A visco-elasto-plastic multi-surface cyclic model

Un modèle visco-élastoplastique cyclique ¨ surfaces multiples

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ABSTRACT: Modeling a visco-elasto-plastic material under cyclic loading has been a big issue due to the complexity of the problem. Single surface nonlinear kinematic hardening models, two yield bounding surface models, multi-segment backstress models showed some success in this field. Still, all these models fail in case of modeling cohesionless soil with pressure sensitivity and softening. In this paper, a visco-elasto-plastic multi-surface model is proposed for cyclic loading. This model is originally proposed by Mroz (1967) and has been extended to conhesionless soil by Prevost (1985). In this research, the model is extended to include the effect of rate of loading. The integration of incremental elasto-plastic equation is carried out by steepest descent return mapping algorithm.The dilatancy of dense sand has been modeled using non-associated flow rule. The non-associated flow has been attained by using Row's stress-dilatancy equation relating angle of internal friction to dilatancy angle by material constants. The viscous behavior is introduced through the incorporation of three component model into the basic constitutive equations. The quality of the simulation is assessed and limitations are discussed.

RÉSUMÉ : Modélisation d'un matériau visco-élasto-plastique sous chargement cyclique a été un gros problème en raison de la complexité du problème. Simple surface cinématique non linéaire des modèles de durcissement, deux modèles de rendement de surface de délimitation, les modèles backstress multi-segments ont montré un certain succès dans ce domaine. Néanmoins, tous ces modèles ne dans le cas de la modélisation des sols sans cohésion avec sensibilité à la pression et adoucissantes. Dans cet article, un visco-élasto-plastique multi-surface du modèle est proposé pour le chargement cyclique. Ce modèle est proposé à l'origine par Mroz (1967) et a été étendu au sol conhesionless par Prévost (1985). Dans cette recherche, le modèle est étendu pour inclure l'effet de la vitesse de chargement. L'intégration progressive de l'élasto-plastique équation est effectuée par dilatance retour la plus grande pente cartographie algorithm.The de sable dense a été modélisé en utilisant la règle d'écoulement non associée. Le flux non associé a été atteint en utilisant stress dilatance Row équation reliant l'angle de frottement interne à l'angle de dilatance par les constantes du matériau. Le comportement visqueux est introduit à travers la constitution de trois modèle de composants dans les équations de base constitutifs. La qualité de la simulation est évaluée et limitations sont discutées.

KEYWORDS: multi-surface, cyclic model, elasto-visco-plasticicty, cohesionless, dilatancy.

1 INTRODUCTION

Mathematical modelling of material behavior was of great interest during the last century due to the progress of digital computer. As a result, several constitutive models were proposed by the researchers to simulate the response of materials under monotonic loading as well as cyclic loading under the general framework of elasto-visco-plasticity. Materials are often subjected to transient and cyclic stresses due to earthquake or other source of dynamic load. A number of models have been introduced by many researchers to simulate this cyclic stress-strain behaviour of materials, such as Prager (1956), Mróz (1967), Hossain *et al.* (2007), Hossain, Siddiquee and Tatsuoka (2005) etc. These cyclic response calculations of the material are modeled through kinematic hardening, isotropic hardening and/or a combination of both. But, most of these cyclic models are unable to reproduce the memory effect of materials, which eventually produces a closed or near-closed hysteretic stress-strain loop. In this paper, multi-surface model has been put forward with the three component model to accommodate the viscous property. The elasto-plastic incremental equations aof multi-surface model are integrated here by return mapping algorithm. The family of multi-surface models proposed by Mroz (1967), Prevost (1975), Kohey and Jamali (1999) and others possess the inherent ability to follow the Masing's law. But most of these models were proposed for pressure independent materials. A pressure dependent one-dimensional formulation was presented in Tatsuoka *el at.*, (2003). In this paper, the cyclic behavior in 3D stress space is simulated by introducing a new framework in which the dimensionless kinematic hardening rate is varied according to the instantaneous stress value at that point along the stress path. When the direction of the loading is reversed, the initial rate of hardening is restored and the rate of variation of hardening is scaled according to modified Masing's law. As a result a closed hysteretic stress-strain loop is obtained due to cyclic loading.

2 CONSTITUTIVE EQUATIONS

2.1. Small Strain Theory

The kinematic hardening rule evolves with the accumulative plastic strain. Since the present model has been developed within the small strain range, the total strain increment can be divided into its elastic and plastic parts as follows:

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{p} \qquad (1)$$

where, $d\varepsilon_{ij}^{e}$ represents the elastic components of strain defined by hooks law, $d\varepsilon_{ij}^{p}$ represents the incremental plastic strain. As the material is pressure sensitive, Hooks law can be envisaged in the form of Bulk's modulus, K and shear modulus G. In this paper, both the modulus is variable and depends on the mean pressure. The plastic strain components are determined by the flow rule and consistency condition.

$$G = G_1 \cdot \left(\frac{p}{p_1}\right)^n \qquad B = B_1 \cdot \left(\frac{p}{p_1}\right)^n \tag{2}$$

Where, n= experimental parameter, p_1=reference normal stress, G_1 and B_1 are also experimental parameters.

2.2. Yield function and plastic potential

The material model used in this paper is a generalized elasto-plastic, pure kinematic hardening one. A simple hyperbolic equation (Tatsuoka *et al*., 1993, Hossain *et al*., 2007) has been used as the evolution function of yield surface. The yield surface used is a generalized Drucker-Prager one given by;

$$f^n = \left(s_{ij} - p\alpha_{ij}\right)\left(s_{ij} - p\alpha_{ij}\right) - m^2 p^2 = 0 \tag{3}$$

, where p is the mean normal stress (i.e., hydrostatic stress component) , s_{ij} is the desiatoric component of stress tensor, α_{ij} is the kinematic deviatoric tensor defining the coordinates of the yield surface center in deviatoric stress sub-space; m is the material parameter defining the opening of the cone, n is the number of yield surfaces.

A plastic potential function (g) is selected such that the deviatoric plastic flow is associative. A non-associative plastic flow rule is used for its dilatational component. So the deviatoric component of plastic potential is defined same as the yield function. The dilatational or plastic volumetric component is defined by Rows' dilatancy relationship given by

$$R = K \cdot d \tag{4}$$

Where, $R = \frac{\sigma_v}{\sigma_h}$ and $d = d\varepsilon_h^p \big/ d\varepsilon_v^p$ for loading and vice-versa for unloading, K is the material constant.

2.3. Kinematic hardening rule

A pure kinematic hardening rule is formulated as follows:

$$p\dot{\alpha}_{ij} = a\mu_{ij} \tag{5}$$

Where, μ_{ij} = deviatoric component of tensor defining the direction of translation of the yield surfaces. a = amount of translation determined through the consistency condition as follows:

If the yield function was isotropic, then it could be described by eq. (6) and (7) and for kinematic surface, by eq. (8) & (9) -

$$f\left(\sigma_{ij}, \kappa\right) = 0 \tag{6}$$

$$\frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \kappa} d\kappa = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \kappa} \lambda = 0 \tag{7}$$

$$f\left(\sigma_{ij}, \alpha_{ij}\right) = 0 \tag{8}$$

$$\frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \alpha_{ij}} d\alpha_{ij} = 0 \tag{9}$$

$$\frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} = \lambda \frac{\partial f}{\partial \kappa} \tag{10}$$

$$\frac{a}{p} \frac{\partial f}{\partial \sigma_{ij}} \mu_{ij} = \lambda H \tag{11}$$

$$a = \lambda p H \frac{1}{\left(\partial f / \partial \sigma_{ij}\right) \mu_{ij}} \tag{12}$$

So

$$\dot{\alpha}_{ij} = \lambda p H \frac{1}{\left(\partial f / \partial \sigma_{ij}\right) \mu_{ij}} \mu_{ij} \tag{13}$$

The yield surfaces are all self-similar conical surfaces in general three-dimensional stress space. The yield surfaces are to be translated by the current stress point upon contact. In order to avoid the overlapping of the surfaces, the direction of translation μ_{ij} of the active yield surface is chosen such that

$$\mu_{ij} = \frac{m'}{m}\left(s_{ij} - p\alpha_{ij}\right) - \left(s_{ij} - p\alpha'_{ij}\right) \tag{14}$$

Where m' and α'_{ij} are the plastic parameters associated with the next outer surface of the nested yield surfaces.

3 RETURN MAPPING ALGORITHM

In this algorithm, elastic trial stress is returned to the current yield surface, following the existing hardening law and flow rule. In this way, the incremental elasto-plastic relation is integrated in a robust way (Simo and Ortiz, 1986). In this particular scheme, stress tensor is divided into two components, deviatoric stress (s_{ij}) and mean stress (p). α_{ij} is designated as angle of the center line of the concentric cone in the context of pure kinematic hardening. Expanding the yield function into a Taylor's series gives Eq. (15)-

$$f\left(s_{ij}, p, \alpha_{ij}\right) = 0 \tag{15}$$

$$f\left(s_{ij}, p, \alpha_{ij}\right) - \frac{\partial f}{\partial s_{ij}} ds_{ij} - \frac{\partial f}{\partial p} dp - \frac{\partial f}{\partial \alpha_{ij}} d\alpha_{ij} = 0 \tag{16}$$

Now considering the followings facts:

$$ds_{ij} = 2G \cdot d\gamma^p = 2G \cdot \lambda \frac{\partial g}{\partial s_{ij}} \tag{17}$$

$$dp = K \cdot d\varepsilon_v^p = K \cdot \lambda D \tag{18}$$

As it has been defined that

$$D = \frac{d\varepsilon_v^p}{d\gamma^p} \text{ and } d\varepsilon_v^p = D \cdot d\gamma^p = D \cdot d\kappa = D \cdot \lambda$$

as it has been known that $d\gamma^p = d\kappa = \lambda$ for plane strain situation.

Combining Eqs. (15), (16) and (17) and using Prager's kinematic hardening rule defined in Eq. (13) the plasticity multiplier can be derived-

$$f\left(s_{ij}, p, \alpha_{ij}\right) - \frac{\partial f}{\partial s_{ij}} 2G \cdot \lambda \frac{\partial g}{\partial s_{ij}} - \frac{\partial f}{\partial p} KD \cdot \lambda - \lambda H = 0 \tag{19}$$

$$\lambda = \frac{f\left(s_{ij}, p, \alpha_{ij}\right)}{\frac{\partial f}{\partial s_{ij}} 2G \cdot \lambda \frac{\partial g}{\partial s_{ij}} + \frac{\partial f}{\partial p} KD \cdot \lambda + H} \tag{20}$$

Using the trial stresses, following integrated elasto-plastic stresses and kinematic hardening parameter are obtained as shown in Eq. (19) and Eq. (20).

$$s_{ij} = s_{ij} - \lambda \cdot 2G \cdot \frac{\partial f}{\partial s_{ij}} \quad (21)$$

$$p = p - \lambda \cdot K \cdot D \quad (22)$$

$$d\alpha_{ij} = \lambda \cdot pH \frac{1}{\left(\partial f / \partial \sigma_{ij}\right) \mu_{ij}} \mu \quad (23)$$

Updated variables are –

$$\sigma_{ij} = s_{ij} + p\delta_{ij} \quad (24)$$

$$\alpha_{ij} = \alpha_{ij}^{prev} + d\alpha_{ij} \quad (25)$$

4 VISCO-PLASTIC FORMULATION

The loading rate effects due to material viscosity on the stress-strain behavior of sand (not due to delayed dissipation of excess pore water) are often very important in geotechnical engineering practice. A number of researchers (Tatsuoka et al. 2002; Tatsuoka, 2004) reported significant loading rate effects observed in laboratory stress-strain tests on sand under drained conditions; i.e., effects of strain rate and its change on the stress-strain relation, creep deformation and stress-relaxation during otherwise monotonic loading (ML) at a constant strain rate.

Within the framework of the general non-linear three-component model (Fig. 1), Di Benedetto et al. (2002) and Tatsuoka et al. (2002) proposed a set of stress-strain models to simulate the effects of material viscosity on the stress-strain behaviour of geomaterial (i.e., clay, sand, gravel and sedimentary softrock). They showed that the viscous property of clean sand (i.e., uniform sand) is different from that of clay in that the viscous effect decays with an increase in the irreversible strain and proposed a specific model to describe the above (i.e., the TESRA model explained below).

In this paper, it is shown that this model can be smoothly implemented in a FE code (Siddiquee *et al*., 1996, Siddiquee *et al.,* 2006). Then, the shear stress – shear (or axial) strain relations obtained from typical drained plane strain compression (PSC) tests performed at fixed confining pressure on clean sands (i.e., Toyoura and Hostun sands), reported by Di Benedetto et al. (2002) and Tatsuoka et al. (2002), that were simulated by the FE code embedded with the TESRA Temporary Effect of Strain Rate and Acceleration) model are reported.

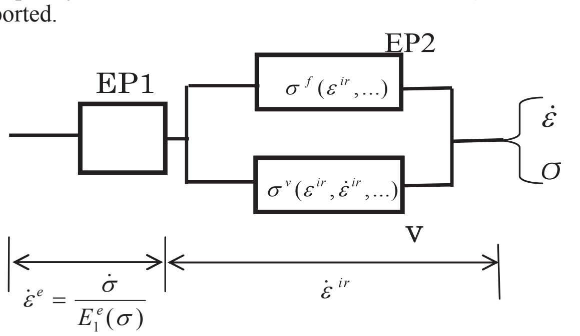

Figure 1 General non-linear non-linear three-component model

Although Di Benedetto et al. (2002) and Tatsuoka et al. (2002) showed at least three different functional forms of the viscous component, σ^v, were proposed. In this paper, the simplest form ("New Isotach") was adopted to describe the loading rate effects of clay-like materials, for which, for primary ML along a fixed stress path, the current value of σ^v is a non-linear function of instantaneous value of $\dot{\varepsilon}^{ir}$ while it is always proportional to the instantaneous value of σ^f as:

$$\sigma^v\left(\varepsilon^{ir},\dot{\varepsilon}^{ir}\right) = \sigma^f\left(\varepsilon^{ir}\right) \cdot g_v(\dot{\varepsilon}^{ir}) \quad (26)$$

$$\sigma = \sigma^f\left(\varepsilon^{ir}\right) \cdot \{1 + g_v(\dot{\varepsilon}^{ir})\} \quad (27)$$

where $g_v(\dot{\varepsilon}^{ir})$ is the viscosity function, which is always zero or positive and given as follows for any strain (ε^{ir}) (19b) or stress (σ^f) history (with or without cyclic loading):

$$g_v(\dot{\varepsilon}^{ir}) = \alpha \cdot [1 - \exp\{1 - (\frac{|\dot{\varepsilon}^{ir}|}{\dot{\varepsilon}_r^{ir}} + 1)^m\}] \quad (\geq 0) \quad (28)$$

where $|\dot{\varepsilon}^{ir}|$ is the absolute value of $\dot{\varepsilon}^{ir}$; and α, $\dot{\varepsilon}_r^{ir}$ and m are positive material constants. According to this model, as far as ML continues along a fixed stress path, the viscous stress component, σ^v, is a unique function of instantaneous values of ε^{ir} and $\dot{\varepsilon}^{ir}$, independent of previous loading history. The term "new" of the model name comes from that, with the original isotach model (Suklje, 1966), the stress σ (therefore σ^v) is a function of instantaneous strain rate, $\dot{\varepsilon} = \partial\varepsilon / \partial t$, not $\dot{\varepsilon}^{ir}$, while, with the new isotach model, σ^v is a function of $\dot{\varepsilon}^{ir}$. This difference results into significant variations in the model behaviour, in particular during stress relaxation with $\dot{\varepsilon}$ = 0 and immediately after a step change in $\dot{\varepsilon}$ during otherwise ML at a constant $\dot{\varepsilon}$.

5 IMPLEMENTATION

The model is implemented in one-element FEM with Plane strain idealization (Siddiquee *et al.,* 1999, 2001a, 2001b). The material data for the initial configuration of the concentric cones of Drager-Prager yield surfaces (Figure 2) are adopted from the research work done by Prevost (1985). The viscous property was set arbitrarily to show the capability of this model by setting the values of the constants of Eq. (28). Here in this paper, α=05 and m=0.5 were adopted. The reference value of loading rate, $\dot{\varepsilon}_r^{ir}$=1.0e-^{8} is set for the analysis. In this paper, α_{ij} is used instead of σ^f in eq. (28) as the model is driven by the movement of α_{ij} .

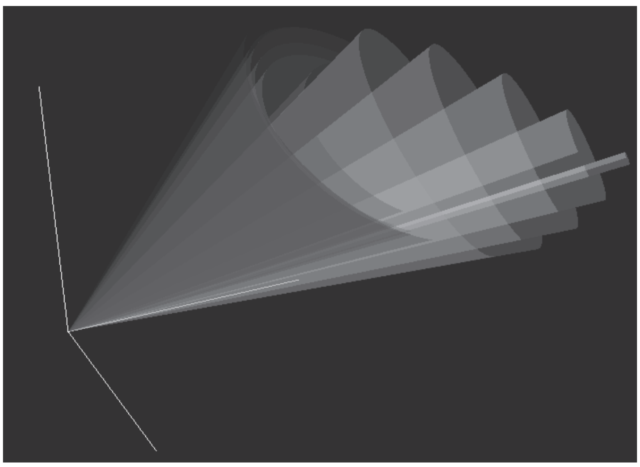

Figure 2. Visualization of concentric cones of Drager-Prager surfaces in Open-GL window.

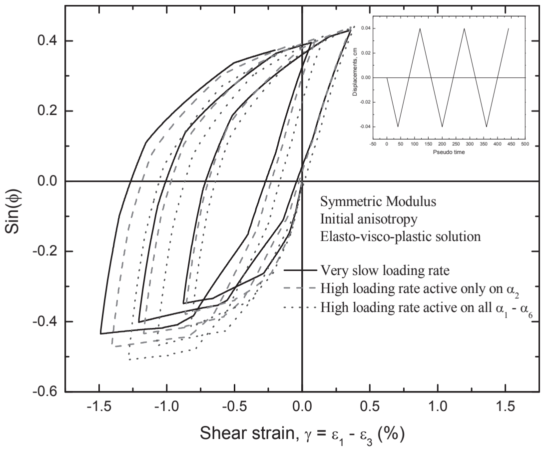

Figure 3. Fixed amplitude cyclic loading simulation showing the variation of Sin(ϕ) versus Shear strain, γ.

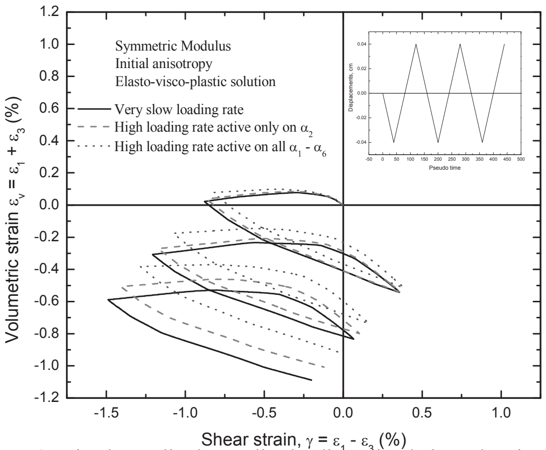

Figure 4. Fixed amplitude cyclic loading simulation showing the variation of Volumetric strain, ε_v versus Shear strain, γ.

6 ANALYSIS AND RESULTS

A fixed amplitude displacement control analysis has been carried out by using dynamic relaxation. There were a total of 10 yield surface nested concentrically as shown in Figure 2. The yield surfaces were activated one after another. The activation of an yield surface is decided by the proximity tolerance (f=0.0001). When two consecutive yield surfaces yields due to loading and difference between them becomes less than the tolerance, then the larger aperture yield surface is activated. In Figure 2, the currently active yield surface is designated by red color. The model is integrated through a return mapping algorithm. The determination of exact value of proximity tolerance is a limitation to this implementation.

The results of the analysis are shown in Figure 3 and 4. Three type of analysis are carried out in this research. First analysis was very slow loading analysis, showed in black colored slid line. The other two analyses are carried out at higher speed of loading. It has been found that viscous effect of loading rate is pronounced much when the viscous formulation is based on all components of initial shear angles (α_{ij}).

7 CONCLUSIONS

A visco-elasto-plastic multi-surface model is developed for cyclic loading. In this research, the original model is extended to include the effect of rate of loading. The integration of incremental elasto-plastic equation is carried out by steepest descent return mapping algorithm.The dilatancy of dense sand has been modeled using non-associated flow rule via direct inclusion of Row's stress-dilatancy relationship.

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