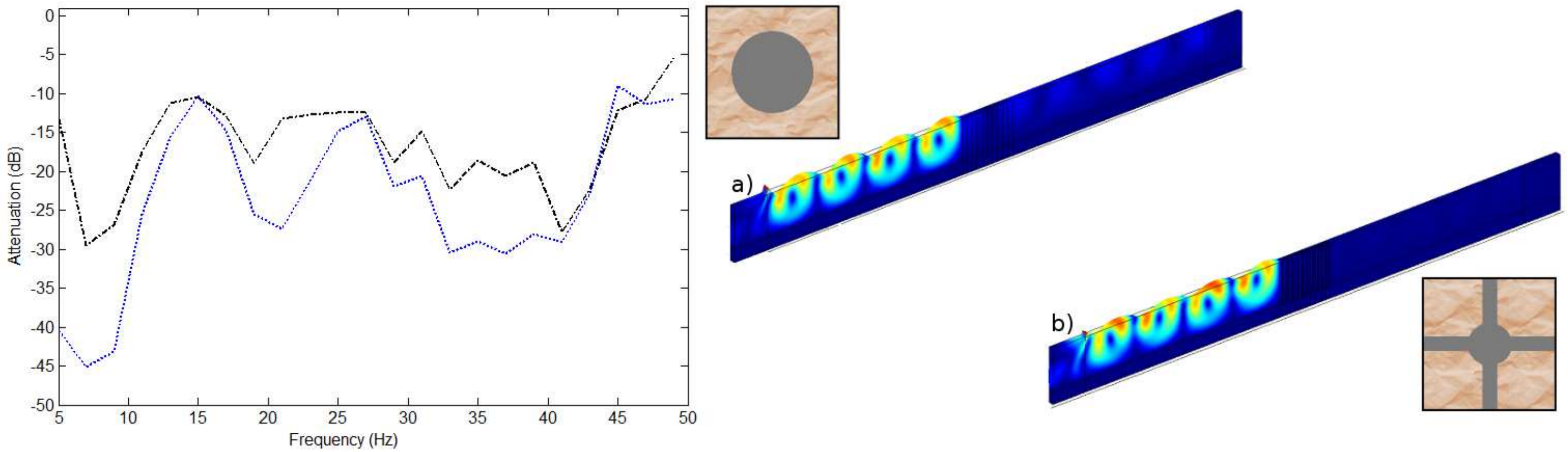
Dispersion curves for idealised cases: (a) clamped bedrock with no columns (b) a clamped perfectly rigid column (i.e. with zero displacement assumed on all its boundary) of radius 0.15 m. These are shown around the edges of the irreducible Brillouin zone $\Gamma X M$.

Real designs



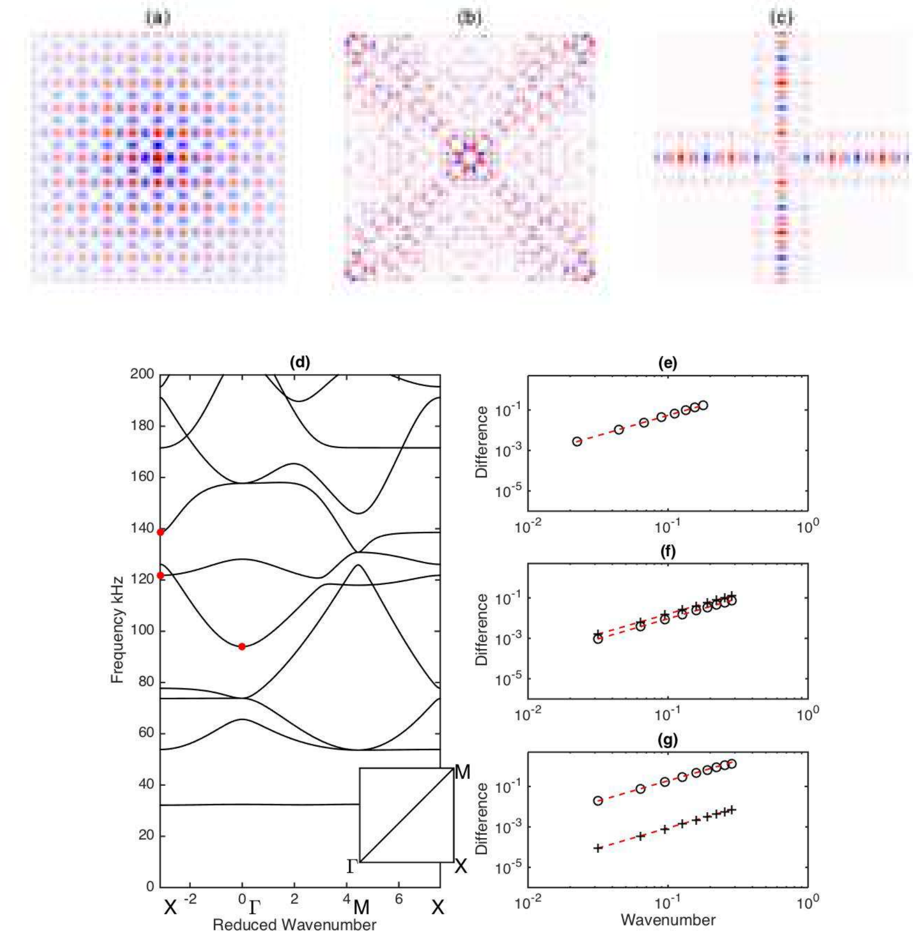
Dispersion curves for realistic cases: (a) steel cylindrical columns of radius 0.6 m clamped to the bedrock (b) steel columns of radius 0.3 m linked to their nearest neighbours via steel plates. These are shown around the edges of the irreducible Brillouin zone $\Gamma X M$

Transmission spectra



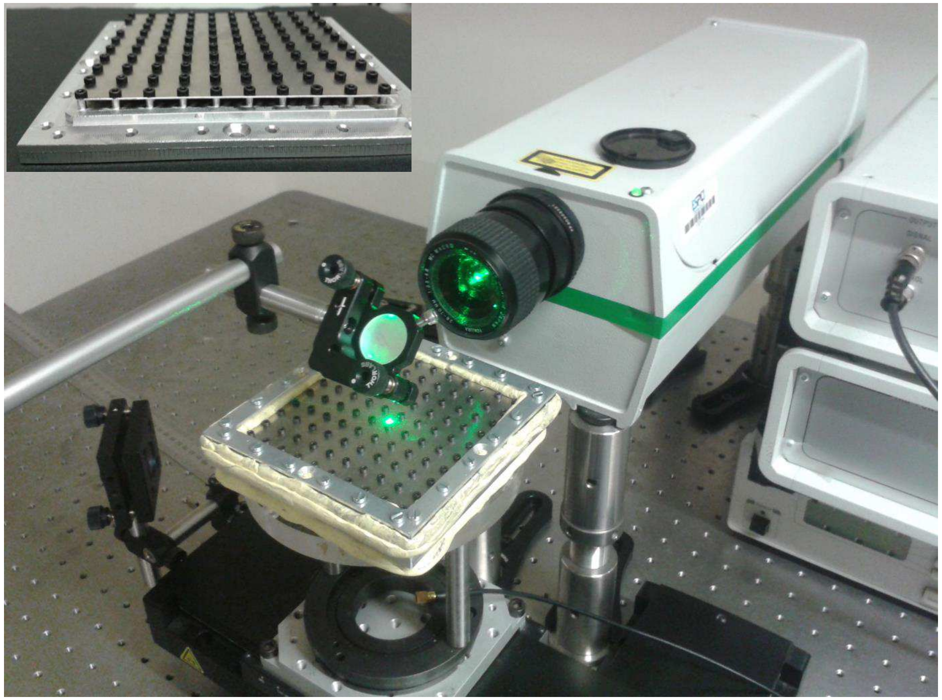
Attenuation (in dB) of incoming Rayleigh waves versus frequency (in Hz) through an array with 10 rows of clamped cylindrical inclusions of radius 0.6 m (upper curve) and clamped cross-shaped inclusions (lower curve). The absolute value of the vertical displacement at 9 Hz for (a) the steel column and (b) a column augmented by attachments to its neighbours; the geometries, which share the same filling fraction, are shown as insets.

Kirchhoff plate approximation/ HFH



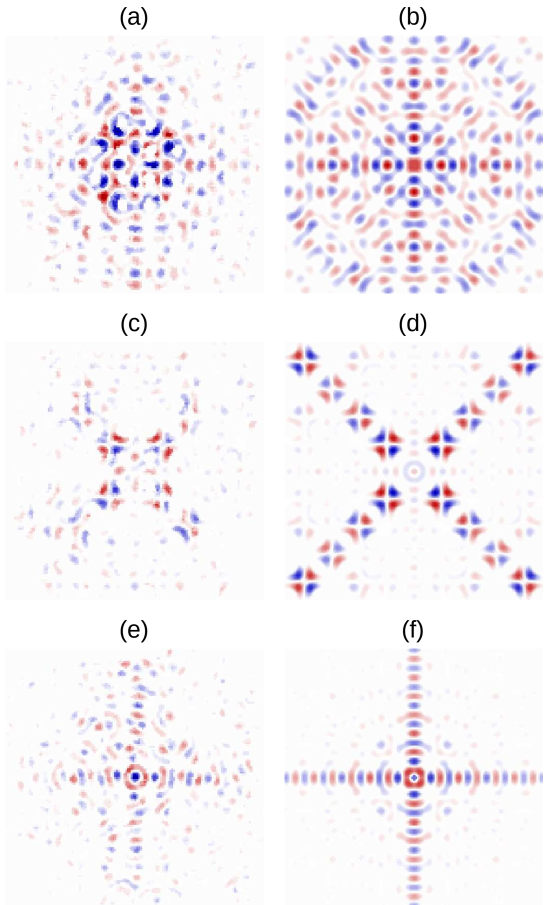
Dispersion curves, frequencies to investigate and HFH predictions.
(a-c) the flexural wave field predicted for a point frequency source placed centrally in the array. (a) 94 kHz, isotropic response as predicted by effective medium tensor components $T_{11} = T_{22} = 1025$; (b) 122 kHz hyperbolic behaviour following from $T_{11} = -180.17$ and $T_{22} = 311.74$; (c) 138 kHz creating a + shape with $T_{11} = -4189.10$ and $T_{22} = 19.51$.

Experiments (pinned elastic plates)



These are delicate experiments, highly frequency dependent. Plates of duraluminium a 10 cm $\times$ 10 cm 0.5 mm-thick vibrating plate ($\rho = 2789$ kg/m^3, E=74 GPa, ν=0.33) (with Sebbah, Lefebvre, Antonakakis, Achaoui, Guenneau)

FDTD simulations versus experimental data



Numerical versus experimental plots of flexural waves excited by a forcing of fixed frequency. (b,d,f) are snapshots in the time domain taken from full FDTD at $\Omega = 95, 110, 119$ kHz with 1 kHz of spectral bandwidth. (a,c,e) are experiments at $\Omega = 90, 105, 128$ kHz with 10 kHz of spectral bandwidth

Concluding remarks

- We have here illustrate three novel metamaterial based designs for mitigating seismic waves and ground vibration - diverting into the bulk, steering on the surface or shielding the structure.
- Much of this work is done in conjunction with Engineering and Physics Departments providing fast, accurate numerics, asymptotic approaches and complementary insight.
- Some is done with industry: Menard (Civil Engineering), EDF, Rolls-Royce, AMEC (in NDE work) or government DSTL.
- Academic papers in these areas, a couple of patent applications, edited books etc.
- For the dynamic anisotropy: Lefebvre et al PRL 2017 experiments and theory. Maling et al Wave Motion 2016 for asymptotics re Maxwell.
- Whispering Bloch Modes covered in Maling et al 2016a,b.