Numerical analysis on prediction for residual deformation of earth structure using rigid plastic dynamic deformation analysis

Étude numérique pour prévoir la déformation résiduelle dun ouvrage en terre à l'aide de l’analyse de la déformation dynamique rigide plastique

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ABSTRACT: Recently, some deformation for against earth structure has been allowed in the current design code. Elastic plastic deformation analysis can evaluate properly of behavior of the ground. But may not be able to evaluate appropriate the amount of residual displacement from problems of initial stress and stress history. From such problem, we propose dynamic deformation analysis based on the rigid plastic constitutive model like limit equilibrium method.

RÉSUMÉ: Récemment, une certaine déformation des ouvrages en terre a été autorisée dans les codes de dimensionnement actuels. L’analyse de la déformation plastique-élastique permet d'évaluer exactement le comportement du sol. Pourtant, elle n’est appropriée pour la mesure de la déformation résiduelle des problèmes de contrainte initiale et d’histoire de contrainte. Face aux problèmes ci-dessus, nous proposons une analyse de la déformation dynamique basée sur un modèle constitutif rigide-plastique comme la méthode d'équilibre limite.

KEYWORDS: *rigid plastic constitutive equation , dynamic deformation analysis , residual deformation*

1 INTRODUCTION

Recently, the stability evaluation is done by a residual deformation from viewpoint of rationalization in the designing earth structure. Example, it has been proposed the elasto plastic deformation analysis method using the elasto plastic constitutive equation as a method to predict the residual deformation. The analysis method can properly evaluate behavior of the ground. But there are some problems as such effect by stress history and initial stress, and setting of analysis parameters. Therefore, may not be able to evaluate properly a residual displacement against conditions of target problem. In addition, it feels limitation of applicability against complex problems of the slope because the governing equation is expressed by incremental equation.

In this study, we developed a rigid plastic dynamic finite deformation analysis based on the rigid plastic finite element method (RPFEM) assuming the rigid plastic theory to the soil material. The RPFEM has been applied to the stability evaluation as such the bearing capacity problems of the earth structure in the geotechnical engineering field. It has advantage that it isn't necessary assuming slip surface, and considering a geometric nonlinearity is easy, and applicability to express the ground characteristic is good. Therefore, it can reasonably express behavior of the earth structure.

In this paper, we will explain about formulation of the proposed method used a rigid plastic constitutive equation. In addition, we will do simulations of the bearing capacity problem in the horizontal ground and the slope. And, we will show applicability to deformation problems by the proposed method from simulation's results.

2 ANALYSIS METHOD

2.1 *The rigid plastic constitutive equation*

We formulated the rigid plastic constitutive equation using the Drucker-Prager yield function from the upper bound theorem of the limit theorems. Here, I_1 is first invariable value of stress tensor. J_2 is second invariable value of deviatoric stress tensor. And, ω and ψ relate to a cohesion and an angle of shear resistance based on the mohr coulomb's failure criterion is coefficients. The tensile stress has been defined positive.

$$f(\boldsymbol{\sigma}) = \omega I_1 + \sqrt{J_2} - \psi = 0 \tag{1}$$

A stress $\boldsymbol{\sigma}$ decompose to a stress $\boldsymbol{\sigma}^{(1)}$ and a stress $\boldsymbol{\sigma}^{(2)}$. Here, the stress $\boldsymbol{\sigma}^{(1)}$ can define by a strain velocity, the stress $\boldsymbol{\sigma}^{(2)}$ can not define by the strain velocity. The stress $\boldsymbol{\sigma}^{(1)}$ is expressed from the associated flow rule. The stress $\boldsymbol{\sigma}^{(2)}$ is expressed using a condition equation (volume change characteristic) on the strain velocity and the indefinite constant. Here, $\dot{\boldsymbol{\varepsilon}}$ is the strain velocity. $\dot{e}$ is a equivalence strain velocity. $\dot{\varepsilon}_v$ is a volume strain velocity. $\boldsymbol{I}$ is a unit tensor.

$$\boldsymbol{\sigma}^{(1)} = \frac{\psi}{\sqrt{3\omega^2+1/2}}\frac{\dot{\boldsymbol{\varepsilon}}}{\dot{e}} \ , \ \dot{e} = \sqrt{\dot{\boldsymbol{\varepsilon}}:\dot{\boldsymbol{\varepsilon}}} \tag{2}$$

$$h(\dot{\boldsymbol{\varepsilon}}) = \dot{\varepsilon}_v - \frac{3\omega}{\sqrt{3\omega^2+1/2}}\dot{e} = \dot{\varepsilon}_v - \eta\dot{e} = 0 \tag{3}$$

$$\boldsymbol{\sigma}^{(2)} = \beta\frac{\partial h}{\partial \dot{\boldsymbol{\varepsilon}}} = \beta\left\{\mathbf{I} - \frac{3\omega}{\sqrt{3\omega^2+1/2}}\frac{\dot{\boldsymbol{\varepsilon}}}{\dot{e}}\right\} \tag{4}$$

Finally, it is obtained the equation (5) as the rigid plastic constitutive equation from the equation (2) and the equation (4). In addition, the condition equation (3) has been incorporated to the constitutive equation (5) by applying the penalty method (κ is the penalty constant) because speed up of calculation.

$$\boldsymbol{\sigma} = \frac{\psi}{\sqrt{3\omega^2+1/2}}\frac{\dot{\boldsymbol{\varepsilon}}}{\dot{e}} + \kappa(\dot{\varepsilon}_v - \beta\dot{e})\left\{\mathbf{I} - \frac{3\omega}{\sqrt{3\omega^2+1/2}}\frac{\dot{\boldsymbol{\varepsilon}}}{\dot{e}}\right\} \tag{5}$$

2.2 *Formulation of the governing equation*

A magnitude of strain velocity in the rigid plastic constitutive equation (a relationship of the stress and the strain velocity) is indefinite value. It has been proposed an analysis method using the constrained condition to restrict a magnitude of displacement velocity in the ultimate bearing capacity problems. But, it can not apply the constrained condition in the deformation analysis. Therefore, we defined the magnitude of displacement velocity using the equation of motion (using the momentum) in this study.

The equation (6) is the equation of motion at the reference configuration. Here, ρ_0, $\ddot{\mathbf{u}}$, $\boldsymbol{\pi}$ is a mass, an acceleration, a nominal stress at the reference configuration. $\mathbf{g}$ is a gravitational acceleration.

$$Div\boldsymbol{\pi} + \rho_0\mathbf{g} = \rho_0\ddot{\mathbf{u}} \tag{6}$$

It is obtained a weak form by to apply the principle of virtual work to the equation of motion (6). In addition, it is obtained the equation (7) as a weak form of equation of motion by to update to the current configuration from the reference configuration besed on the updated lagrange method. Here, V, S_σ is a volume boundary, a sress boundary at the current configuration. ρ, $\boldsymbol{\sigma}$ is a mass, a ttrue stress at the current configuration.

$$\begin{aligned}\int_V \boldsymbol{\sigma} : div\delta\dot{\mathbf{u}}dV + \int_V \rho\ddot{\mathbf{u}}\cdot\delta\dot{\mathbf{u}}dV \\ = \int_{S_\sigma} \mathbf{t}\cdot\delta\dot{\mathbf{u}}dS + \int_V \rho\mathbf{g}\cdot\delta\dot{\mathbf{u}}dV\end{aligned} \tag{7}$$

As shown in the equation (5), the rigid plastic constitutive equation has characteristic that the true stress is decided from the boundary problems at the current configuration because there are not effect by the stress history. The rigid plastic dynamic deformation analysis is formulated as shown in the equation (8) by to apply the rigid plastic constitutive equation to the equation (7). Here, it need to repetition calculation because the equation (8) is the nonlinear equation having a acceleration (displacement acceleration), a displacement velocity and a displacement. Therefore, we carried out analysis by the direct substitution method in this study. In addition, the proposed method applied the implicit method by the wilson' s θ method because the equation (8) has the acceleration (displacement acceleration) and the displacement velocity of unknown.

$$\begin{aligned}&\int_V \left[\frac{\psi}{\sqrt{3\omega^2+1/2}}\frac{\dot{\boldsymbol{\varepsilon}}}{\dot{e}}\right] : div\delta\dot{\mathbf{u}}dV \\ &+ \int_V \left[\kappa\left(\dot{\varepsilon}_v - \beta\dot{e}\right)\left\{\mathbf{I} - \frac{3\omega}{\sqrt{3\omega^2+1/2}}\frac{\dot{\boldsymbol{\varepsilon}}}{\dot{e}}\right\}\right] : div\delta\dot{\mathbf{u}}dV \\ &= \int_{S_\sigma} \mathbf{t}\cdot\delta\dot{\mathbf{u}}dS + \int_V \rho\mathbf{g}\cdot\delta\dot{\mathbf{u}}dV - \int_V \rho\ddot{\mathbf{u}}\cdot\delta\dot{\mathbf{u}}dV \\ &\qquad for \quad \forall\delta\dot{\mathbf{u}}\end{aligned} \tag{8}$$

The equation of motion expresses dynamic behavior of the ground against action force. There is a characteristic that an equilibrium of disagreement between the external load and the internal stress by the inertia force is satisfied from viewpoint of the static equilibrium equation. Therefore it can keep the equilibrium by the inertia force against the external load of if the ground can not bear. However, there is a possibility to occur sudden deformation because of large acceleration (displacement acceleration) due to the inertia force. Therefore, it is expressed the velocity effect of bearing capacity with dynamic behavior by the inertia force even if the ground's strength is constant.

3 VERIFICATION OF ANALYSIS METHOD

This chapter will show result of the limit bearing capacity analysis and the dynamic deformation analysis against the analysis condition at the Table 1 and the horizontal ground model at the Figure 1. This calculation does not consider the body force. And, it does not update coordinate from the reference configuration based on the infinitesimal deformation theory to compare to the theoretical solution.

The Prandtl has shown the theoretical solution $(2+\pi)c$ in the limit bearing capacity of the horizontal ground. The theoretical solution of the limit bearing capacity is obtained 102.83 kPa from the Table 1.

Next, we show the equivalence strain velocity distribution and the collapse mode of the ground at the Figure 2 as result of the limit bearing capacity analysis using the rigid plastic constitutive equation (5). The collapse mode expresses by to use a displacement which multiplied a displacement velocity to any time. It showed that has been obtained similar collapse mode when compared to the Prandtl's theoretical collapse mode.

In addition, this analysis obtained 104.87 kPa as the limit bearing capacity.

Next, we show the result (a relationship of the loading and the displacement acceleration, the displacement velocity, the displacement) of the dynamic deformation analysis used the rigid plastic constitutive equation (5) of the proposed method at the Figure 3. We applied a loading velocity of 10.0 kPa/sec (a time interval Δt of 0.1 sec/step) as the analysis condition. It is

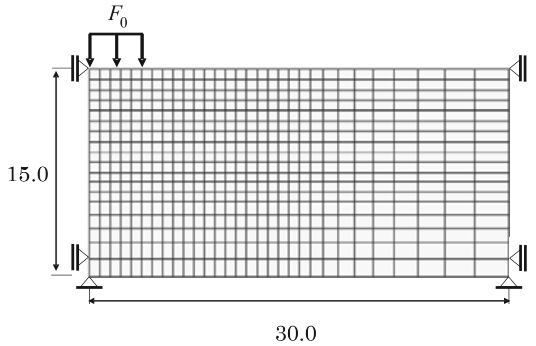


Figure 1. Analysis model [Length unit : m]

Table 1. Analysis condition

Angle of shear resistance ϕ [°]	0.0
Cohesion c [kPa]	20.0
Unit weight γ_t [kN/m^3]	0.0
Initial loading F_0 [kPa]	20.0

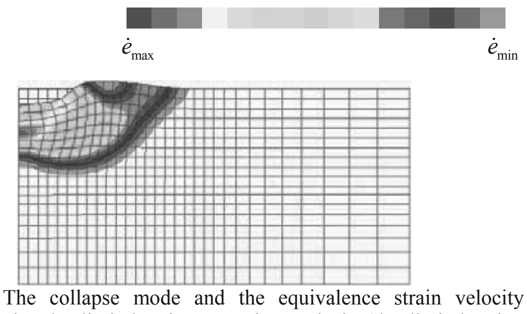


Figure 2. The collapse mode and the equivalence strain velocity distribution by the limit bearing capacity analysis (the limit bearing capacity is 104.87 kPa)

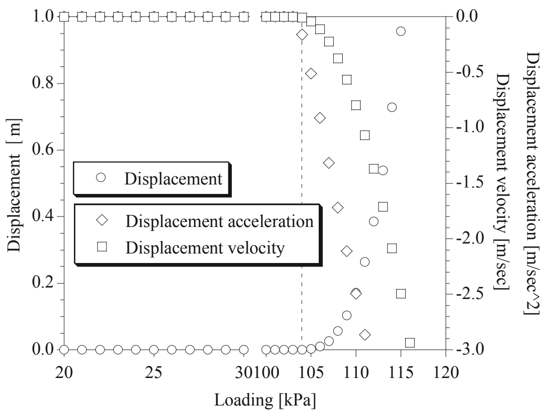
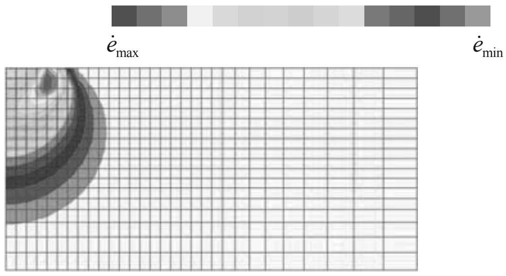
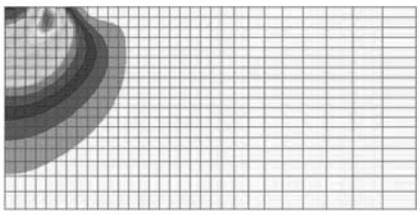


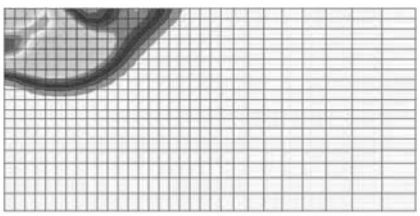
Figure 3. Relationship of the loading and the displacement acceleration, the displacement velocity, the displacement



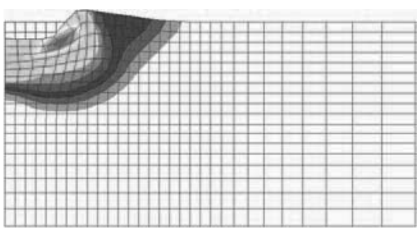
(a) Loading：84.0 kPa



(b) Loading：99.0 kPa



(c) Loading：105.0 kPa



(d) Loading：116.0 kP

Figure 4. The collapse mode and the equivalence strain velocity distribution by the dynamic deformation analysis

shown to occur deformation if the loading exceeds 105.0 kPa (Figure 3). It indicated that the proposed method can obtain similar result against the Prandtl's theoretical solution and the limit bearing capacity analysis because this loading value is the limit bearing capacity. In addition to that, we show the equivalence strain velocity distribution and the deformation mode against each loading at the Figure 4. It indicated that occur the equivalence strain velocity of the bulb form at the initial loading. After it indicated the similar deformation mode against the Prandtl's theoretical collapse mode with increase of the loading.

It indicated that the rigid plastic dynamic deformation analysis can evaluate properly against the limit bearing capacity problems of the horizontal ground from the above simulation results.

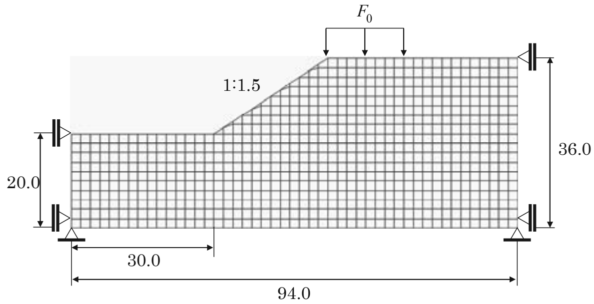


Figure 5. Analysis model [Length unit : m]

Table 2. Analysis condition

Angle of shear resistance ϕ [°]	10.0
Cohesion c [kPa]	50.0
Unit weight γ_t [kN/m^3]	18.0
Initial loading F_0 [kPa]	180.0

4 VERIFICATION OF EFFECT BY THE LOADING HISTORY

This chapter will verify applicability of the proposed method against deformation behavior by the loading history such as increase or decrease. We show the analysis model at the Figure 5. This model has inclination of slope of 1 : 1.5. The loading applied to top of slope as the loading velocity of 10.0 kPa/sec (time interval Δt of 0.1 sec/step). The boundary condition of displacement gave the restraint condition of the model bottom and the horizontal restraint condition of the model side. The parameter assumed the cohesive soil (the Table 2).

4.1 *The limit bearing capacity analysis*

We show result of the limit bearing capacity analysis at the Figure 6. This Figure shows the equivalence strain velocity distribution and the collapse mode. Here, this collapse mode is expressed from displacement which multiplied displacement velocity to any time. It obtained the collapse mode which shows the slip line (the large shear zone of the equivalence strain velocity) of the circular arc form toward the toe of slope from the top of slope as result of the limit bearing capacity analysis. And it obtained 195.94 kPa as the limit bearing capacity.

4.2 *The bearing capacity deformation analysis*

Next, we show result of the deformation analysis considering the loading history against the bearing capacity problem of slope at the Figure 5. We carried out three cases of the case [1] constant increase, the case [2] keep after constant increase, the case [3] decrease after constant increase, as the analysis cases. In addition, we carried out comparison of calculation based on the infinitesimal deformation theory or the finite deformation theory because it verify effect of the geometry form. Both theories obtained 196 kPa as the same limit bearing capacity against the limit bearing capacity analysis's result because the displacement increased after the loading exceeds the loading 196 kPa. We show the collapse mode based on the finite deformation theory in the case [1] at the Figure 7. This collapse mode is expressed from the displacement which it is obtained from the deformation analysis. This collapse mode obtained

similar result against the collapse mode obtained by the limit bearing capacity analysis showing at the Figure 6.
Next, we show a relationship of the loading and the displacement based on both theories to the Figure 8 and the Figure 9. Firstly, we explain result based on the infinitesimal deformation theory at the Figure 8. If the case [1], it indicated the rapid increase inclination of the displacement by occurrence of the acceleration (displacement acceleration) with the loading speed by continuing the loading after it exceeds the limit bearing capacity. If the case [2], it indicated similar increase inclination against the case [1]. But it indicated the increase inclination smaller than the case [1]. If the case [3], it indicated the inclination which keep the constant displacement (the residual displacement) with the unloading after the displacement increased. This behavior shows to occur the strain velocity in the ground by to receive effect of previous motion after the unloading. So, it is conceivable that the displacement is kept by to occur the acceleration of opposite direction to converge by a gap of the loading as the dynamical reason.
Secondly, we explain result based on the finite deformation theory at the Figure 9. If the case [1], it indicated the increase inclination of the displacement after it exceeds the limit bearing capacity like result of the infinitesimal deformation theory. However it indicated the gentle increase inclination against the infinitesimal deformation theory. If the case [2], it indicated the inclination which keep the constant displacement after increased the displacement by to keep the constant loading like the case [3] of the infinitesimal deformation theory. This inclination is different inclination against the case [2] of the infinitesimal deformation theory. It is conceivable that deformation decreased because increased the limit bearing capacity of the ground by to occur effect of embedment with deformation of the ground by the loading as this reason. If the case [3], it indicated the inclination which keep the constant displacement like the case [3] of the infinitesimal deformation theory. However it indicated the inclination the residual displacement is smaller than the infinitesimal deformation theory from effect of embedment. It is proved that it can evaluate effect of the geometry form by based on the finite deformation theory from all analysis cases.
It has been shown applicability of the finite deformation analysis by the proposed method from this chapter's result. But the proposed method has the problem that it can't calculate by to occur distortion of the finite element by shear deformation with deformation of the ground. Example, such as the case [1] of the finite deformation theory. Therefore it need to improve so that can be applied to large deformation calculation of the ground. Example, such as the remesh techniques. We are going to improve this problem in the future.

5 CONCLUSIONS

We developed the rigid plastic dynamic deformation analysis using the rigid plastic constitutive equation to predict the residual deformation of the earth structure. The proposed method has characteristic that it can be done deformation analysis in the stress boundary problems which to apply in the rigid plastic constitutive equation is difficulty. Therefore the proposed method can do the residual deformation analysis by collapse of the earth structure. We compared the Prandtl's theoretical solution and the limit bearing capacity analysis in the horizontal ground to verify applicability of the proposed method. And we were carried out simulation in the slope to show that it can evaluate properly deformation behavior of the ground against the loading history. We showed that it can evaluate properly problems such as effect of the geometry form by using the proposed method from these result.

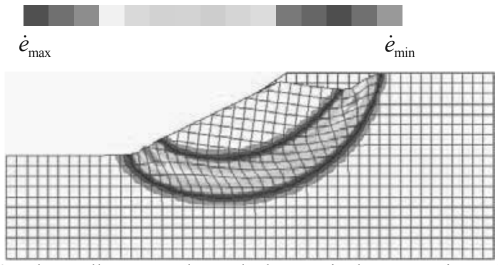


Figure 6. The collapse mode and the equivalence strain velocity distribution by the limit bearing capacity analysis (the limit bearing capacity is 195.94 kPa)

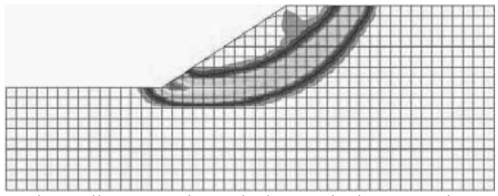


Figure 7. The collapse mode and the equivalence strain velocity distribution by the dynamic deformation analysis (the limit bearing capacity is 196.0 kPa)

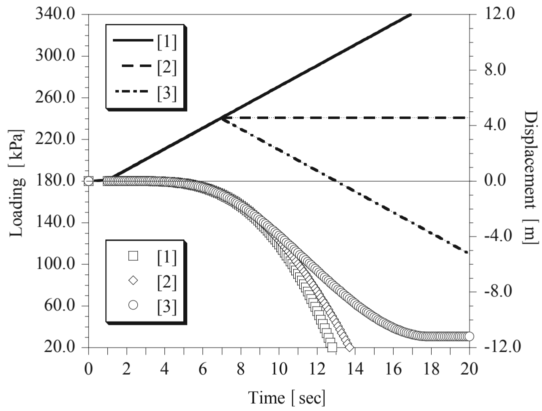


Figure 8. Difference of the residual displacement by the loading history in the infinitesimal deformation theory

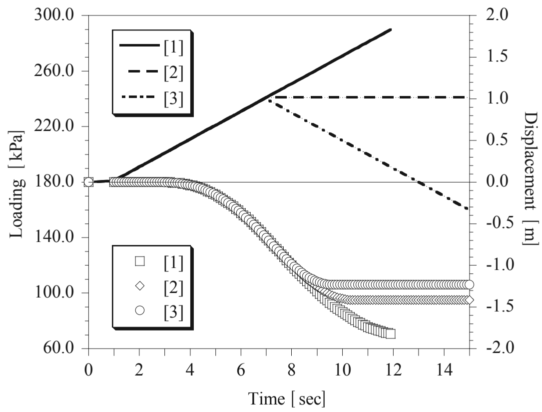


Figure 9. Difference of the residual displacement by the loading history in the finite deformation theory

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