Application of Genetic Algorithms with Hill Climbing Procedure to a Constitutive Model for Hard Soils and Soft Rocks

Application des algorithmes génétiques avec la méthode de gradient à un modèle constitutif pour sols durs et roches tendres

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ABSTRACT: For engineering applications, the complex behaviour of hard soils / soft rocks can be modelled using advanced constitutive models, although they require a great number of parameters. The application of Genetic Algorithms with a local search technique has proven to be a useful tool to be used in their determination. A constitutive model for hard soil / soft rocks was used to fit the experimental results measured in tests performed in marl specimen. This model was implemented in the explicit finite difference code FLAC and its calibration was done using a Genetic Algorithm with Hill Climbing procedure implemented in MATLAB. The use of the two programs with complete distinct objectives (MATLAB to the fitting process and FLAC to the numerical calculations) provides great flexibility to the implementation of any constitutive model to reproduce the results from experimental tests.

RÉSUMÉ : Pour les applications en ingénierie, le comportement complexe des sols durs / roches tendres peut être modélisé à l'aide de modèles constitutifs avancés même s'ils nécessitent un grand nombre de paramètres. L'emploi des algorithmes génétiques avec une technique de recherche locale, s'est avéré un outil utile pour la détermination de ces paramètres. Un modèle de comportement pour les sols durs/ roches tendres a été utilisé pour ajuster les résultats expérimentaux mesurés lors de tests sur des échantillons de marne. Ce modèle a été mis en œuvre dans le code aux différences finies explicite FLAC et son calibrage a été fait en utilisant un algorithme génétique avec la procédure "Hill Climbing" installé dans MATLAB. L'utilisation conjointe de ces deux programmes avec des objectifs complètement différents (MATLAB pour l'ajustage des paramètres et FLAC pour les calculs numériques) donne une grande flexibilité à cette technique pour la mise en œuvre de modèles constitutifs pour reproduire les résultats d'essais.

KEYWORDS: Genetic Algorithm, Hill Climbing procedure, constitutive model, hard soils / soft rocks, marls.

1 INTRODUCTION

In general, marls are classified as hard soils / soft rocks, HSSR, and exhibit evolutive behaviour, since their mechanical and hydraulic properties are strongly affected by suction and stress changes, related with plastic strain, wetting / drying cycles and others weathering processes (Cardoso 2009). Evident physical degradation results from these changes.

HSSR are often treated as bonded materials in which links (cements or other physical connections) provide additional strength and stiffness to the soil structure. Progressive rupture of these bonds, caused by stress and suction changes, affects irreversibly the hydro-mechanical behaviour of these materials. Several constitutive models for HSSR can be found in the literature (Nova 2005, Gens and Nova 1993, Kavvadas and Belokas 2001, Hashiguchi 2009). Most of the existing models use bond degradation as a function of the accumulated plastic strains and damage laws to represent the behaviour of these materials.

An extensive experimental programme was developed in order to characterise the physical, mechanical and hydraulic properties of Abadia marls, dated from the Upper Jurassic. These geomaterials occurred and were used in the A10 motorway (Arruda dos Vinhos, Portugal). For the numerical modelling of structures formed with these marls, a nine parameter constitutive model based on the two yield surfaces concept presented by Gens and Nova (1993) was developed and applied. For the evaluation of these parameters a genetic algorithm (GA) was implemented.

Previous GA application for parameter determination purposes allowed demonstrating the efficiency and flexibility of this procedure (Pal et al. 1996). Also the association of a local search technique, like Hill Climbing (HC), improved the convergence of GA (Taborda et al. 2011).

In this paper, a GA with an embedded HC procedure was implemented in MATLAB to fit a HSSR constitutive model to the marls experimental results. The constitutive model formulation is herein presented and was implemented in the explicit finite difference code FLAC (through the programming language C++).

2 CONSTITUTIVE MODEL FOR HSSR

In the constitutive model presented in this section all the stresses considered are effective stresses. This model has two yield surfaces, f_u and f_b, based on modified Cam Clay yield surface (see Figure 1), defined as:

$$f_u = \frac{\mathbf{s} : \mathbf{s}}{c^2} + p(p - p_{cu}) \tag{1}$$

$$f_b = \frac{\mathbf{s} : \mathbf{s}}{c^2} + (p + p_t)(p - p_{cb}) \tag{2}$$

where f_b represents the bonded material yield surface (current yield surface), f_u the idealised yield surface of the unbounded material corresponding to the limiting case of destructured soil, $p = \mathrm{tr}(\boldsymbol{\sigma})/3$ and $\mathbf{s} = \mathrm{dev}(\boldsymbol{\sigma})$ are the mean and deviatoric parts of the effective stress tensor $\boldsymbol{\sigma}$, respectively, c is a material parameter, p_{cu} and p_{cb} are the yield mean stress of the unbonded and bonded materials, respectively, and p_t is the yield mean stress in tension of the bonded material.

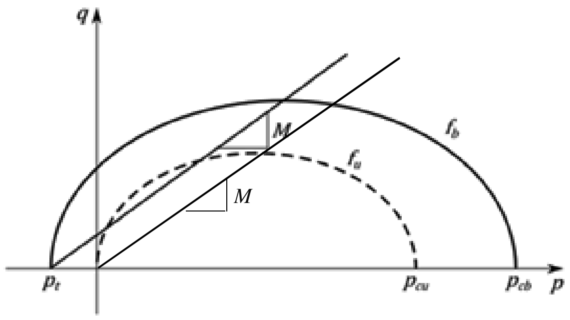

Figure 1. Representation of the model's yield surfaces, in a (*p*,*q*) space.

When $c = \sqrt{2/3}\,M$, the modified Cam Clay yield surface is obtained for f_u. The two yield surfaces concept follows the work presented by Gens and Nova (1993).

The elastic behaviour is governed by the following hypoelastic model

$$\mathrm{d}\boldsymbol{\sigma}^e = \mathbb{D}^e : \mathrm{d}\boldsymbol{\varepsilon}^e = K\mathrm{d}\varepsilon_v^e\mathbf{I} + 2G\mathrm{d}\mathbf{e}^e \tag{3}$$

$$\mathbb{D}^e = K\mathbf{I}\otimes\mathbf{I} + 2G\left(\mathbb{I} - \frac{1}{3}\mathbf{I}\otimes\mathbf{I}\right) \tag{4}$$

$$K = \frac{p}{\kappa} \tag{5}$$

$$G = \frac{3}{2}\frac{1-2\upsilon}{1+\upsilon}K \tag{6}$$

where $\mathrm{d}\boldsymbol{\sigma}^e$ is the increment of the elastic effective stress tensor, $\mathbb{D}^e$ the fourth order elastic stiffness tensor, K and G the elastic bulk and shear modulus, respectively, υ the Poisson's ratio, $\mathrm{d}\varepsilon_v^e$ the elastic volumetric strain increment, $\mathrm{d}\mathbf{e}^e$ the deviatoric part of the elastic strain tensor increment and κ the swelling lines slope.

An associated plastic flow law is considered. Therefore, the plastic strain rate, $\mathrm{d}\boldsymbol{\varepsilon}^p$ takes the form

$$\mathrm{d}\boldsymbol{\varepsilon}^p = \Lambda\frac{\partial f_b}{\partial\boldsymbol{\sigma}} \tag{7}$$

where Λ is the plastic multiplier.

This model has two hardening laws: (i) the first one is related with the behaviour of the unbonded yield surface, f_u, and (ii) the second one is related with the decrease of the space between both yield surfaces as a result of debonding.

Based on the work presented by Nova (2005), the bonding effects can be quantified by a parameter b, defined as

$$\frac{p_{cb}}{p_{cu}} = 1 + b \Leftrightarrow b = \frac{p_{cb}-p_{cu}}{p_{cu}} = \frac{p_m}{p_{cu}}, \tag{8}$$

and the value of p_t is defined as

$$p_t = \alpha p_m \tag{9}$$

where $p_m = p_{cb} - p_{cu}$ represents the part of the yield mean stress corresponding to the bond effects and α defines the value of p_t as a function of p_m.

According to the same author, the hardening laws of the p_{cu} and p_m take the form of Eqs. 10 and 11, where parameter ρ depends on the bond fragility, since it measures the rate at which the bonds are broken and the soft rock is transformed into a destructured soil, λ is the normal compression line slope of the unbounded material, ξ controls the value of dilatancy at failure, $\mathrm{d}\varepsilon_v^p$ is the plastic volumetric strain increment and $\mathrm{d}\mathbf{e}^p$ is the deviatoric part of the plastic strain tensor increment.

$$\mathrm{d}p_{cu} = \frac{p_{cu}}{\lambda-\kappa}\left(\mathrm{d}\varepsilon_v^p + \xi\|\mathrm{d}\mathbf{e}^p\|\right) \tag{10}$$

$$\mathrm{d}p_m = -\rho p_m\left|\mathrm{d}\varepsilon_v^p\right| \tag{11}$$

Through the consistency condition, defined as

$$\begin{aligned}\mathrm{d}f_b &= \frac{\partial f_b}{\partial\boldsymbol{\sigma}}:\mathrm{d}\boldsymbol{\sigma} + \frac{\partial f_b}{\partial p_{cb}}\mathrm{d}p_{cb} + \frac{\partial f_b}{\partial p_t}\mathrm{d}p_t = \frac{\partial f_b}{\partial\boldsymbol{\sigma}}:\mathrm{d}\boldsymbol{\sigma} + \\ &+ \frac{\partial f_b}{\partial p_{cb}}\left(\frac{\partial p_{cb}}{\partial p_m}\mathrm{d}p_m + \frac{\partial p_{cb}}{\partial p_{cu}}\mathrm{d}p_{cu}\right) + \frac{\partial f_b}{\partial p_t}\frac{\partial p_t}{\partial p_m}\mathrm{d}p_m = 0\end{aligned} \tag{12}$$

the plastic multiplier, Λ, is obtained by

$$\Lambda = \frac{\frac{\partial f_b}{\partial\boldsymbol{\sigma}} : \mathbb{D}^e : \mathrm{d}\boldsymbol{\varepsilon}}{\frac{\partial f_b}{\partial\boldsymbol{\sigma}} : \mathbb{D}^e : \frac{\partial f_b}{\partial\boldsymbol{\sigma}} + M_p} \tag{13}$$

with

$$M_p = -\left[\frac{\partial f_b}{\partial p_{cb}}\left[(1+\alpha)H_m + H_c\right] + \frac{\partial f_b}{\partial p_t}\alpha H_m\right] \tag{14}$$

$$H_m = -\rho p_m\left|\mathrm{tr}\left(\frac{\partial f_b}{\partial\boldsymbol{\sigma}}\right)\right| \tag{15}$$

$$H_c = \frac{p_{cu}}{\lambda-\kappa}\left[\mathrm{tr}\left(\frac{\partial f_b}{\partial\boldsymbol{\sigma}}\right) + \xi\left\|\mathrm{dev}\left(\frac{\partial f_b}{\partial\boldsymbol{\sigma}}\right)\right\|\right]. \tag{16}$$

The increment of the effective stress tensor, $\mathrm{d}\boldsymbol{\sigma}$, is defined as

$$\mathrm{d}\boldsymbol{\sigma} = \mathbb{D}^e : \left(\mathrm{d}\boldsymbol{\varepsilon} - \Lambda\frac{\partial f_b}{\partial\boldsymbol{\sigma}}\right). \tag{17}$$

This constitutive model requires the definition of nine parameters: six constants (κ, λ, M, υ, ρ and ξ) and three initial values ($p_{cb,ini}$, $p_{cu,ini}$ and α_{ini}), in addition to the initial stress and strain tensor, $\boldsymbol{\sigma}_{ini}$ and $\boldsymbol{\varepsilon}_{ini}$, respectively.

3 MARL'S PROPERTIES AND EXPERIMENTAL TESTS

An extensive set of experimental tests was performed to characterise the behaviour of the marls studied in unsaturated states (Cardoso 2009, Muralha et al. 2011). The tests performed on rock specimens for characterising the mechanical properties of this marl are unconfined compression tests, Brazilian splitting tests, oedometric tests and isotropic compression tests followed by triaxial tests. Unsaturated states were achieved using vapour equilibrium as controlled suction technique (Cardoso, 2009). Only the results of the triaxial tests performed under constant suction $s = 39$ MPa ($RH = 75\%$) are analysed in this paper.

The model adopted was used to this unsaturated material after computing effective stresses using Eq. 18 proposed by Alonso et al. (2010)

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma}_{net} + sS_r{}^a\mathbf{I} \tag{18}$$

where, $\boldsymbol{\sigma}'$ is the effective stress tensor, $\boldsymbol{\sigma}_{net} = \boldsymbol{\sigma} - p_a\mathbf{I}$ the net stress tensor, $\boldsymbol{\sigma}$ the total stress tensor, p_a the air pressure, *s* the suction, S_r the degree of saturation and a the parameter that corrects the global degree of saturation to discount the microstructural degree of saturation, which quantifies the immobile water within the soil that does not affect volume changes. For $s = 39$ MPa, $S_r = 35\%$ considering the water retention properties of the marls determined by Cardoso (2009). For this material $a = 4.56$. Assuming $p_a = 0$, $\boldsymbol{\sigma}' = \boldsymbol{\sigma} + 325.1\mathbf{I}$ [kPa].

The stress paths adopted in these tests consisted of a first stage where isotropic compression was applied, followed by the increment of the axial stress until shear failure. Two unloading / reloading cycles were applied in the isotropic compression phase under the confinement mean stress corresponding to the beginning of the shear phase. Five tests were performed under this constant suction, having different confining pressures: 4 MPa (tests 3 and 4), 8 MPa (tests 6 and 8), and 12 MPa (test 12) (Muralha et al. 2011). Only test 12 is analysed in this paper because it differs from the others due to the fact that one unloading / reloading cycle was applied in shear phase. This cycle is important to characterise the shear stiffness in the elastic range necessary for the constitutive model. The curves corresponding to the two loading stages are shown in Figure 2. For this test $p = 12$ MPa at the beginning of the shear phase.

4 MODEL CALIBRATION

The parameters for the constitutive model were determined by fitting the experimental results found in test 12 using a GA and a HC procedure implemented in MATLAB. The constitutive model was implemented in FLAC.

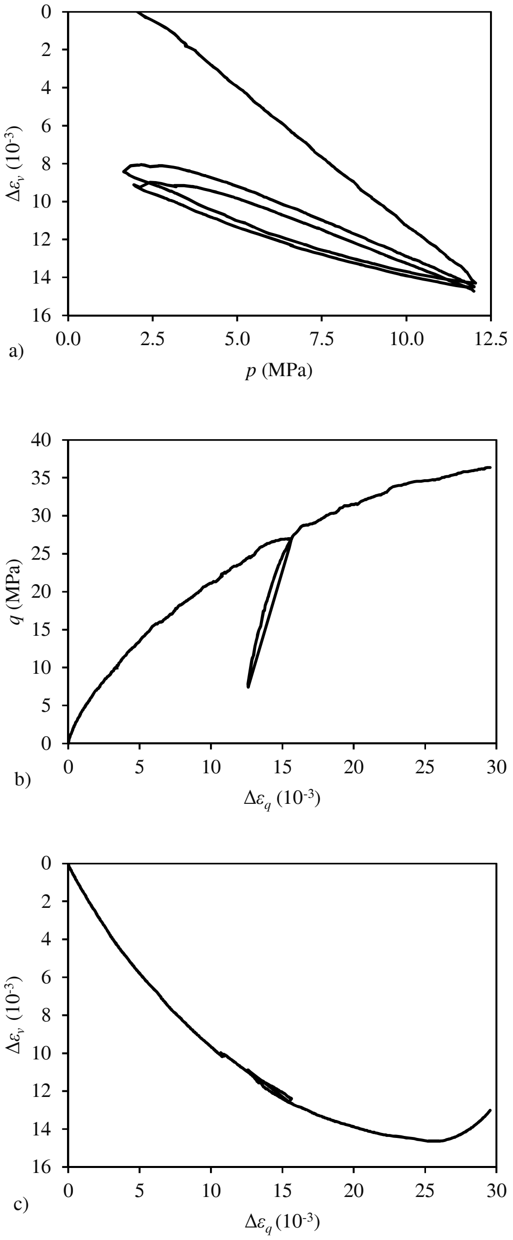

Figure 2. Experimental data of the test 12 defined in total stresses: a) isotropic phase, b) and c) shear phase.

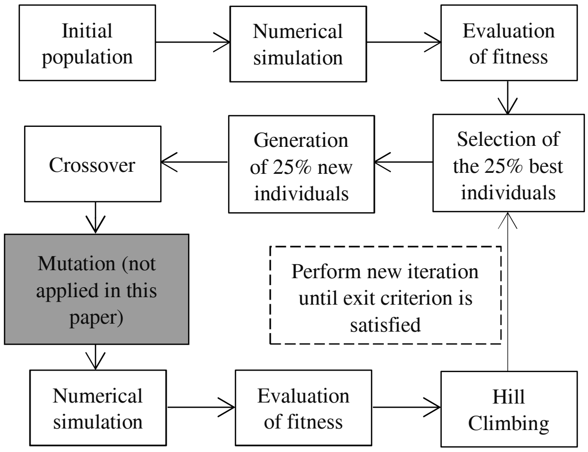

Figure 3. Scheme for implementing HC procedure into the GA in MATLAB with the numerical simulations in FLAC.

Table 1. Limits of the search area of each parameter.

Parameter	Max	Min	Parameter	Max	Min
κ	0.007	0.002	ρ	3.0	0.0
λ	0.030	0.008	$p_{cb,ini}$	8000	3000
M	1.85	1.20	b_{ini}	3.0	0.0
ν	0.35	0.25	α_{ini}	0.5	0.0

GA are assumed to be global search methods used to solve optimisation problems (see Pal et al. 1996). They employ concepts from the theory of natural evolution, such as selection, mutation, inheritance and crossover. HC procedure is a local direct search optimisation technique which, starting from a given initial solution, attempts to improve it by randomly altering its characteristics. The linkage between GA and HC was done according to the recommendations presented in Renders and Bersini (1994) and Taborda et al. (2011).

A simplified diagram illustrating the implementation process is presented in Figure 3. The algorithm starts with the definition of the initial population. For this problem, the initial population was randomly generated and was composed by 256 individuals. Each individual had 8 parameters (8 genes), which are the parameters of the model previously defined. In fact they should be 9 parameters but ξ was considered to be zero because the experimental curves measured in the shear phase tended to a horizontal line (Figure 2b and c). The limits of the search area of each parameter are presented in Table 1. The limits of b_{ini} ensure $p_{cb,ini} > p_{cu,ini}$ (Eq. 8).

Mutation, usually introduced in GA methods to avoid their early convergence into a unique solution, is replaced by coupling HC to GA because HC modifies the genes in each iteration.

The numerical simulation of each individual was performed by FLAC with the purpose to reproduce test 12 in both isotropic compression and shear stages. The unloading / reloading cycles from the two stages were considered.

The evaluation of each individual was made by measuring the area between the numerical and the experimental curves defined in spaces (i) $\varepsilon_v - p$, (ii) $\varepsilon_q - q$ and (iii) $\varepsilon_q - \varepsilon_v$, where

$$\varepsilon_q = \sqrt{\frac{2}{3}}\|\mathbf{e}\| \tag{19}$$

represents the deviatoric strain. The results were considered on dimensionless spaces to avoid scale effects. The 25% best individuals were selected to the next iteration.

The following iteration started with the random generation of more 25% of individuals, always using the limits presented in Table 1. After a random process where pairs of parents were formed, the crossover of the individuals was done with “alternate with variable probability crossing” method (Taborda et al. 2008, cited by Azeiteiro 2008). The other 50% of individuals were generated by this manner, using the information of their progenitors.

FLAC was used to test the 75% new individuals, which were evaluated using the procedure of measure the areas between the numerical and the experimental curves. After this, a new HC was applied only to the 25% best individuals, and the rest of the procedure was repeated until 150 iterations. A visual validation of the results was done.

The final results are presented in Figure 4 for all the spaces considered, namely (i) $\varepsilon_v - p$, (ii) $\varepsilon_q - q$ and (iii) $\varepsilon_q - \varepsilon_v$. The final parameters are in the Table 2. A good agreement was found between the numerical and the experimental data of test 12.

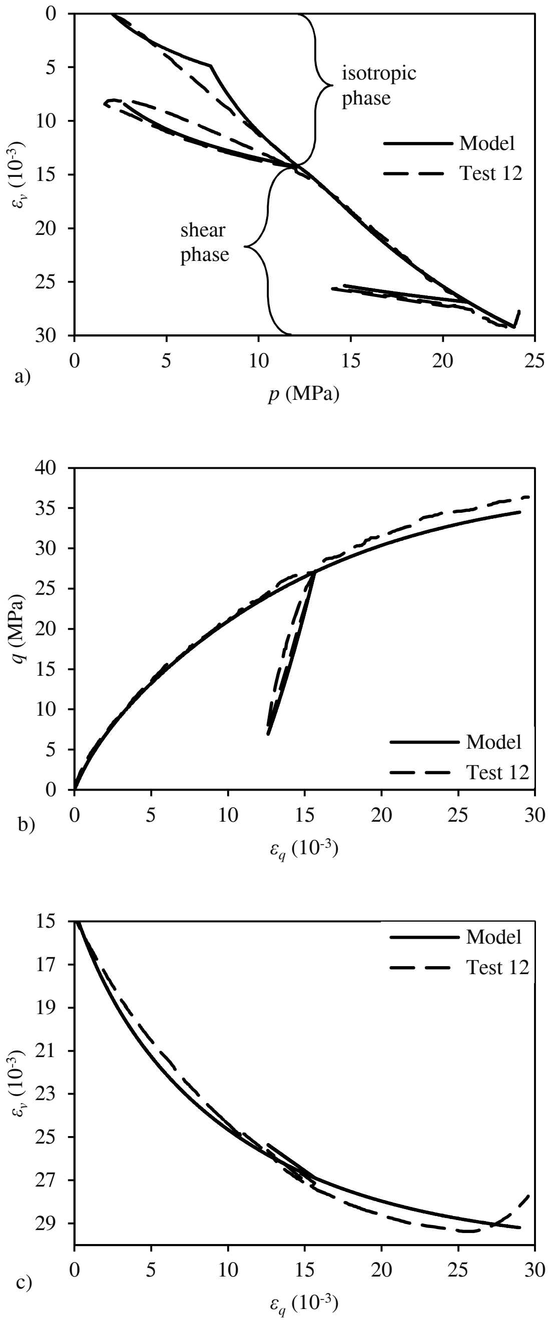

Figure 4. Comparison between the results obtained by the constitutive model and the test 12: a) $\varepsilon_v - p$, b) $\varepsilon_q - q$ and c) $\varepsilon_q - \varepsilon_v$.

Table 2. Final parameters for the model.

Parameter	Value	Parameter	Value
κ	0.0041	ρ	2.059
λ	0.0111	$p_{cb,ini}$	7396.5
M	1.414	$p_{cu,ini}$	2480.1
ν	0.284	α_{ini}	0.304

5 CONCLUSIONS

The constitutive model for HSSR used to simulate the behaviour of Abadia Marls requires 9 calibration parameters. Their calibration was done by fitting numerical results to experimental curve using a Genetic Algorithm with Hill Climbing procedure implemented in MATLAB. The physical meaning of the parameters was ensured by a proper definition of their search limits.

The simulation of the triaxial test on one specimen subjected to isotropic compression until reaching 12MPa, followed by shear until almost 35MPa, was done using FLAC. The comparison between the numerical and the experimental results shows a good agreement. This proves that this type of algorithms can be used in the determination of numerical parameters of constitutive models.

The link between MATLAB and FLAC used as complementary numerical tools provides great flexibility to this type of algorithms, for the determination of parameters of constitutive models with results from any experimental test.

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