An elastic-viscous-plastic modeling of time-dependent behaviors of overconsolidated clays

Un modèle élasto-visco-plastique pour les argiles surconsolidés

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ABSTRACT: The instant normal compression line is proposed by analyzing the existing theories and experimental results. Based on the creep law, the relationship between the aging time and the overconsolidation parameter is built. With the reloading equation of the UH model used to calculate the instant compression deformation, a one-dimensional stress-strain-time relationship is proposed. Furthermore, the characteristic rate that is a function of the overconsolidation parameter is defined. Then a three-dimensional elastic-viscous-plastic constitutive model is suggested by incorporating the equivalent time into the current yield function of the UH model. The proposed model can describe not only creep, rate effect and other viscous phenomena, but also shear dilatancy, strain softening and other behaviors of overconsolidated clays. Besides, it needs only one additional parameter (the coefficient of secondary compression) to consider the creep law compared with the modified Cam-clay model.

RÉSUMÉ : Une ligne de compression normale instantanée est proposée en se basant sur les théories existantes et sur les résultats expérimentaux. Basée sur la loi de fluage, la relation entre le temps de vieillissement et le paramètre de surconsolidation est établie. En utilisant l'équation de rechargement du modèle UH pour calculer la déformation de compression instantanée, on propose une relation contrainte-déformation-temps unidimensionnelle. Par ailleurs, on définit le taux caractéristique qui est fonction du paramètre de surconsolidation. Ensuite, un modèle tridimensionnel visco-élasto-plastique est proposé en intégrant un temps équivalent dans la fonction de charge actuelle du modèle UH. Le nouveau modèle peut décrire non seulement l'effet de fluage, l'effet de taux et d'autres phénomènes visqueux, mais également dilatance, adoucissement et d'autres comportements d'argiles surconsolidés. En outre, il n'y a besoin que d'un seul paramètre supplémentaire (le coefficient de compression secondaire) dans la loi de fluage par rapport au modèle Cam-clay modifié.

KEYWORDS: creep, viscoplasticity, stress-strain, overconsolidation, three-dimensional.

1 INTRODUCTION

Viscosity refers to the time-dependency of stress-strain relationship of clays. The phenomena related to viscosity include creep and rate effect which would change the engineering properties of clays and result in such engineering problems as ground settlements and landslides. Therefore, numerous studies on viscosity have been conducted and large numbers of constitutive models have been developed. The models considering viscosity could be built with the empirical method, the viscoelastic method or the viscoplastic method. However, the empirical models are strictly limited to the specific boundary and loading conditions, and the linear viscoelastic models are not well valid for the behaviors in the range of large strains. Hence, more and more studies are focusing on the viscoplastic models (e.g., Kutter and Sathialingam 1992, Yin and Graham 2002, Yin et al. 2010). Nevertheless, most of the models proposed so far are only applicable to normally consolidated (NC) and slightly overconsolidated (OC) clays. Although a few viscoplastic models have been extended to describe viscosity of OC clays (e.g., Kutter and Sathialingam 1992), they are all complex and have plethoric parameters. Hence it is necessary to build a new elastic viscoplastic model for OC clays.

2 UH MODEL

The UH model (Yao et al. 2009) is a constitutive model for OC clays based on the modified Cam-clay model (MCC). As shown in Figure 1, there are two yield surfaces in the UH model, i.e., the current yield surface and the reference yield surface. The current yield surface is the one where the current stress point (p, q) lies. p is the mean effective stress and q the generalized deviatoric stress. The reference yield surface passes the reference stress point $(\bar{p},\bar{q})$ which is the corresponding point of (p, q) at the same stress ratio η ($\eta=q/p$). The UH model defines the overconsolidation parameter R to reflect the degree of overconsolidation of clays. R is the ratio of the current stress to its corresponding reference stress. A smaller value of R corresponds to a larger OCR (overconsolidation ratio). The expression of R is

$$R = \frac{p}{\bar{p}} \tag{1}$$

In the UH model, the potential failure stress ratio M_f is suggested to reflect the strength of OC clays. M_f is expressed as follows (Yao et al. 2012).

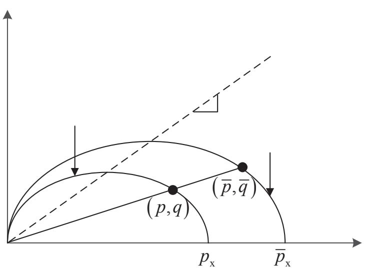

Figure 1. Current and reference yield surfaces of the UH model.

$$M_f = 6\left[\sqrt{\frac{\chi}{R}\left(1+\frac{\chi}{R}\right)}-\frac{\chi}{R}\right] \qquad \chi = \frac{M^2}{12(3-M)} \tag{2}$$

where M is the stress ratio at the critical state. Based on the concept of the potential failure ratio, the UH model changes the hardening parameter of MCC into the unified hardening parameter H that is expressed as

$$H = \int dH = \int \frac{M_f^4 - \eta^4}{M^4 - \eta^4} d\varepsilon_v^p \quad (3)$$

where ε_v^p is the total plastic volumetric strain.

By combining the elliptical yield function and the unified hardening parameter, the current yield function of the UH model can be written as

$$f_1 = \ln\frac{p}{p_{x0}} + \ln\left(1 + \frac{\eta^2}{M^2}\right) - \frac{1}{c_p}H = 0 \quad (4)$$

where p_{x0} is the initial intersection point of the current yield surface and axial p. $c_p=(\lambda-\kappa)/(1+e_0)$. λ is the slope of the normal compression line (NCL) in e-ln$p$ plane, κ the slope of rebound curve in e-ln$p$ plane, e the void ratio and e_0 the initial void ratio. Because the reference surface represents the normally compressed states of clays, the UH model adopts the yield function of MCC as the reference yield function.

$$f_2 = \ln\frac{\bar{p}}{\bar{p}_{x0}} + \ln(1 + \frac{\eta^2}{M^2}) - \frac{1}{c_p}\varepsilon_v^p = 0 \quad (5)$$

where $\bar{p}_{x0}$ is the initial intersection point of the reference yield surface and axial p.

3 INSTANT NORMAL COMPRESSION LINE

3.1 *Instant compression and delayed compression*

As shown in Figure 2, in tests the isotropic compression represented by NCL experiences some test time because of the limitation of permeability. If clays creep for the same time from different points on NCL, such as C and C′, the final states will shape a line parallel to NCL, i.e., CD=C′D′ (Bjerrum 1967). With the creep time changing, the line of final states will move and form a series of parallel lines, as shown as lines "1 Day", "10 Days" and "100 Days". In light of this phenomenon, it is inferred that above NCL there must be a compression line for which the creep time is "0". The line reflects the normally and instantaneously compressed characteristics of clays. Thus, it is called instant normal compression line (INCL) in this paper.

If remolded clays prepared in tests are assumed to be not influenced by time, their states can be represented by point A on INCL in Figure 2. ACD is the consolidation process which is traditionally divided into primary compression AC and secondary compression CD. However, the time of the primary compression of soft clays is so long that creep also exists in the primary compression. So the consolidation was divided into instant compression and delayed compression in accordance with the concept of INCL. That is to say, ACD is equivalent to ABD that includes instant compression AB and delayed compression BD. In the instant compression, the total stresses are supposed to be imposed on the clay skeleton instantaneously. In the delayed compression effective stresses are constant, and the clay deforms only due to the time.

3.2 *One-dimensional (1-D) rate effect*

As shown as in Figure 3, the curves of isotropic compressions at various constant strain rates are all parallel to INCL. Moreover, when the volumetric strain rate is larger, the position of the loading curve is higher, i.e., the apparent preconsolidation pressure p_{ac} is increasing with the strain rate. Figure 4 illustrates the results of 1-D oedometer consolidation tests with constant strain rates on several kinds of clays (Yin et al. 2010). $\dot{\varepsilon}_v$ is the volumetric strain rate and p_0 the initial mean effective stress.

The results are fitted by the hyperbolic function. The positive correlation between p_{ac} and the volumetric strain rate is obvious. However, the change of p_{ac} is decreasing gradually with the strain rate increasing. When the strain rate is large enough, the change of p_{ac} is small. Therefore, this paper assumed that the value of p_{ac} will not vary when the rate is large enough. That is, while the rate keeps increasing, the loading curve in Figure 3 is not going to move upwards indefinitely, and there will be a bounding line which is actually INCL.

4 1-D EVP RELATIONSHIP

According to the concept of INCL and the theory of instant compression and delayed compression, the total volumetric strain is additively decomposed into the following:

$$\varepsilon_v = \varepsilon_v^s + \varepsilon_v^t = \varepsilon_v^{se} + \varepsilon_v^{sp} + \varepsilon_v^{tp} = \varepsilon_v^e + \varepsilon_v^p \quad (6)$$

where ε_v^s is the volumetric strain under effective stresses, i.e. the instant strain including elastic strain ε_v^{se} and plastic strain ε_v^{sp} . ε_v^t denotes the volumetric strain influenced by time effects, i.e., the delayed strain. The strain due to time effects is irreversible, so $\varepsilon_v^t = \varepsilon_v^{tp}$. ε_v^e is the elastic volumetric strain that only occurs on account of stresses. ε_v^p is the plastic volumetric strain containning ε_v^{sp} and ε_v^{tp} .

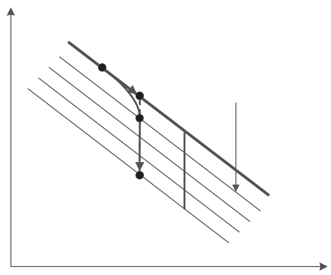

Figure 2. Division of isotropical consolidation process.

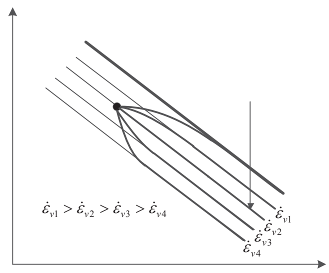

Figure 3. Schematic graph of 1-D rate effect.

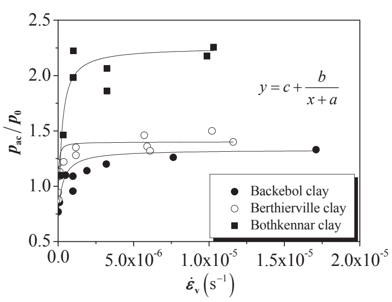

Figure 4. Relationships between apparent preconsolidation pressure and volumetric strain rate (Yin et al. 2010).

4.1 *Instant strain*

The proposed model adopts the function of the reloading line of the UH model to calculate the instant strain. Assuming η in eq. (4) to be zero and considering the elastic strain, the 1-D stress-strain equation of the UH model can be expressed as

$$d\varepsilon_v^s = \left(\frac{\kappa}{1+e_0} + \frac{\lambda - \kappa}{1+e_0} \frac{M^4}{M_f^4} \right) \frac{dp}{p} \quad (7)$$

If states of clays are initially on INCL, the reloading line will coincide with INCL.

4.2 Delayed strain

Dots in Figure 5 represent the data of an oedometer consolidation test on clays (Zhu 2000). The process before C belongs to the primary compression AC in Figure 2, and the one after C is the secondary compression CD. In the semi-logarithmic coordinate system, the data points of CD form a line approximately. So the formula of the creep can be expressed as

$$e = e_\lambda - C_{\alpha e} \ln(t+1) \quad (8)$$

where e_λ is the void ratio of the point on INCL at current p, t the time and $C_{\alpha e}$ the coefficient of secondary compression.

Clays develop from normal consolidation to overconsolidation with creeping. The time effects on clays are equivalent to making clays stiff and aged. So the elapsed time of the creep is called the aging time. It was pointed out that the creep rate is dependent on the current state and independent of the paths (Yin et al. 2002). Therefore, the state of clays can be represented by one path, e.g. the path of creep. That is, the state of clays is able to be described by the aging time. Besides, in e-ln$p$ plane, R can also reflect the state of clays. Hence, the aging time and R are related with each other. The relationship between them can be derived as

$$t_a = R^{-\alpha} - 1 \quad (9)$$

where t_a denotes the aging time and $\alpha = (\lambda - \kappa)/C_{\alpha e}$.

The delayed strain increment is derived from eqs. (8) and (9), as shown as follows:

$$d\varepsilon_v^{tp} = \frac{C_{\alpha e}}{1+e_0} \frac{dt_a}{t_a + 1} = \frac{C_{\alpha e}}{1+e_0} R^\alpha dt \quad (10)$$

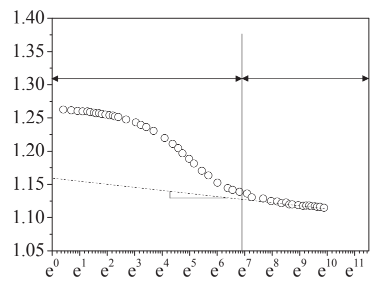

Figure 5. Experimental data of isotropic consolidation tests (Zhu 2000).

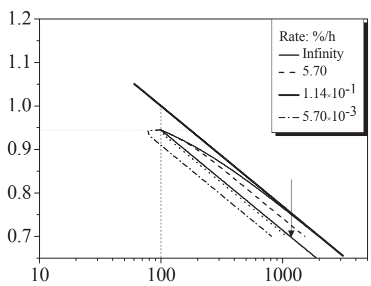

Figure 6. Predicted curves of isotropic compressions at constant rates.

where dt_a is the aging time increment, and dt the real time increment. Although the aging time is not equal to the real time, the increments of them are the same.

4.3 1-D stress-strain-time relationship

By combining eqs. (6), (7) and (10), the 1-D stress-strain-time relationship of OC clays can be expressed as

$$d\varepsilon_v = \frac{\kappa}{1+e_0} \frac{dp}{p} + \frac{\lambda - \kappa}{1+e_0} \frac{M^4}{M_f^4} \frac{dp}{p} + \frac{C_{\alpha e}}{1+e_0} R^\alpha dt \quad (11)$$

If clays are loaded instantaneously, i.e., $dt=0$, then the relationship will be changed into the stress-strain relationship of the UH model.

4.4 1-D prediction

Figure 6 shows the predicted results of isotropic compressions on OC clays (initial R=0.5) at constant rates of void ratio. The row "1-D" in Table 1 illustrates the values of parameters used in the prediction.

4.4.1 Characteristic rate

In Figure 6 the compression curves of different rates are parallel to INCL finally. When the rate is larger, as curve "5.70 %/h" shows, clays show overconsolidated behaviors. When the rate is slower, as curve "1.14×10^{-1} %/h" shows, clays behave in a similar way of underconsolidated clays. If clays just behave as normal consolidation when being compressed with a certain rate, then the certain rate will be defined as the characteristic rate. The characteristic rate is a function of R.

$$\dot{e}_{cr} = C_{\alpha e} \frac{\lambda}{\lambda - \kappa} R^\alpha \left(1 - \frac{M^4}{M_f^4} \right)^{-1} \quad (12)$$

When the states of clays are on INCL, M_f=M so that the characteristic rate is infinite, which means the instant compression curve goes back to INCL at last.

4.4.2 Relaxation

If clays are being isotropically compressed at a very slow rate, as curve "5.70×10^{-3} %/h" in Figure 6, on the curve there will be a part where e is almost invariant but p is decreasing. At this time, the behavior of clays is similar to relaxation. The difference between this type of curves and other curves is that the beginnings of the former lie on the left of the creep path, which indicates that the creep path is a boundary determining whether the relaxation exists. Consequently, during the isotropic compression, if the current strain rate of clays is smaller than the creep rate, then the loading curve will exhibit the relaxation feature.

Table 1. Parameters adopted in the predictions.

Parameters	λ	κ	$C_{\alpha e}$	M	ν	$e_{\lambda 0}$	$p_{\lambda 0}$(kPa)
1-D	0.1	0.02	0.0100	1.35	-	1.00	100
3-D	0.2	0.04	0.0046	1.27	0.1	1.26	60

5 EVP UH MODEL

5.1 Hardening parameter

Compared with the UH model, the EVP UH model considers the viscoplastic strain of clays. Consequently, according to eqs. (3) and (6), the hardening parameter of the EVP model should be composed of the instant hardening parameter H^s and the delayed hardening parameter H^t.

$$H = \int \frac{1}{\Omega} \left(d\varepsilon_v^{sp} + d\varepsilon_v^{tp} \right) = H^s + H^t \quad (13)$$

where $\Omega = (\mathrm{M}^4 - \eta^4)/(\mathrm{M}_\mathrm{f}^4 - \eta^4)$.

5.2 *Yield functions*

The parameter of time effects is assumed to be $\bar{t}$ and incorporated into the current yield function. So the current yield function of the EVP UH model is assumed as

$$f_3 = \ln\frac{p}{p_{x0}} + \ln\left(1 + \frac{\eta^2}{\mathrm{M}^2}\right) + \bar{t} - \frac{1}{\mathrm{c_p}}\left(H^\mathrm{s} + H^\mathrm{t}\right) = 0 \qquad (14)$$

Because the yield function is also workable in the isotropic compression, the function of $\bar{t}$ can be obtained by substituting η=0 into eq. (14).

$$\bar{t} = \frac{\mathrm{C}_{\alpha\mathrm{e}}}{\lambda - \kappa}\int \frac{\mathrm{M}_\mathrm{f}^4}{\mathrm{M}^4} R^\alpha \mathrm{d}t \qquad (15)$$

where $\bar{t}$ is not the real time but the equivalent time, indicating the time effects on clays. In the proposed model, INCL represents the instantaneously normally consolidated states and its position is not affected by time. Therefore, the EVP model still takes eq. (5) as the reference yield function. However, when time effects are considered, the plastic volumetric strain consists of both the strains due to stresses and time, i.e., $\varepsilon_\mathrm{v}^\mathrm{p} = \varepsilon_\mathrm{v}^\mathrm{sp} + \varepsilon_\mathrm{v}^\mathrm{tp}$.

5.3 *Constitutive relationship*

The time has no effects on the elastic law. So the elastic strain increments are still calculated by Hooke law.

$$\mathrm{d}\varepsilon_\mathrm{v}^\mathrm{e} = \frac{3(1-2\nu)}{E}\mathrm{d}p \qquad \mathrm{d}\varepsilon_\mathrm{d}^\mathrm{e} = \frac{2(1+\nu)}{3E}\mathrm{d}q \qquad (16)$$

where ν is the Poisson's ratio. The elastic modulus is $E = 3(1-2\nu)(1+e_0)p/\kappa$. The plastic strain rate can be expressed as:

$$\begin{aligned} \dot{\varepsilon}_{ij}^\mathrm{p} &= \Phi \frac{\partial f}{\partial \sigma_{ij}} \\ \Phi &= \Phi^\mathrm{s} + \Phi^\mathrm{t} \\ &= \mathrm{c_p}\Omega\left(\dot{p} + \frac{2\eta}{\mathrm{M}^2 - \eta^2}\dot{q}\right) + \frac{\mathrm{C}_{\alpha\mathrm{e}}}{1+e_0}\frac{\mathrm{M}_\mathrm{f}^4}{\mathrm{M}^4}\frac{(\mathrm{M}^2+\eta^2)^2}{\mathrm{M}_\mathrm{f}^4 - \eta^4} pR^\alpha \end{aligned} \qquad (17)$$

Φ^s and Φ^t are the time-independent and time-dependent plastic factor, respectively. $\dot{p}$ and $\dot{q}$ are the rates of stresses.

5.4 *Shear creep*

Figure 7 shows calculated stress paths of the undrained creeps with constant shear stresses. In the predictions, clays are instantaneously loaded first. Then the shear stresses remain unchanged in order to make clays creep under time effects. The paths in Figure 7(a) and Figure 7(b) are for low stress ratios (q/p<M**Erreur ! Source du renvoi introuvable.**) and high stress ratios (q/p>M), respectively. The values of parameters adopted are shown in the row "3-D" of Table 1.

6 VERIFICATION

The predicted results by the proposed model are compared with the test data of Hong Kong Marine Deposits (Zhu 2000). The parameters are displayed in the row "3-D" of Table 1.

Figure 8 shows the predicted and measured results of the triaxial undrained compressions at various constant strain rates on clays having the same initial OCR. Figure 9 illustrates the predicted and measured results of tests at the same constant strain rate (1.5 %/h) on clays having different initial OCRs. Because the confining pressures imposed on clays of various OCRs are different from each other, the results are normalized.

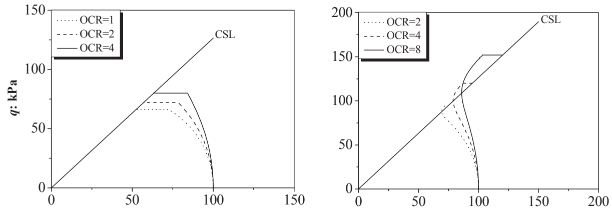

Figure 7. Stress paths of shear creeps: (a) at low stress ratios, and (b) at high stress ratios.

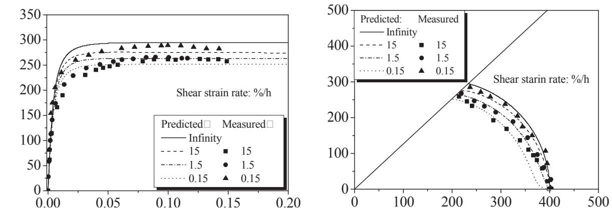

Figure 8. Comparison between predicted and measured results of triaxial undrained compressions at various rates.

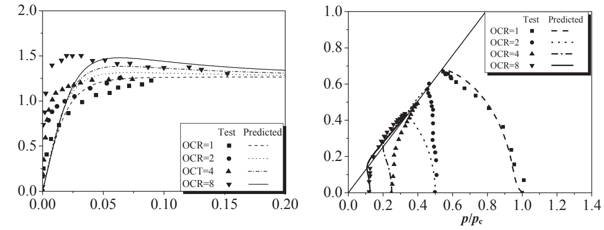

Figure 9. Comparison between predicted and measured results of triaxial undrained compressions on clays having different initial OCRs.

7 CONCLUSIONS

This paper established a new EVP framework by assuming the existence of the instant normal compression line. Furthermore, the 1-D EVP relationship was proposed by adopting the reloading line of the UH model as the instant compression line. Then with the equivalent time incorporated into the current yield function of the UH model, the 3-D EVP model was built. Compared with MCC, the proposed model needs only one additional parameter (the coefficient of secondary compression).

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