Rapid Drawdown Analysis using Strength Reduction

Analyse d’abaissement rapide utilisant la force de réduction

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ABSTRACT: The undrained shear strength during rapid drawdown is controlled by the properties of the embankment fill material and the consolidation stresses prior to drawdown. Current design methods use limit equilibrium analyses to evaluate both the consolidation stresses and the stability of the slope after drawdown. The method described in this paper uses the finite element method to calculate the consolidation stresses throughout the slope during steady state seepage before drawdown. Undrained shear strengths are calculated for all nodes in the model based on the major principal effective consolidation stresses and the results of ICU triaxial tests. The undrained strength of each element in the model is determined by interpolation from the strengths at the surrounding nodes. Using these strengths and an elastic-plastic constitutive model, the stability of the slope is evaluated by the strength reduction method. Back analysis of rapid drawdown failures suggests that undrained strengths from ICU tests should be reduced by 30% for the rapid drawdown condition.

RÉSUMÉ : La résistance du sol non drainé pendant l´abaissement rapide est contrôlée par les propriétés des matériaux de remplissage du remblai et des contraintes de consolidation avant l´abaissement. Les méthodes de design actuels l´analyse d´équilibre limité pour évaluer aussi bien la consolidation des contraintes que la stabilité des pentes après l´abaissement. La méthode décrite dans cet article utilise la méthode des éléments finis pour calculer les contraintes de consolidation tout au long de la pente pendant l´infiltration en état permanent avant l´abaissement. Les résistances du sol non drainé sont calculées pour tous les nodules du modèle en fonction des majeur principal consolidation stress et des résultats d´essais triaxiales ICU. La résistance du sol non drainé pour chaque élément du modèle est déterminée par l´interpolation des résistances aux nodules environnants. En utilisant ces résistances et un modèle constitutif élastique-plastique, la stabilité de la pente est évaluée par la méthode de la réduction des résistances. La rétro-analyse des défaillances des abaissements rapides semble indiquer que les résistances non drainés d´essais ICU devrait diminuer d´un 30% pour conditions d´abaissement rapide.

KEYWORDS: rapid drawdown, finite element, strength reduction, total stress analysis, earth dams, slope stability

1 INTRODUCTION

Rapid drawdown (RDD) has long been recognized as one of the critical design conditions for the upstream or riverside slope of dams and levees. The rapid drawdown condition occurs when the water level adjacent to a slope or embankment is lowered quickly after a long period of being elevated either at the normal operating level for a dam or in the case of levees, during a prolonged flood. Rapid removal of the supporting water load from the upstream face of the embankment, combined with changes in pore pressure, results in an undrained unloading condition in which total stresses decrease, but shear stresses within the embankment increase. Both effective stress and total stress methods have been developed to analyze stability during rapid drawdown and are discussed in the following sections.

1.1 Effective stress methods

The principal difficulty with effective stress methods is that the pore pressures during the drawdown must be known, and drawdown is an undrained loading condition. Estimating pore pressures during undrained loading is a difficult and uncertain undertaking.

Bishop (1954) proposed the $\overline{B}$ method to estimate pore pressures at the end of drawdown. The $\overline{B}$ method assumes that the changes in pore pressure during drawdown are equal to the changes in major principal stress. Li and Griffiths (1988) approximated the pore pressures at the end of drawdown by means of transient seepage analyses. Lane and Griffiths (2000) used assumptions similar to the $\overline{B}$ method along with finite element strength reduction analysis.

These effective stress methods result in pore pressures at the end of drawdown that do not reflect the tendency of the soil to dilate or compress. Thus they result in the same pore pressures at the end of drawdown for poorly compacted and well-compacted soils. In reality, the pore pressures at the end of drawdown for poorly compacted soils are much higher than for well-compacted soils, because well-compacted soils tend to dilate under the increased shear stresses during drawdown. Thus neither the $\overline{B}$ method nor the transient seepage analysis method, which do not reflect the quality of compaction of the fill, can provide a useful evaluation of stability during drawdown, and should not be used for this purpose.

Berilgen (2007) computed pore pressures during drawdown using an elastic-plastic constitutive model that included the effects of shear dilation. This procedure would be expected to result in more realistic estimates of pore pressure at the end of drawdown, but unfortunately requires analyses using complex constitutive relationships.

1.2 Total stress methods

Total stress methods do not require pore pressures at the end of drawdown to be estimated. The effect of these pore pressures is instead accounted for in the undrained strengths of the compacted soil. Well-compacted soil is stronger than poorly compacted soil, reflecting the fact that the pore pressures due to undrained loading are smaller when the soil is compacted well. This advantage has led to the adoption of total stress analyses in the United States. Total stress analyses have been developed through the work of Lowe and Karafiath (1960), Duncan et al. (1990), and the U. S. Army Corps of Engineers (2003).

These methods relate the undrained strength of the soil determined from consolidated-undrained laboratory tests to the effective stresses in the embankment before drawdown. As developed by Lowe and Karafiath (1960), Duncan et al. (1990), and the U. S. Army Corps of Engineers (2003), the undrained strength was related to the stresses along the trial failure surface, which were determined by limit equilibrium analyses. Limit equilibrium analyses were used because the finite element method was largely unavailable when the method was developed. Today, with finite element capabilities more routinely available, it seems more logical to use finite element analyses to evaluate the stress state prior to drawdown, as described here. The principal steps in the total stress method described here are:

- Evaluate the consolidation stresses in the embankment using finite element analyses, modeling steady seepage conditions with the water level high;
- Use these stresses, with the results of consolidated-undrained triaxial tests, to determine undrained strengths throughout the embankment; and
- Determine the factor of safety after drawdown by the finite element strength reduction method.

2 PROPOSED METHOD OF ANALYSIS

The geometry of the embankment being analyzed is represented by a finite element model. The model should include appropriate boundary conditions, mesh density, element type, etc.

The long-term effective consolidation stresses control the undrained strength during drawdown. The consolidation stresses within the embankment are determined using a finite element model that includes steady state seepage and long-term boundary loads, such as the reservoir water. At this stage, all of the soils are modeled using linear elastic stress-strain properties.

Determination of the appropriate undrained shear strength for RDD is the most important and the most complex step in the analysis. The undrained strength, s_u, of a compacted soil can be related to the major effective consolidation stress, σ'_{1c}, and other factors, such as the minor principal consolidation stress, anisotropic strength and deformation characteristics, compaction prestress effects, and the degree of principal stress rotation from consolidation to failure. If strengths are being determined using samples taken from an existing earth embankment, the additional factors of disturbance and recompression will also influence the measured strengths.

Isotropically consolidated undrained, ICU, triaxial compression tests on specimens compacted to the same relative compaction as the soil in the field are relatively easy to perform, but they do not replicate all of the factors mentioned earlier, such as stress rotation, anisotropy, and compaction prestress, which also influence the undrained strength. In the proposed method, the effects of these factors are included by applying an empirical adjustment factor, R, to the strengths measured in ICU tests, i.e. the adjusted strength is expressed as

$$s_{u-ADJ} = \left(R/100\right) \cdot s_{u-ICU} \quad (1)$$

where:

$s_{u\text{-}ADJ}$ = undrained strength adjusted for the influence of the factors noted above; and

$s_{u\text{-}ICU}$ = undrained strength measured in ICU laboratory tests.

The value of the empirical factor R must be determined by back analysis of RDD failures. Based on the two available, well-documented case histories, the value of R was found to be 70. Additional well-documented case histories of RDD failure would make it possible to refine this value. Some of the laboratory tests from the cases analyzed here were performed on samples taken from the embankments. The value of R determined for these cases may include effects of disturbance and recompression, which would not be reflected in tests on samples compacted in the laboratory.

The adjusted undrained strength is used in the analysis of stability after drawdown. The model geometry from the consolidation stress analysis is used along with modified constitutive and strength properties to calculate the factor of safety by the strength reduction method.

For the stability analysis, undrained strengths are assigned to those portions of the model where negligible drainage will occur during drawdown. Drained strength parameters are assigned to the portions of the zones where drainage will occur. These include zones of materials with high permeability and areas near the surface of the slope where the drainage path is short. The depth of this drained zone along the slope surface can be estimated using one-dimensional consolidation theory.

This paper follows the recommendations of Griffiths and Lane (1999) and uses non-convergence as the failure criterion in the strength reduction analysis.

3 EXAMPLES

The proposed method is compared to the limit equilibrium method, using the RDD failures at Pilarcitos Dam and Walter Bouldin Dam as benchmark cases.

3.1 *Pilarcitos Dam*

Pilarcitos Dam is a 23.8 m high homogenous earth dam built from compacted sandy clay with a total unit weight of 21.2 kN/m^3. The lower 17.7 m of the upstream slope is inclined at 2.5H:1V, and the upper 6.1 m is inclined at 3H:1V. The long-term water level was 1.8 m below the crest.

A rapid drawdown slide occurred in 1969 after the reservoir level was lowered 10.7 m over the course of 43 days. This case has been considered by a number of researchers, including Wahler and Associates (1970) and Duncan et al. (1990).

Laboratory strength tests were performed on samples from the embankment by Wahler and Associates (1970). A drained zone 0.46 m thick (measured perpendicular to the slope face) was used for the drawdown analysis. This depth corresponds to 90% dissipation of excess pore pressure in 43 days, based on an assumed coefficient of consolidation of 46 cm^2/day.

The Pilarcitos Dam finite element model was created using the software Phase^2 v.8.011. A rigid foundation was assumed and the nodes along the base of the embankment were fixed. The consolidation stress analysis assumed linear elastic stress strain behavior with E = 10.8 MPa for both the consolidation stress and drawdown analyses. Poisson's ratio, ν, was assumed to be 0.42 for the drained portion of the embankment, and 0.49 for the undrained portion. The drained zones in the drawdown analysis used a drained friction angle of 45° based on the Wahler and Associates (1970) tests. The stresses prior to drawdown were calculated in three-stages, using effective stress analyses. Gravity loads within the embankment were applied in the first stage. The boundary loads of the water in the reservoir were applied in the second stage. In the third stage, pore pressures corresponding to steady state seepage were assigned to the nodes in the embankment.

The ICU triaxial compression test data obtained by Wahler and Associates (1970) was used to express undrained strength as a function of σ'_{1c} as shown in Figure 1. The ICU strength (solid) line was fitted to these points and also to match the general trend in the drained secant friction angle and pore pressure response observed in the tests. The strength line fits the data below 200 kPa well with the exception of one outlier, which developed lower pore pressures during shear. The adjusted undrained strengths were calculated using R = 70 as explained earlier.

The values of σ'_{1c} at each node were exported from the FE analysis into a spreadsheet. The undrained strength was calculated for each node using the R = 70 adjusted strength envelope shown in Figure 1. The undrained strength for each element was then calculated by the FE software using TIN interpolation from the nodal values.

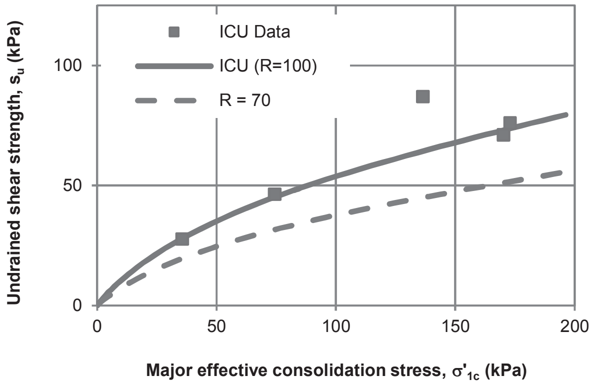

Figure 1. Undrained strengths for Pilarcitos Dam analyses

Elastic-plastic stress strain behavior was used for the drawdown analysis. Following the recommendation of Griffiths and Lane (1999), a dilation angle of 0° was assumed. The drawdown analysis was performed in two stages. In the first stage, the initial body forces, stresses, and boundary conditions for the normal operating conditions were applied. In the second stage, the water level and reservoir loading were reduced to the drawdown levels and a strength reduction analysis was performed to evaluate the factor of safety. The critical strength reduction factor, SRF_{crit}, was calculated for the adjusted strength with R = 70.

In Table 1, the strength reduction factor of safety is compared to the limit equilibrium factor of safety calculated by the Duncan, Wright, and Wong (DWW) method (1990) . It can be seen that, with a value of R = 70, the finite element strength reduction method is in close agreement with the widely-accepted DWW method.

Table 1. Summary of RDD stability analyses for Pilarcitos Dam

Method of Analysis	Factor of Safety
Finite Element Strength Reduction with R = 70	1.01
DWW Limit Equilibrium	1.04

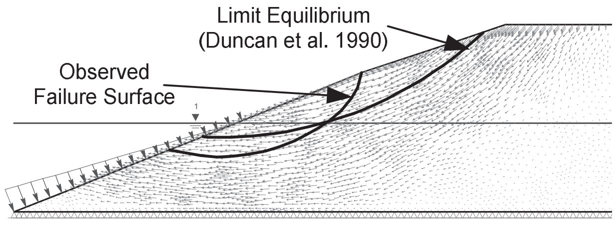

Figure 2. Nodal displacement vectors from Pilarcitos Dam strength reduction analysis, SRF_{crit} = 1.01, R = 70

The nodal displacement vectors shown in Figure 2 illustrate the failure mechanism predicted by the strength reduction analysis. The proposed method predicts a relatively deep failure zone that intersects the base of the embankment, whereas the slip surface observed in the field encompasses a considerably smaller portion of the embankment. The cause of this difference is not known.

3.2 *Walter Bouldin Dam*

In 1975, a rapid drawdown failure occurred at Walter Bouldin Dam in Alabama when the water level in the reservoir dropped 9.8 m over the course of 5-1/2 hours. This catastrophic drawdown rate was caused by the failure and breach of a different section of the dam. The dam cross-section at the location of the RDD failure is shown in Figure 3. The soils were poorly compacted, which led to both the breach and the RDD failure.

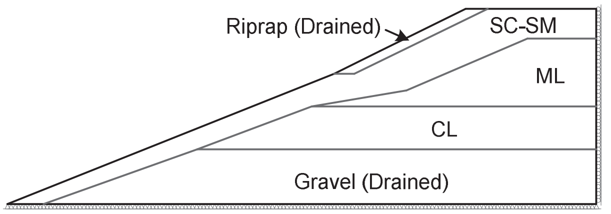

Figure 3. Cross-section of Walter Bouldin Dam

For the finite element consolidation stress analysis, the soils were all assigned linear elastic stress strain properties with E = 47.9 MPa. The riprap and base layer of clayey sandy gravel were assigned ν = 0.3. The other three soils were assigned ν = 0.35. Pore pressures corresponding to the full reservoir height of 14.3 m above the base of the embankment were assigned throughout the cross-section, assuming a horizontal piezometric surface since the information required to perform a steady-state seepage analysis was not available. The nodes along the base of the embankment were fixed while the downstream boundary was restrained in the horizontal direction.

The undrained strengths in Figure 4 are based on ICU triaxial tests performed by Whiteside (1976).

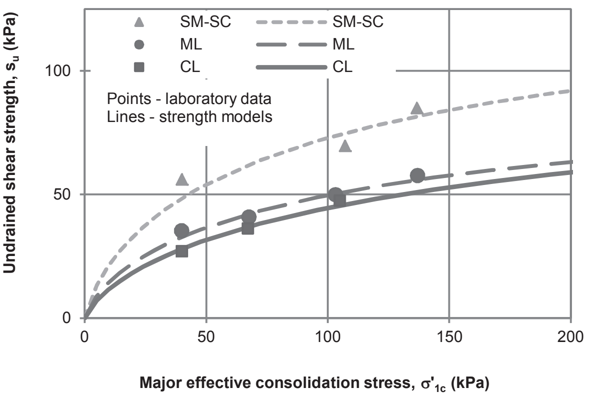

Figure 4. Undrained strengths (R=100) for Walter Bouldin Dam

The value of σ'_{1c} at each node calculated in the consolidation analysis was exported into a spreadsheet, and undrained strengths were calculated using the ICU strengths shown in Figure 4. Adjusted strength values for each node were calculated using the data in Figure 4, with R = 70. The adjusted undrained strength of each element in the finite element mesh was computed using TIN interpolation.

The factor of safety against slope instability was calculated using the strength reduction method, as explained previously. The results are summarized in Table 2. It can be seen that the agreement is good between factors of safety calculated by the

DWW method and the finite element strength reduction method with R = 70, as it was for Pilarcitos Dam.

Table 2. Summary of RDD stability analyses for Walter Bouldin Dam

Method of Analysis	Factor of Safety
Finite Element Srength Reduction with R = 70	1.05
DWW Limit Equilibrium	1.02

In Figure 5, the observed failure surface and the critical circle from limit equilibrium are superimposed on the nodal displacement vectors from the finite element analysis. The methods generate essentially the same result. Both result in deeper rupture zones than observed in the field. Again, the cause of this discrepancy is not known.

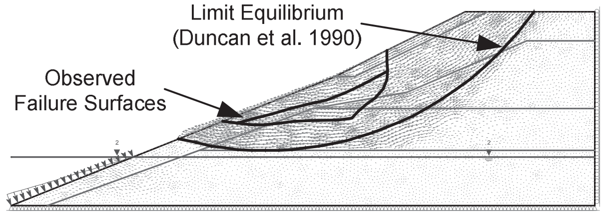

Figure 5. Nodal displacement vectors from Walter Bouldin Dam strength reduction analysis, $SRF_{crit} = 1.05$, R = 70

4 ADVANTAGES AND DISADVANTAGES

Compared to the widely accepted limit equilibrium procedure for rapid drawdown stability analysis, the proposed finite element method has a number of strengths and advantages:

- It follows the conventional approach for analysis of rapid drawdown and other short-term loading problems by using total stress stability analysis.
- The use of the finite element method to determine the consolidation stress state is an improvement over the use of limit equilibrium methods for this purpose. In 1960, when Lowe and Karafiath developed their groundbreaking method, using limit equilibrium to calculate consolidation stresses was the only choice. Today, however, with finite element analyses becoming widely available, it is logical to use the finite element method for calculation of consolidation stresses.
- Representing undrained strength as a function of σ'_{1c} alone is a simple means of including the most important factor controlling undrained strength – the major principal consolidation stress. It makes use of ICU triaxial tests, which are easy to perform, widely used, and relatively inexpensive.
- The reduction factor R is a simple means of adjusting for differences between the ICU laboratory tests and field conditions, namely unequal major and minor principal consolidation stresses, anisotropic strength and deformation characteristics, compaction prestress effects, and principal stress rotation from consolidation to failure.
- The method could be just as easy (or easier) to implement as limit equilibrium methods once a specific module is programmed into commercial finite element codes.

Disadvantages of the proposed method include:

- Finite element strength reduction analysis may not yet be readily available in all geotechnical engineering organizations.
- The recommended value of R = 70, although consistent with available experience at Pilarcitos Dam and Walter Bouldin Dam, is based on only two case histories.
- The shallow failure mechanism observed in RDD failures is not predicted by the analyses, and the reason for this discrepancy is not known.

5 CONCLUSIONS

A total stress representation of strength is appropriate for undrained problems because the very great difficulty in predicting pore pressures during undrained loading makes it infeasible to use effective stress analyses for these cases. The proposed method uses a total stress representation of undrained strength.

Undrained strength of the embankment soil is characterized as a function solely of the major effective consolidation stress. Other factors, such as anisotropic consolidation, principal stress rotation from consolidation to failure, plane strain conditions, and laboratory recompression, which also affect undrained strength, are included in the method through an empirical adjustment factor, R.

Based on the two best-documented case histories for RDD (Pilarcitos Dam and Walter Bouldin Dam), it is concluded that a value of R = 70 is appropriate for both cases. As further cases become available, they should be examined closely to determine if R = 70 is appropriate in those cases also.

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