

# The Design of Filter Materials and their Importance in Geotechnical Engineering

## La conception de matériaux filtrants et leur importance en géotechnique

Messerklinger S.

Consulting Engineers, Poyry Energy Ltd., Zurich, Switzerland

**ABSTRACT:** The presence of water has a major influence on the design of soil structures as it reduces the effective stresses and hence shear resistance, and applies seepage forces in case of flow. This key topic is well known to every geotechnical engineer and the design principle for soil structure is to drain groundwater, infiltrated surface water or seepage water in a controlled manner from the soil. However, for soil structures whose purpose is to retain water, such as embankment dams impounding a reservoir, or dikes for flood protection along rivers and channels, both sealing and draining have to be ensured by the structures. With simple construction measures such as filter and drainage zones incorporated in earth structures composed of selected and treated materials, the stability and safety of these structures can be improved considerably. This paper discusses seepage control measures as well as the selection and design of appropriate filter materials.

**RÉSUMÉ :** La présence d'eau a une influence majeure sur la conception d'ouvrages géotechniques, car elle réduit les contraintes effectives et la résistance au cisaillement et donc applique des forces d'infiltration en cas de débit. Le sujet abordé est bien connu des ingénieurs en géotechnique. Le principe de conception des ouvrages est de drainer les eaux souterraines, les eaux de surface infiltrées ou les eaux d'infiltration de manière contrôlée à partir du sol. Toutefois, pour les ouvrages dont le but est de retenir l'eau, comme les barrages de retenue en remblai ou les digues de protection contre les inondations le long des rivières et des canaux, l'étanchéité et le drainage doivent être intégrés dans les ouvrages. Avec de simples mesures constructives telles que de des zones de filtrage et de drainage dans des ouvrages en terre, et composées de matériaux sélectionnés et traités, la stabilité et la sécurité de ces ouvrages est considérablement améliorée. Le présent document examine les mesures de contrôle d'infiltrations dans les ouvrages et la sélection et conception de matériaux filtrants appropriés.

**KEYWORDS:** filter design, seepage control, embankment dam failure

## 1 INTRODUCTION

### 1.1 *Effect of water in soils*

Soils are composed of single particles. The loads are transferred at the particle contacts with normal and shear forces<sup>1</sup>. The maximum shear force which can be transferred at the particle contact is proportional to the effective normal force at the contact, as defined by the total interparticle force and the pore water pressure, should the soil be saturated. The porewater pressures can correspond to the (a) hydrostatic head, should the soil skeleton be submerged, or (b) to an excess pressure which exceeds the hydrostatic head. Excess pressures develop for example (a) in loose deposits of low permeable granular soils, such as silts and fine sands, during an earthquake event (see e.g. Messerklinger et al., 2011a) or (b) by the application of an external load, e.g. during construction work, on compressible and low permeable soils such as clays and silts.

Summarizing: The water of a submerged soil skeleton reduces the effective interparticle forces and hence the shear resistance of the soil.

If the water in the soil skeleton is flowing with a velocity ( $v$ ) at a hydraulic gradient ( $i$ ), forces due to water flow are applied on the soil particles. These flow forces on the soil particles act in addition to the pore water pressures. The flow forces ( $F$ ) act in the flow direction. Their magnitude is  $F=i \cdot \gamma_w \cdot A$  where  $\gamma_w$  is the unit weight of the water and  $A$  is the cross-sectional area (in

flow direction) of the soil body the water is flowing through. This is the average force on a soil body due to water flow at a hydraulic gradient of  $i$ . However, the flow forces acting on a single particle vary significantly. The flow velocity of the water in the pore space depends on the pore diameter and increases approximately with the square of the pore diameter. If the pore diameter changes, the pore flow velocity will also change. However, in permeability tests only the overall soil permeability, as defined by the permeability coefficient ( $k$ ), is determined.

Summarizing: In case a hydraulic gradient is applied to the water in a submerged soil skeleton, the water will flow around the single particles, which applies flow forces in flow direction. These flow forces depend on the hydraulic gradient and are independent of the volume of water flowing through the soil.

### 1.2 *Effect of water on natural or man-made soil structures*

These effects of water on the soil influence the stability of soil structures, irrespective of whether they are natural or man-made. Natural soil structures are for example: (i) soil slopes; (ii) in-situ soils surrounding a man-made excavation or (iii) soil foundations of buildings or embankment dams. Man-made soil structures are for example the embankment dams<sup>2</sup> themselves. Subsequently, the effect of water on the stability and safety of such structures is discussed.

<sup>1</sup> For clays interparticle forces can act in addition to contact forces. The contact forces are defined by the self-weight of the soil and the external loads. Interparticle forces are e.g. (i) electromagnetic attractions, which are commonly called van der Waals forces or (ii) electrostatic repulsive or attractive forces at double layers.

<sup>2</sup> Note: Embankment dams can be used for different purposes such as road embankments, landfills, off-shore embankments e.g. for sea water intakes, shore protection or embankments surrounding reservoirs.

### 1.2.1 Soil slopes

Natural soil slopes are subjected to the natural groundwater flow conditions, and are formed of the given in-situ soil material which may have predefined slip surfaces with reduced shear strength, due to previous slides. These were subjected to earthquakes and weather conditions typical in the region and have correspondingly an overall factor of safety for slope stability of somewhat above one.

Constructing in or on natural soil slopes either reduces the resisting forces, e.g. by excavation, or applies driving forces, e.g. when structures are founded on the slope. For slopes within or next to man-made structures an overall factor of safety of well above one is desirable. Hence, the stability of natural slopes next to or within the construction area normally has to be improved.

The stability of slopes can be improved by man-made structures which apply resisting forces such as anchors, piles, etc. Or the slope stability can be improved by lowering the groundwater level in the slope to increase the effective stress and hence shear resistance at the drained soil. The water level can be lowered by drainage, e.g. with borings filled with filter and drainage materials and free or pumped outflow (see e.g. Messerklinger 2012).

### 1.2.2 Soils surrounding a man-made excavation

The excavation in saturated soil can be surrounded by an (i) impermeable or (ii) a permeable wall. For an impermeable wall, both the earth pressure and the water pressure act on the wall, and the pressure can be in the order of two to three times that for a permeable wall, on which only the effective earth pressure is acting. Lowering the water level behind the wall, e.g. by pumped wells or by drainage into the excavation pit, reduces the loads on the wall. However, this imposes a hydraulic gradient on the in-situ soil surrounding the excavation. This hydraulic gradient applies flow forces on the particles of the soil and these forces can cause transport of fine soil particles within the coarse soil particle skeleton for internally unstable soils (criteria see Chap. 2). At the surface where the water leaves the soil body, e.g. at the pumped drainage well or at other drainage points, the soil can be eroded unless the surface is protected with a filter and drainage material.

### 1.2.3 Soil foundations of a dam impounding a reservoir

With the impounding of the reservoir a hydraulic gradient and water pressure are applied on the soil foundation. The increased hydrostatic water pressure reduces the effective stresses and hence strength of the soil. The imposed hydraulic gradient applies flow forces onto the soil particles which can cause erosion within the soil skeleton or at the surface of the soil body where the water flows out of the soil. A layer of filter material at the water exit below a layer of drainage material will prevent erosion of soil particles and increase the effective stress. Examples are presented in Messerklinger et al. 2010 and 2011b.

### 1.2.4 Man-made embankment dams for reservoirs

With the impounding of the reservoir the hydraulic gradient and the water pressure are applied onto the man-made earth fill. Man-made earth structures allow for the placement of filter and drainage zones within the dam body. This is normally supported by a zone of reduced permeability (e.g. clay, concrete, asphalt, geomembrane) which reduces the volumes of water flowing through the structure (see Messerklinger 2011c). The incorporation of filter materials assures the stability and safety of embankment dams.

### 1.3 Summary

Water has a major influence on the stability and erosion resistance of natural and man-made soil structures as it reduces the effective stress and hence shear strength of the soil and applies forces in case water is flowing through the soil.

Hence, draining the water out of the soil structures improves their stability or the stability of structures built on or in the soil.

However, draining of the soil has to be done in a controlled manner. The hydraulic gradients and hence flow forces applied on the soil particles must not erode particles within the soil skeleton or at the surface.

For natural soils, this is assured by limiting the hydraulic gradient. For man-made structures the erosion is controlled by filter zones incorporated in the soil structure. The design of suitable filter materials is discussed in the next chapter.

## 2 DESIGN OF FILTER MATERIALS

For the design of state-of-the-art filter materials, the following six aspects are considered: (a) Filter ability (b) Internal stability (c) Self healing (d) Material segregation (e) Drainage capacity (f) Material durability.

### 2.1 Filter ability

With the identification of effective stresses in soils by Terzaghi and his co-workers in the early thirties of the last century, (Terzaghi 1936) a new era in soil mechanical engineering was initiated. This was the time when the effects of water on soil were investigated in depth, and resulted in the development of the consolidation theory (Terzaghi & Fröhlich 1936).

At the same time, Bertram (1940) proposed the criterion  $D_{15\text{filter}}/d_{85\text{base soil}} \leq 6$  for soil filters based on laboratory investigations. This filter criterion was later modified to  $D_{15\text{coarse-side filter}}/d_{85\text{fine-side base soil}} \leq 4$  and a drainage criterion of  $D_{15\text{fine-side filter}}/d_{85\text{coarse-side base soil}} \geq 4$  was added by Terzaghi and Peck (1948), (Fig.1). These filter and drainage criteria were used for decades and are still today subjects lectured on to the bachelor and master students.

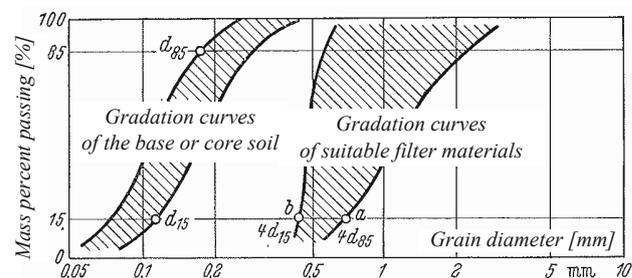


Figure 1: Filter and drainage criteria from Terzaghi & Peck (1948).

The filter design was reconsidered after incidents at and failures of major dam structures. E.g. after the Balderhead dam incident, where core material was eroded from an open fracture in the core zone into the filter material so causing sinkholes at the dam crest (Vaughan et al. 1970), Peter Vaughan and his co-workers searched for what they called the “perfect filter”. The idea was to hold back the smallest grain of a core material even under severe conditions such as concentrated seepage flow at high hydraulic gradients through e.g. a crack in the core. The approach towards the criterion was not via the gradation curve, such as adopted before by Terzaghi and his co-workers, but by the permeability coefficient of the filter material. Vaughan believed that “...effectiveness of a filter may be defined by its permeability with more generality than by its grading.” (Vaughan & Soares 1982, p.17). They proposed a linear correlation between the permeability coefficient ( $k$  in m/s) and the filtered particle diameter of  $k = 6.1E-6 \cdot \delta^{1.42}$  ( $\delta$  in  $\mu\text{m}$ , Note: The particle size of clays with flocculated structure is the floc-size.).

At the same time, James Sherard was investigating the cracking and failure of embankment dams built in the United States (Casagrande 1950, Sherard et al. 1963, Bertram 1967). In 1973 he wrote (p. 272): “... at present it is well known that cracks have developed in the impervious sections of many dams

...". He identified that the cracking was mainly caused by differential settlement of homogenous clay dams or by hydraulic fracturing of the core material due to the water pressure after impounding of the reservoir.

Numerous filter tests were performed (Sherard et al. 1984a), and based on the slot test data (Sherard et al. 1984b) four soil categories with four individual filter criteria were identified:

- 1.) Sandy silts and clays ( $d_{85b}$ : 0.1-0.5 mm):  $D_{15f}/d_{85b} \leq 5$
- 2.) Fine-grained clays ( $d_{85b}$ : 0.03-0.1 mm):  $D_{15f} \leq 0.5$  mm
- 3.) Fine-grained silts ( $d_{85b}$ : 0.03-0.1 mm):  $D_{15f} \leq 0.3$  mm
- 4.) Exceptionally fine soils ( $d_{85b} < 0.02$  mm):  $D_{15f} \leq 0.2$  mm

With the non-erosion filter test the filter criteria were further developed and termed criteria for "critical filter" (Sherard & Dunnigan 1985, 1989) as distinct from the "perfect filter" discussed above. For the critical filters four categories were defined based on the fines content ( $<0.075$  mm, sieve 200) of the base soil (or core material). The fines content was determined on a gradation curve with a maximum grain diameter of 4.75 mm (sieve 4). For base soils with a maximum grain size exceeding 4.75 mm, the gradation curve was regraded to  $\leq 4.75$  mm in order to determine whether the base soil falls into category 1, 2 or 4. Whether the base soil falls into category 3 was determined on the original, non-regraded curve. For each of the 4 categories a filter criterion was defined (Tab. 1). These criteria still apply today. The current design approach is to use the conservative values of these criteria, as given in the right column of Tab. 1.

Table 1. Filter criteria.

Soil group	Fines content $<0.075$ mm	Filter criterion determined by tests after Sherard & Dunnigan (1989)	State-of-the-Art criteria in dam engineering
1	85-100	$D_{15f} = 7d_{85b}$ to $12d_{85b}$	$D_{15f} \leq 9d_{85b}$
2	40-80	$D_{15f} = 0.7$ to 1.5 mm	$D_{15f} \leq 0.7$ mm
3	0-15	$D_{15f} = 7d_{85b}$ to $10d_{85b}$ *	$D_{15f} \leq 4$ to $5 d_{85b}$ †
4	15-40	Intermediate between group 2 and 3	Intermediate between group 2 and 3

\*For subrounded grain shape 7 and for angular grains 10.

†Incorporates a factor of safety of two.

## 2.2 Internal stability

For filter materials to be internally stable means that within the soil skeleton the small particles do not move due to water flow forces. All soil particles should remain at their position even for water flow at high ( $\gg 1$ ) hydraulic gradients such as occur at a fracture in the sealing zone of an embankment. A good definition of internal stability is given e.g. by Kenney & Lau (1985): "Internal stability of granular material results from its ability to prevent loss of its own small particles due to disturbing forces such as seepage and vibration." In more recent literature, the term internal stability is used in a much broader sense<sup>3</sup>. However, in this paper the term will be used for the filter material design, and the internal stability of natural soils (in the foundation or dam fill) will be discussed at the end of this chapter.

Concerning the formation of sinkholes at the crest of zoned embankment dams, James Sherard (1979) studied the phenomenon and recommended use of a method proposed by

<sup>3</sup> Fell and his co-workers in Australia (e.g. Foster and Fell 1999) discussed internal erosion by investigating the erosion process within the soil structure. They divided the erosion process into four steps: (i) initiation (ii) continuation (iii) progression (iv) breaching/ failure. The term internal erosion was divided into four sub-categories: (a) Concentrated leak erosion (b) Backward erosion (c) Contact erosion (d) Suffusion. These four sub-divisions were taken over by the latest ICOLD Bulletin "Internal Erosion of existing Dams and their Foundations." Hence, what was previously termed internal stability of filters now falls into the sub-category (d) Suffusion.

Prof. Victor de Mello (1975) for the investigation of gap-graded soils, in order to assess the internal stability of filter materials.

In this method, which is also called "retention ratio criterion", the gradation curve of the filter material is divided into two curves at a selected grain diameter ( $d_s$ ), gradation curves for the portions finer and coarser than  $d_s$ , respectively. For the two gradation curves the retention ratio ( $R_R$ ) is calculated from the Terzaghi filter criterion:  $R_R = D_{15f}/d_{85b}$ . This is repeated for different values of  $d_s$ . All grains are considered to be stable if they satisfy the criterion  $R_R \leq 7 \div 8$  for subrounded grains or  $R_R \leq 9 \div 10$  for angular grains. The grain diameters ( $d_s$ ) for which the retention ratio exceeds the given limits are potentially unstable and can be eroded by the water flow. Using this criterion to identify stable materials shows that gradation curves with a more or less straight line in the semi-logarithmic plot are stable.

Experimental investigations performed by Kenney & Lau (1985 and 1986) lead to a strict criteria in which the gradation curve of the fine part of the filter material ( $0 < M\% < 30$ ) should be on the more uniform side of the Fuller curve and the gradation curve of the coarser part of the filter material ( $30 < M\% < 100$ ) should be on the more uniform side of a straight line in the semi-logarithmic plot with a uniformity coefficient of  $C_u \leq 12$  (Kenney & Lau 1986).

With this criterion, rather uniform filter materials are defined as internally stable. Such materials can be produced for man-made structures but they are rare in nature e.g. in soils present in the foundation of dams. Hence, for the assessment of natural soils with respect to internal stability, the approach is not to define the gradation but the critical hydraulic gradient. These studies were first done in Russia with the start of the construction of large run-of-river power plants in the 1920s (e.g. Pavlovsky 1922). Patrashev (1965) proposed a suffusion criterion and Pravedny (1976) a criterion for contact erosion. These criteria are not further discussed in the present paper as they are not applied for the design of man-made filter materials.

## 2.3 Self healing

Self-healing means that cracks which can form in the filter zone due to e.g. differential settlement, etc. do not stay open but close in case of water flow. Hence, the filter material must not have cohesion. This is assured by limiting the content of non-plastic ( $I_p < 5\%$ ) fines to less than 5% (the latest ICOLD Bulletin on CFRD's, No. 141, allows 7% of fines). The sand-castle test (Vaughan & Soares 1982), confirms that the selected filter material meets the self-healing requirements.

## 2.4 Material segregation

When the filter material segregates, meaning that the coarser particles separate from the finer particles, the filter zone can no longer fulfill its purpose of preventing fine particles moving from the core to the filter zone or within the filter zone, because the segregated coarse grained components do not form a filter to the adjacent materials. Hence, the segregation of filter materials has to be avoided. Whether a material segregates depends on the handling and placement methods and on the gradation of the material. In the 50's and 60's of the last century, segregated material zones were improved manually. Later, the focus was put on the selection of appropriate gradation curves. One of the first discussions on segregation criteria is given in Sherard et al. (1984b) where they proposed a coarse boundary for filter materials (see also Fig. 2). It was generally agreed that a high content of sand and a small maximum grain size reduces the segregation. Based on observations and laboratory investigations (e.g. Sutherland 2002) a stricter criterion was presented by Milligan (2003), which specifies that wetted filter material with a gradation finer than the limit curve given in Figure 2 should be selected. The latter criterion is nowadays commonly applied.

## 2.5 Drainage capacity

The Terzaghi criterion  $D_{15f}/d_{85b} \geq 4$  still applies and Sherard recommends  $D_{15f} \geq 0.2$  mm.

## 2.6 Material durability

The durability of filter materials is typically investigated with standard tests such as the Los Angeles abrasion test (ASTM C535) or the wet and dry strength variation (typical limit  $\leq 35\%$ ). However, for important dam structures a mineralogical and chemical investigation of the dam material is recommended. This can highlight if the material has inclusions of (i) swelling clay minerals or (ii) minerals which dissolve in water, e.g. gypsum or carbonate rocks. Latter materials cannot just dissolve but also re-cement at the particle contacts and create true cohesion. Materials with carbonate and sulphide content should be used with care for dam filter materials.

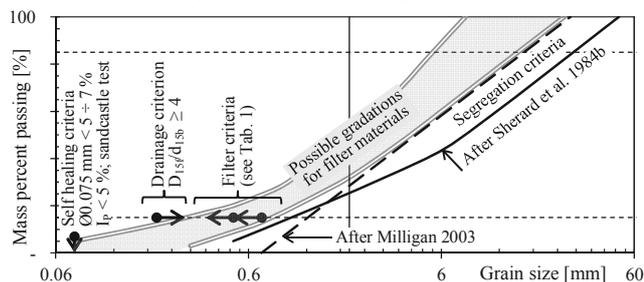


Figure 2: Summary of filter criteria.

## 3 CONCLUSIONS

Water has a major influence on the stability and erosion resistance of natural and man-made soil structures. Draining the water out of the soil structure improves its stability. However, draining of the soil has to be done in a controlled manner without erosion. This is achieved with filter materials placed in or on the soil structures. Filter materials have to have certain properties which are described by filter criteria, significant development of which took place during recent decades. Nowadays, these filter criteria are composed of six different parts and for each of these criteria are defined which have been discussed in this paper in detail.

Despite of all the efforts in filter design a significant number of failures still occurs due to erosion. Embankment dam failures are given e.g. in the ICOLD Bulletin "Internal Erosion of Existing Dams and their Foundation". About 4 of 10'000 large dams fail per year and 2 of these failures are caused by internal erosion. Overall about one embankment dam in 180 fail during its lifetime.

A recent example is the failure at the Prudencia hydro-power plant in Panama, where a homogenous dam made of residual soils failed at the contact to a concrete gravity structure. Neither in the dam body or at the dam toe, nor at the contact to the rigid concrete wall, was any filter material placed. This supported the failure mechanism which was triggered in the first place by leakage in the geo-membrane sealing (Messerklinger, 2013).

Although the design of filter materials and their application to soil structures is taught in undergraduate classes and is well known to geotechnical engineers, the lack of the design and placement of filter materials still causes numerous failures. Hence, further efforts on the selection of appropriate filter materials and their incorporation in soil structures are essential.

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