

# Validation of computational liquefaction in plane strain

## Validation de liquéfaction simulée en déformation plane

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**ABSTRACT:** This paper presents a validation of computational liquefaction in plane strain for a user-defined model that closely replicates liquefaction across a wide spectrum of soils. The results of triaxial and plane strain compression tests on Changi sand (used in large reclamation projects in Singapore) are utilized for property determination and validation respectively. The stress-strain behaviour is computed using a user-defined implementation of the NorSand general plasticity model in the FLAC numerical platform. Reasonable matches are obtained between the plane strain data and the computed responses, both in terms of stress-path and stress-strain behaviour. This accomplishes one necessary step towards allowing engineering practice for liquefaction damage assessment/mitigation to use a convenient computational platform anchored in a proper mechanics-based representation of soil behaviour.

**RÉSUMÉ:** Cet article présente une validation de liquéfaction simulée en déformation plane dans le cas d'un modèle original qui reproduit de près la liquéfaction dans une large gamme de sols. Les résultats d'essais triaxiaux et de compression en déformation plane sur du sable Changi (employé dans les grands projets de remblaiement à Singapour) sont utilisés respectivement pour la détermination de ses propriétés et pour la validation du modèle. Le comportement contrainte-déformation est calculé en introduisant le modèle de plasticité général NorSand dans le code de calcul FLAC. Les tests de validation et les simulations équivalentes présentent des résultats plutôt proches, que ce soit en termes de parcours de contrainte ou de comportement contrainte-déformation. Les pratiques de construction concernant les estimations/atténuations des dégâts dus à la liquéfaction peuvent ainsi être facilitées par l'utilisation de cette approche numérique associée à une représentation mécanique correcte du comportement des sols.

**KEYWORDS:** sand, liquefaction, plane-strain, constitutive modelling.

## 1 INTRODUCTION

Earthquakes remain an ever-present hazard, with soil liquefaction continuing to be a dominant mechanism in the consequent infrastructure damage and losses (e.g. the recent Christchurch events). Although the geologically-based "NCEER" method (Youd et al. 2001) underlies most current earthquake hazard reduction practice in geotechnical engineering, the limitations and flawed physics of the NCEER approach are becoming increasingly recognized. These limitations can be overcome by adopting advanced constitutive models, possible in engineering practice with the 'user defined model' facility of commercial numerical analysis platforms (e.g. as available in the popular FLAC and Plaxis platforms).

The past decade has also witnessed a greatly increased demand for metals, an economic trend that seems unlikely to change as 'BRIC' group living standards continue to approach those in the 'developed' world. Metals must be mined, and one result of mining is vast quantities of ground rock – 'tailings', which are produced as part of ore extraction. Usual mining practice is to impound tailings using dams. A new trend with tailings is to reduce their water content during deposition and "stack" the tailings above the retaining dam – economically attractive, but with the obvious potential for large scale release of these waste materials to the environment if a liquefaction-driven flowslide develops (e.g. failure of the Merriespruit Tailing Dam analysed by Fourie et al. 2001).

Liquefaction can be triggered by various mechanisms (Jefferies and Been 2000, Chu et al. 2003) but, regardless of trigger, the greatest damage (deformation) arises when the post-

liquefaction strength is less than the pre-liquefaction stress state – a situation captured in laboratory tests that focus on static liquefaction.

This paper presents a plane strain validation using a critical state based model that closely replicates liquefaction across a wide spectrum of soils. Plane strain approximates the conditions that arise in most geotechnical construction, certainly far more so than the triaxial paths that underlie current geotechnical understanding. Here, conventional triaxial compression tests are used for property determination, and subsequently static liquefaction tests in plane strain are used as the validation case. The tested material is Changi sand, a sand used at large reclamation projects in Singapore (Wanatowski and Chu 2007, 2012). The stress-strain behaviour is computed using a user-defined implementation of the NorSand general plasticity model (Jefferies 1993, Jefferies and Shuttle 2002) in the FLAC numerical platform (Shuttle and Jefferies 2005). Reasonable matches are obtained between the plane strain data and the computed responses, both in terms of stress-paths and stress-strain behaviour. This accomplishes one necessary step to allow engineering practice for liquefaction damage assessment/mitigation to use a convenient computational platform anchored in a proper mechanics-based representation of soil behaviour.

## 2 LABORATORY EXPERIMENTS

### 2.1 *Changi sand*

Changi sand is a subangular marine dredged silica sand used for the Changi land reclamation project in Singapore. The Unified

Soil Classification System describes the sand as a medium grained, poorly graded, clean sand. Changi sand contains approximately 12% of shells and has been used in a number of experimental studies. Its index properties are given in Wanatowski and Chu (2006, 2007).

## 2.2 Experimental set-up

All plane strain tests were performed in a plane-strain apparatus developed by Wanatowski and Chu (2006). The plane-strain condition was imposed by two metal vertical platens, fixed in position by two pairs of horizontal tie rods. The lateral stress in the  $\varepsilon_2=0$  direction (i.e. intermediate principal stress,  $\sigma_2$ ) was measured by four submersible pressure cells with two on each vertical platen.

The plane-strain testing system was fully automated. A digital hydraulic force actuator was mounted at the bottom of a loading frame to apply axial load. The actuator was controlled by a computer via a digital load/displacement control box. The control box adjusted the movement of the base pedestal to achieve a desired rate of load or rate of displacement so that either deformation-controlled or load-controlled loading mode could be applied. The vertical load was measured by an internal load cell. A pair of miniature submersible linear variable differential transformers (LVDT) was used to measure the vertical displacement. An external LVDT was also used to measure the axial strain when the internal LVDTs run out of travel. The cell pressure was applied through a digital pressure/volume controller (DPVC). Another DPVC was used to control the back pressure from the bottom of the specimen while measuring the volumetric change at the same time. The free-end technique (Rowe and Barden 1964) was adopted to reduce the boundary frictions and to delay the occurrence of non-homogeneous deformations. For details of the testing arrangement, see Wanatowski and Chu (2006).

The triaxial experiments were carried out using a fully automated triaxial testing system described by Chu and Leong (2001). The testing system comprised of a computer, a triaxial machine, a hydraulic actuator, and three digital pressure/volume controllers (DPVCs) and a data-logger. The dimensions of the triaxial specimen were 100 mm in diameter by 200 mm in height. As in the plane-strain apparatus, the free-end technique (Rowe and Barden 1964) was adopted in all the tests to minimize the bedding errors and to delay the occurrence of non-homogeneous deformations.

## 2.3 Results

The initial conditions of three isotropically consolidated undrained (CIU) triaxial compression tests conducted on very loose Changi sand are summarized in Table 1, where  $\sigma_1'$ ,  $\sigma_2'$  and  $\sigma_3'$  are the vertical, zero strain horizontal, and in-plane horizontal principal effective stresses respectively,  $p_0'$  is the mean effective consolidation pressure and  $e_0$  is the initial void ratio.

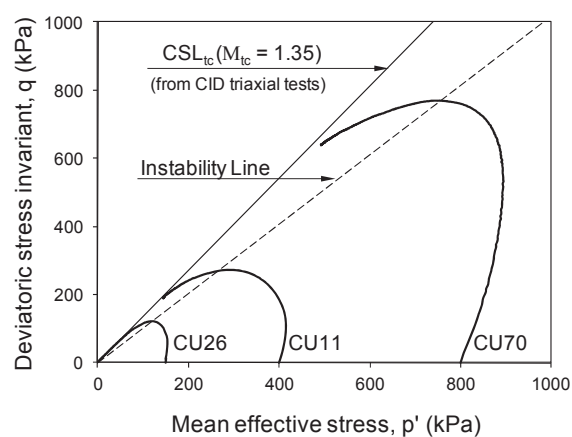
Table 1. Summary of sample conditions prior to undrained shearing.

Type	Name	$\sigma_1'$	$\sigma_2'$	$\sigma_3'$	$p_0'$	$e_0$
CIU	CU26	150	150	150	150	0.888
CIU	CU11	400	400	400	400	0.887
CIU	CU70	800	800	800	800	0.880
CK <sub>0</sub> U	U04	139.5	91	59.5	97	0.935
CK <sub>0</sub> U	U05	300	147	142	196	0.915
CK <sub>0</sub> U	U06	458	246	189	298	0.899

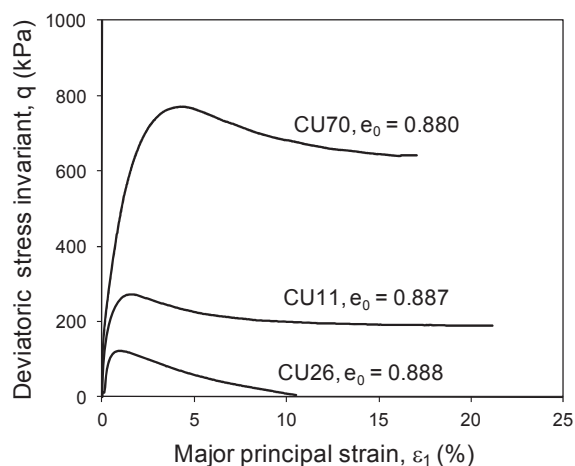
The effective stress paths of these tests are shown in Figure 1(a). The critical state line in triaxial compression ( $CSL_{tc}$ ) determined from drained tests on very loose sand (Wanatowski

and Chu 2007, 2008) is also shown in Figure 1(a) and gives a critical stress ratio,  $M_{tc} = 1.35$ , corresponding to a friction angle of 33.4°. As shown in Figure 1(a), in all tests the effective stress paths increased gradually towards the peak state and then traced down towards the CSL suggesting flow liquefaction behaviour. By connecting the peak points of the undrained stress paths shown in Figure 1(a), the instability line (IL) as defined by Lade (1993) can be determined. The zone bounded by the IL and the CSL has been called the zone of instability in which loose sand will become unstable when an undrained condition is imposed.

The stress-strain curves of the three CIU triaxial tests are shown in Figure 1(b). All the stress-strain curves show a similar response; that is a sharp increase in deviatoric stress to a peak followed by strain softening. It can be seen from Figure 1(a) that all the undrained effective stress paths were approaching the CSL determined from CID tests on very loose sand (Wanatowski and Chu 2006, 2007, 2008). Therefore, the CSL shown in Figure 1(a) can be considered the same as the steady-state line (SSL) for axisymmetric conditions (Poulos et al 1985).



(a)



(b)

Figure 1. Results of CIU triaxial tests on loose Changi sand.

Table 1 also summarizes the initial conditions of three  $K_0$  consolidated undrained (CK<sub>0</sub>U) plane-strain compression tests carried out on loose Changi sand. The effective stress paths and the stress-strain curves of these tests are presented in Figures 2(a) and 2(b), respectively. The critical state line in plane strain ( $CSL_{psc}$ ) as determined by drained CK<sub>0</sub>D plane-strain tests on loose Changi sand (Wanatowski and Chu 2006, 2007) is also

shown in Figure 2(a). Its slope in the deviatoric stress invariant ( $q$ ) versus mean effective stress ( $p'$ ) plane is  $M_{psc} = 1.16$ , corresponding to a friction angle of  $36.0^\circ$ . It can be seen from Figure 2(b) that strain softening occurred in all the tests. Therefore, using the same definition as for the triaxial tests, the instability line for plane-strain conditions can be drawn through the peak points of the undrained effective stress paths, as shown in Figure 2(a). Similar to the triaxial  $CSL_{tc}$  (see Figure 1a), the value  $M_{psc}$  under plane-strain conditions does not appear affected by stress level within a narrow stress range. It can also be seen from Figure 2(a) that all the effective stress paths approach the  $CSL_{psc}$ . Therefore, the  $CSL_{psc}$  can be considered the same as the SSL, as established earlier in Figure 1(a) for axisymmetric conditions (Wanatowski and Chu 2007, 2008).

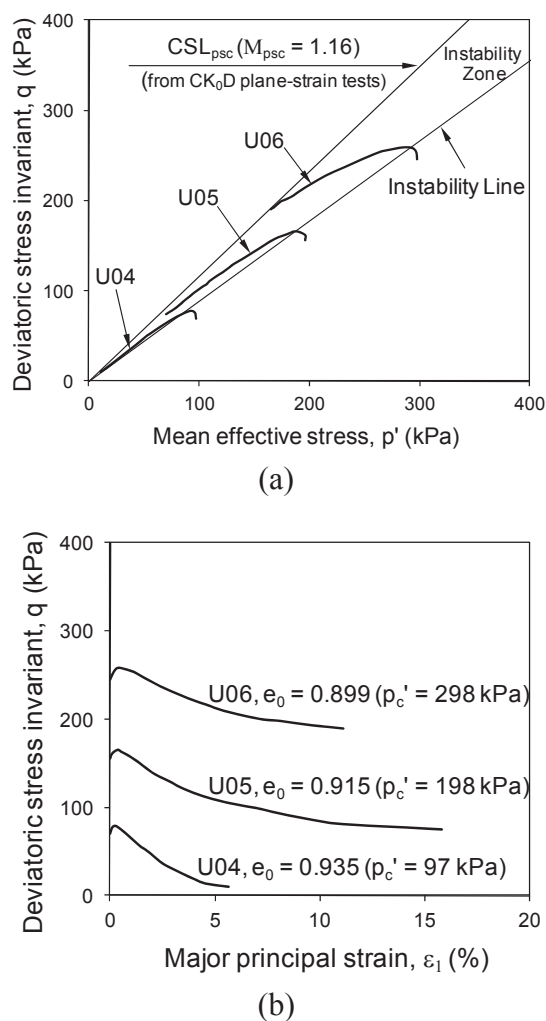


Figure 2. Results of  $CK_0U$  plane-strain tests on loose Changi sand.

### 3 NUMERICAL MODELLING

#### 3.1 *NorSand* model

*NorSand* is an isotropically hardening-isotropically softening critical state model that captures a wide range of particulate soil behaviour. It has two particular features, both controlled by the state parameter ( $\psi$ ): i) accurate representation of dilatant soils using an internal cap to the yield surface (the locus of this cap is the “Hvorslev” surface); and, ii) the yield surface generally does not intersect the critical state, but only moves to that condition with shear strain. The *NorSand* parameter set is common to

other critical state models, but with the addition of a plastic hardening modulus ( $H$ , required because the yield surface is decoupled from the critical state) and a dilatancy parameter ( $\chi$ ) which scales the dilation rate to  $\psi$ . Cam Clay (Schofield and Wroth, 1968) is a “special” case of *NorSand* requiring particular choice of plastic hardening modulus and initial state. Originally derived for triaxial compression (Jefferies 1993), *NorSand* was extended for 3D stress states by Jefferies and Shuttle (2002); there have been further minor revisions to capture the evolution of the critical friction ratio with strain (Jefferies and Shuttle 2005).

#### 3.2 Calibration

All of the eight *NorSand* parameters for Changi sand were determined from standard triaxial tests following the procedures reported in Jefferies and Shuttle (2005). The *NorSand* CSL was determined from loose CID triaxial tests. All other parameters were determined from dense CID triaxial tests. As a single *NorSand* parameter set is applicable to the full range of densities and initial states, the calibration involves obtaining a reasonable fit to all drained tests (rather than the best fit a particular test). A typical fit is shown in Figure 3 for test CD04 ( $p'_0 = 150$  kPa,  $e_0 = 0.654$ ); as seen here, the fits to the volumetric strain evolution is usually excellent as is the shear stiffness to peak strength, but the post-peak strength tends to exceed that measured (a presumed consequence of the test data being determined from overall deformation of the whole sample). For Changi Sand, the parameter set is:  $M_{tc} = 1.35$ ,  $\Gamma = 0.75$  at 1 kPa,  $\lambda_{10} = 0.106$ ,  $N = 0.5$ ,  $\chi = 4.4$ ,  $H = 124\text{--}880\psi$ ,  $G/p'_0 = 200$  to  $700$ ,  $\nu = 0.2$ .

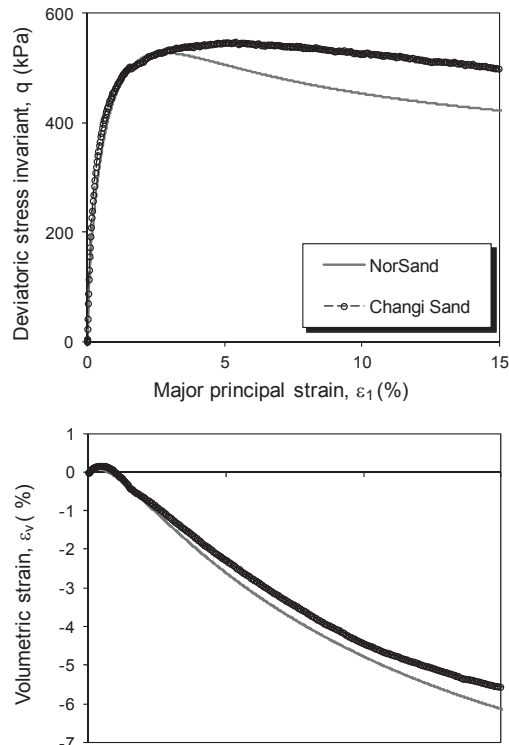


Figure 3. Calibration of *NorSand* for Changi sand to triaxial test CD04 ( $H = 280$ ,  $G/p'_0 = 700$ ).

#### 3.3 Validation

*NorSand* was validated in plane strain by using the parameter set from the triaxial calibration (above) to predict the behaviour of very loose undrained plane strain tests. The fit to  $CK_0U$  plane

strain test U05 is shown in Figure 4 with both the stress-strain and effective stress-paths being shown. The initial stress state was rather close to the instability limit so that only a small increase in the deviatoric stress caused a transition into a static liquefaction situation.

A rather good fit between the measured and computed stress-strain behaviour is evident on Figure 4 with the brittle strength loss being closely modelled. However, a small offset is apparent in the fully-liquefied strength.

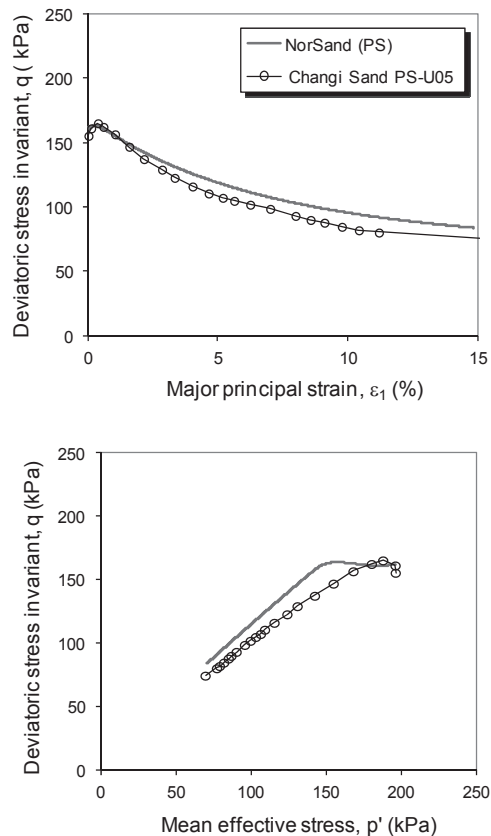


Figure 4. NorSand simulation of loose Changi sand in plane strain.

The match of simulation to measured behaviour of Changi sand is not as good for the stress-path (Figure 4). The sand shows instability at a lower slope to the instability line than the simulation, although the peak undrained strength itself is very closely predicted. This mismatch of the stress-path may seem surprising given the good fit to the stress-strain response, but in fact the mismatch in the stress-path all develops in the initial 0.2% of so of  $\epsilon_1$  and that is not readily seen in Figure 4. There are a range of possible issues when considering the mismatch between NorSand and measured data: broadly, there are potentially both experimental and/or theoretical errors. In the case of test errors, there could be differences between the reported and actual sample void ratio, over-idealization in the CSL, or indeed inaccuracies in the stress sensors themselves (for  $\sigma_2$  in particular).

Regarding theoretical errors, there are two areas for concern: stress-dilatancy and work-hardening. Stress-dilatancy follows in part from the Second Law of thermodynamics and would seem robust, but there are legitimate concerns about the coefficient  $M$ . NorSand includes both an idealization for how  $M$  varies with  $\sigma_2$  (i.e. Lode angle) as well as a particular idealization for the evolution of  $M$  with state parameter (i.e. strain). Both idealizations are open to refinement. However, it is thought that the more likely case of the mismatch between computed and measured stress-paths lies in the hardening law as it is this hardening limit that actually controls the slope of the

instability line. NorSand presently projects the hardening limits seen in dense samples linearly to loose states; further investigation of hardening limits for both loose states and plane strain conditions is warranted.

#### 4 CONCLUSIONS

Despite plane strain being widely accepted as a good analogy for many field slope failures, plane strain remains a relatively unusual test condition. Undrained plane strain testing that replicates liquefaction is even rarer, and provided an opportunity to assess the ability of a “good” mechanics based model (i.e. one whose properties are invariant with stress level and void ratio), to predict the behaviour of very loose undrained plane strain tests. The selected constitutive model, NorSand, was calibrated to standard triaxial tests that are similar to those available from commercial testing laboratories.

The match between the plane strain experimental and predicted NorSand response matches rather well, providing confidence in this mechanics-based methodology. What is intriguing is that this match has been achieved with very simple idealizations of the underlying physics of soil behaviour.

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