General Report of TC103 Numerical Methods

Rapport général du TC103 Méthodes numériques

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ABSTRACT: This general report summarizes 52 papers being included in the TC103 Session on Numerical Methods. Instead of summarized each paper, we have provided an overall view of these papers. A master table (Table 3) is given for all 52 papers in terms of the types of numerical methods empolyed by different authors, together with the full references given at the end of the paper (paper number follows in alphabetic order). The numerical methods used include finite element method (FEM), finite difference method (FDM), material point method (MPM), smoothed particle hydrodynamics (SPH), neural network (NN), genetic algorithm (AG), and finite volume method (FVM). The failure models used in studies include Mohr-Coulomb failure criterion, Drucker-Prager plastic potential, Cam clay model, Matsuoka-Nakai failure model, and Hoek-Brown failure criterion. These numerical analyses have been applied to model piles, tunnels, retaining walls, slopes, levees, tailings impoundment, and breakwaters. Errors of and methods of vailidation for finite element method given by Brinkgreve and Engin (2013) was summarized briefly. Future challenges in numerical methods are outlined.

RÉSUMÉ : Ce rapport général résume les 52 articles inclus dans la session du TC103 sur les méthodes numériques. Au lieu de résumer chaque article, nous avons fourni une vue d'ensemble de ces documents qui sont décrits dans un tableau principal en fonction des types de méthodes numériques employées par différents auteurs, avec les références complètes en fin d'article (numéros d'articles selon un ordre alphabétique). Les méthodes numériques utilisées sont la méthode des éléments finis (FEM), la méthode des différences finies (FDM), la méthode du point matériel (MPM), l'hydrodynamique de particules lissé (SPH), les réseaux de neurones (NN), l'algorithme génétique (AG) et la méthode des volumes finis (FVM). Les modèles de rupture utilisés dans les études comprennent le critère de Mohr-Coulomb, le potentiel plastique de Drucker-Prager, le modèle Cam-Clay, le modèle de rupture de Matsuoka-Nakai et le critère de rupture de Hoek-Brown. Ces analyses numériques ont été appliquées aux pieux modèles, aux tunnels, aux murs de soutènement, aux pentes, aux digues, aux résidus miniers et aux brise-lames. Les erreurs et les modalités de validation pour la méthode des éléments finis donnée par Brinkgreve et Engin (2013) a été brièvement résumées. Certains défis à venir dans les méthodes numériques sont présentés.

KEYWORDS: Numerical Methods, Finite Element Method, Finite Difference Method, Mesh-free Method, SPH, MPM

1 INTRODUCTION. FIRST LEVEL HEADING

A total of 52 papers were submitted to the Technical Session of TC103: Numerical Methods. However, it should be noted that there is a number of papers included in this TC103 that are actually somewhat falled into the borderline between TC103 and other techncial committees, including Tc101 Laboratory Testing, TC 104 In-situ Testing, TC203 Earthquake, TC204 Underground Construction, TC207 Soil-Structure-Retaining Wall, TC208 Slope Stability, TC211 Ground Improvement, TC202 Transportation, and TC301 Historical & Case Studies. Some of these 52 papers included here could well be classified into these TC sessions. Indeed, it is very difficult to find a paper which is purely devoted to "Numerical Methods" without addressing real problems, such as slopes, piles, transportation, and underground constructons. The Sesssion Chairman is Prof. H. Nicot, and Session General Reporter is Prof. K.T. Chau. The main purpose of this general report is to summarize the issues that these 52 technical papers addressed. Some essential issues raised by these papers or related issues will be given at the end of this general report.

It is not an easy job to summarize a wide variety of papers within a short paper like this. Due to limitation of time and of printing space, it is impossible for me to review all 52 papers in great details and present them here. In addition, my summary to be given here will evitably be constrained by my personal educational background, my previous research works and my current interests, on different topics and issues. Neverthless, if I do not do full justice to any author, please offer me your forgiveness and understanding.

Table 1 summarizes the number of papers from each continent. Note that we only take the continent of the leading author of each papers. Geographically, 23 papers are from Europe, 15 from Asia, 8 from North America, 4 from Australia, 1 from South America and 1 from Africa. It appears that Europe is most active in ISSMGE meeting but it can likely be influenced by the location of the conference being at Paris, France. Proximity always makes travel commitment easier. Table 2 compiles the country distribution of all 52 papers. A total of 29 conutries were represented, including 5 from Canada, 4 from Australia, Japan and Norway, 3 from Spain and UK, 2 from Iran, France, The Netherlands, Russia, Singapore and USA, and 1 from each of the following countries and regions: Bangladesh, Brazil, China (Mainland), China (Taiwan), Croatia, Denmark, Egypt, Finland, Germany, India, Indonesia, Ireland, Mexico, Portugal, South Korea, Switzerland, and Thailand.

Note that this distribution may not be accurate since there are collaborations from different universities and different countries. We only count the country of the leading author. In addition, most of these papers are from universities but there are also some from consultant firms.

Table 1. Paper distributions among continents

Continent	Paper No.
Africa	1
Asia	15
Australia	4
Europe	23
North America	8
South America	1

Table 2. Geographical distribution of 52 papers

Country	Paper No.	Country	Paper No.
Australia	4	Ireland	1
Bangladesh	1	Japan	4
Brazil	1	Mexico	1
Canada	5	Netherlands	2
China, Mainland	1	Norway	4
China, Taiwan	1	Protugal	1
Croatia	1	Russia	2
Denmark	1	Singapore	2
Egypt	1	South Korea	1
Finland	1	Spain	3
France	2	Switzerland	1
Germany	1	Thailand	1
India	1	UK	3
Indonesia	1	USA	2
Iran	2		

2 CLASSIFICATION OF PAPERS UNDER DIFFERENT ASPECTS

To help readers to search for information from this Session on Numerical Methods, different aspects of these papers are grouped in a tabular form under different aspects. More specifically, a brief summary of all 52 papers is given in Table 3. The first column compiles the the types of numerical methods empolyed by various authors, second cloumn summarizes the software used if any, and the third column provides the types of problems that these numerical methods were proposed for.

The paper number actually follows the appearing order of these papers in the reference section. For example, P1 is Balkumar et al. (2013) whereas P52 is Yoneda (2013), etc.

Table 3. Classification of 52 papers into different aspects.

Paper No.	Method	Software used	Applications
P1	FEM	Plaxis	Pile
P2	FDM	FLAC	Reinforced earth
P3	-	-	Constitutive modeling
P4	-	-	Constitutive modeling
P5	FDM	FLAC	Soil-pile interaction
P6	FEM	-	Validation of FEM
P7	FDM	FLAC	Stone column
P8	SPH	-	Retaining wall
P9	FEM	Plaxis	Seismic pile
P10	FEM	GeoFEA	Large scale analysis on PC
P11	FEM	Plaxis	Hardening soil model
P12	FEM	-	Strain localization
P13	FEM	ABAQUS	Pile movement
P14	FEM	Plaxis	Excavation
P15	FVM	Riemann solver	Levee erosion
P16	FEM	ABAQUS	Pile driving
P17	FEM	-	Soil modeling
P18	FEM	CORONA	Earth structures
P19	RFEM	RFLA	Undrained bearing capacity
P20	-	-	Tailings
P21	FEM/FDM	GA	Coastal aquifers
P22	SPH	-	Levee erosion
P23	Galerkin	-	Soil-structure interaction
P24	FDM	FLAC	Soil-pile-structure interaction
P25	LEM	MUESA	Pore pressure
P26	FEM	SVSLOPE-3D	slope
P27	FEM	ABAQUS	Bridge/ Soil-structure
P28	FDM	FLAC	Tunnel
P29	FEM	-	Retaining wall
P30	FDM	FLAC/GA	Constitutive law for soft rock
P31	Analytic	-	penetrators
P32	-	-	Embankments/ Chemical-soil
P33	FEM	ABAQUS	Tunnel
P34	FEM	ABAQUS	Retaining wall
P35	NN	-	Pile-settlement
P36	FEM	-	Cyclic model/ constitutive
P37	FEM	FREW	Tunnel
P38	FEM	-	Breakwaters
P39	FDM	FLAC-2D	Desication crack
P40	FEM	-	Pile
P41	FEM	Iwan model	Deep excavation
P42	FEM	Plaxis-3D	Pile/tunnel
P43	-	-	Backfill
P44	FEM	-	Soil-structure interaction
P45	FDM	FLAC	Liquefaction
P46	NN	-	Single pile
P47	FEM	GEOASLA	Soil-water
P48	-	-	Constitutive modeling
P49	FEM	ABAQUS	Embankment
P50	MPM	-	Slope
P51	FEM	-	Pore water pressure
P52	FEM	COTHMA	Methane hydrate

Note that the following abbreviations are used: FEM = Finite Element Method, FDM = Finite Difference Method, FVM = Finite Volume Method, SPH = Smoothed Particle Hydrodynamics, MPM = Material Point Method, RFEM = Random Finite Element Method, GA = Genetic Algorithm, NN = Neural Network, and LEM = Limit equilibrium. Table 3 also provides a quick reference guide to all 52 papers, depending on the interest of the readers.

2.1 *Numerical methods used*

2.1.1 *Finite difference method*

The origin of finite difference method (FDM) probably traces back to the time of Leibniz and Euler (e.g. Euler's method in 1768), and subsequently evolves into different techniques (e.g. Runge-Kutta method). The FDM becomes more established since 1928 after the Courant-Friedrichs-Lewy (CFL) stability condition was derived for hyperbolic type of partial differential equations (Courant et al. 1928). Its day-to-day application, of course, starts with the popularization of computers, especially personal computers. For the use of finite difference method, there are 8 papers empolyed FDM and the sofware used is called FLAC (either 2D or 3D). They include Bennani et al. (2013), Breugnot et al. (2013), Bryson and El Naggar (2013), Kwon et al. (2013), Mayoral et al. (2013), Pereira et al. (2013), Stirling et al. (2013), and Wanatowski et al. (2013). The full name of FLAC is Fast Lagrangian Analysis of Continua, and it is a continuum code for modelling soil, rock and structural behaviour. It employs the explicit finite difference formulation and suites for modelling multistage geomechanical problems, such as sequential excavation, backfilling and loading. The formulation can accommodate large displacements and strains and non-linear material behaviour, even if yield or failure occurs over a large area or if total collapse occurs.

2.1.2 *Finite element method*

Finite element method (FEM) is a numerical technique for finding approximate solutions to boundary value problems. It uses either the weighted residue method (e.g. Galerkin method) or the variational methods via the use of calculus of variations (e.g. Rayleigh-Ritz method) to minimize an error function (in the case of weighted residue approach) or a functional (energy in the case of variational approach) in a global sense and produces a stable solution. FEM encompasses all the methods for connecting many simple element equations over many small subdomains, named finite elements, to approximate a larger domain. Similar to FDM, the origin of FEM is not so straightforward. As summarized by Oden (1987), it probably dates back to the time of Hrennikoff (1941) and Courant (1943). However, this idea was not further pursued then since computers were still largely unavailable. Decades later, the term “finite element method” was coined by the renowned structural and earthquake engineer Clough in 1960 when he involved in the design of wings of Boeing aeroplane (Clough, 1960, 1980). The first finite element book is “The Finite Element Method in Structural ad Continuum Mechanics” by O.C. Zienkiewicz (1967). At least 29 of these papers stated explicitly that finite element method (FEM) was employed. Among the available commerical FEM softwares, the most popular ones are ABAQUS and PLAXIS. Papers used ABAQUS include Elkady (2013), Hamann and Grabe (2013), Lyngs et al. (2013), Rezaei et al. (2013), Sadrekarimi and Monfared (2013), and Yapage et al. (2013). The name and logo of ABAQUS are based on the abacus calculation tool and is a product of Dassault Systemes Simulia Corp. founded in USA. Those used PLAXIS include Balakumar et al. (2013), Chang et al. (2013), Dong and Anagnostou (2013), Everaars and Peters (2013), and Mirmoradi and Ehrlich (2013). PLAXIS is a FEM software intended for 2-Dimensional and 3-Dimensional geotechnical analysis of deformation and stability of soil structures, as well as groundwater and heat flow, in geo-engineering applications such as excavation, foundations, embankments and tunnels. Some researchers used custom-made or more specialized FEM softwares, such COTHMA (Yoneda 2013), FREW (Smith et al. 2013), GeoFEA (Chaudhary et al. 2013), CORONA (Hoshina and Isobe 2013), SVSLOPE (Lu et al. 2013), EQWEAP (Chang et al. 2013), and MUESA (Lehtonen and Lansivaara 2013). In addition, Yesuf et al. (2013) did not give the name of the FEM that you used. A special form of FEM called random FEM (or RFEM) is used by Huang et al. (2013) in considering bearing capacity of clay with nonhomogeneous properties. Although FEM is probbaly the most popular numerical methods used in geotechnical problems, it is not suitable for problems suffering from very large deformation such that the mesh is highly distorted.

2.1.3 *Smoothed particle hydrodynamics*

For the less conventional numerical techniques, smoothed particle hydrodynamics (SPH) was used by Kamalzare et al. (2013) and Bui et al. (2013). The SPH method belongs to mesh-free technique which has been widely adopted in other areas of mechanics. Smoothed particle hydrodynamics (SPH) is a computational method used for simulating fluid flows. It was developed by Gingold and Monaghan in 1977 (Gingold and Monaghan 1977) and Lucy in 1977 (Lucy 1977) initially for astrophysical problems. It is a mesh-free Lagrangian method in which the coordinates move with the fluid, and the resolution of the method can easily be adjusted with respect to variables such as the density. This technique can handle very large deformation and are more suitable for post-failure analysis. In particular, Kamalzare et al. (2013) used SPH method to invetsiagte levee’s erosion due to overtopping of water; and Bui et al. (2013) considered post-failure simulations of retaining walls using SPH.

In fact, SPH has been used in other geomechanics analysis. For example, McDougall and Hungr (2004, 2005) had developed a SPH model for debris flow simulations for 3-D terrain. Both erosion and entrainment have been incorparoted into their model. This SPH approach appears better than the traditional FDM appraoch used by Chau and Lo (2000)

2.1.4 *Material point method*

A numerical technique called material point method (MPM) was empolyed by Yerro et al. (2013) to model static-dynamic transition of slope failure. This MPM tecnique is particular useful in modeling large deformation problems, such as landslides, runouts or anchor pull-out. This formulation uses a dual description of the media by using Lagrangian material points and an Eulerian numerical mesh. The MPM, is an extension of the Particle-in-cell Method (a method developed in Los Alamos National Laboratory in 1957) in computational fluid dynamics to computational solid dynamics, and is a Finite element method (FEM)-based particle method. It is primarily used for multiphase simulations, because of the ease of detecting contact without inter-penetration. It can also be used as an alternative to dynamic FEM methods to simulate large material deformations, because there is no re-meshing required by the MPM. It was originally proposed by Sulsky et al. (1995).

2.1.5 *Neural networks*

There are three papers adopted neural networks (NN) models in considering capacity of piles and constitutive modeling. In particular, Hashash et al. (2013) presented the integration of self-learning simulations (SelfSim), which is based on neural network based material model, with laboratory testing to extract soil-behavior, whereas Shahin (2013) and Wardani et al. (2013) considered the load-settlement and ultimate bearing capacity of a single pile respectively. These NN model can be considered as a kind of artificial intelligence. An artificial neural network, often just named a neural network, is a mathematical model inspired by biological neural networks. A neural network consists of an interconnected group of artificial neurons, and it processes information using a connectionist approach to computation. In most cases a neural network is an adaptive system changing its structure during a learning phase. Neural networks are used for modeling complex relationships between inputs and outputs or to find patterns in data.

2.1.6 *Genetic algorithm*

Pereira et al. (2013) implemented the explicit finite difference code FLAC and its calibration was done using a Genetic Algorithm (GA) with Hill Climbing procedure implemented in MATLAB. The use of these two programs with complete distinct objectives (MATLAB to the fitting process and FLAC to the numerical calculations) provides great flexibility to the implementation of any constitutive model to reproduce the results from experimental tests. Javadi et al. (2013) incorporated numerical modelling of seawater intrusion with an genetic algorithm (GA) to examine different scenarios to control seawater intrusion including different combinations of abstraction, desalination and recharge. A Genetic Algorithm (GA) is a search heuristic that mimics the process of natural evolution. It is routinely used to generate solutions to problems of optimization and solution searching.

2.1.7 *Finite volume method*

"Finite volume" refers to the small volume surrounding each node point on a mesh, resulting from discretization of the body. In the finite volume method, volume integrals in a partial differential equation that contain a divergence term are converted to surface integrals, using the divergence theorem. These terms are then evaluated as fluxes at the surfaces of each finite volume. Because the flux entering a given volume is identical to that leaving the adjacent volume, these methods are conservative. FVM is best for solving conservative law in integral form and can solve for discontinuous solutions. The most fundamental hyperbolic wave problem with a jump discontinuity is called the Riemann problem (LeVeque, 2002). In fact, most of the current finite volume methods make use of the Riemann problem as the building block, and therefore FVM literally uses Riemann solver. Most of the FVM solution schemes used nowadays are of the Godunov-type (Godunov, 1959, LeVeque, 2002). Another advantage of the finite volume method is that it is easily formulated to allow for unstructured meshes. Fujisawa and Murakami (2013) presented a three-dimensional numerical analysis of embankment breaching based on the finite volume method, with a Riemann solver, in solving the shallow water equations for computing the overflow onto the embankments and the changes in configuration of the embankment profiles based on the erosion rates of the embankment materials as a function of the bed shear stress. As mentioned above, the erosion of levees has been considered by Kamalzare et al. (2013) using smoothed particle hydrodynamics (SPH). Therefore, it is recognized that numerical simulations of the coupling between geomaterials and water become more important in the case of geohazards, such as strom surges and tsunami.

2.2 *Failure criteria*

2.2.1 *Mohr-Coulomb failure model*

Among the 52 papers, 14 of them adopted the Mohr-Coulomb failure model in their numerical simulations. They include Biru et al. (2013), Chaudhary et al. (2013), Dong and Anagnostou (2013), Everaars and Peters (2013), Hoshina and Isobe (2013), Kwon et al. (2013), Lehtonen and Lansivaara (2013), Lu et al. (2013), Lyngs et al. (2013), Mayoral et al. (2013), Rezaei et al. (2013), Sturm (2013), Yapage et al. (2013), and Yerro et al. (2013). This failure model remains the most popuplar one.

2.2.2 *Plastic potential of Drucker-Prager*

The plastic potential of Drucker-Prager was employed by 7 authors, including Biru et al. (2013), Bui et al. (2013), Dong and Anagnostou (2013), Hoshina and Isobe (2013), Lehtonen and Lansivaara (2013), Sadrekarimi and Monfared (2013), and Siddiquee and Islam (2013). The Drucker-Prager model allows for non-associated flow rule, which is more appropriate for geomaterials.

2.2.3 *Matsuoka-Nakai failure model*

In considering the calibration of models for kakriritic rocks, Dong and Anagnostou (2013) replaced the original Mohr-Coulomb yield surface by the Matsuoka-Nakai criterion, which does not have the yield vertex as the Mohr-Coulomb model does. Thus, potential singularity problem at sharp corner on yield surface can be avoided (Chau, 2013).

2.2.4 *Hoek-Brown failure criterion*

When considering the blasting problems for granite, Yerro et al. (2013) used both Mohr-Coulomb as well as Hoek-Brown failure crietria in their MPM simulations. The Hoek-Brown failure model has a nonlinear failure envelop (comparing to the straight line envelop of Mohr-Coulomb) that appears to fit triaxial tests data for rocks better.

2.2.5 *Cam clay model*

Cam clay model has been adopted in Wanatowski et al. (2013) for soil-liquefaction, in Yao et al. (2013) for elastic-viscous-plastic modeling in overconsolidated clays, in Yamada and Noda (2013) for delayed failure in natural clays, in Pereira et al. (2013) in modeling hard soils. As demonstrated by Davis and Selvadurai (2002) and Chau (2013), cam clay model is a natural consequence of balancing the rate of plastic work and rate of dissipation (which is assumed as a linear function of shear strain rate) with normality assumed.

2.3 *Analysis by limit equilibrium*

A number of studies have adopted the results of limit equilibrium (LEM) for comparison with their numerical results. These include Hoshina and Isobe (2013) Lehtonen and Lansivaara (2013), Lu et al. (2013), Sadrekarimi and Monfared (2013), and Bui et al. (2013).

2.4 *Soil versus rock*

Among the 52 papers nearly all of them are related to problems on soils, and only 4 papers considered the deformation in rocks. These authors are Dong and Anagnostou (2013), James et al. (2013), Pereira et al. (2013), and Yerro et al. (2013).

2.5 *Types of loading conditions*

2.5.1 *Earthquake loadings*

Kwon et al. (2013) considered the soil-pile-structure interaction under earhquake loadings. Lyngs et al. (2013) considered soil-structure interaction for the seismic analyses of the Izmit Bay Bridge. When considering the performance of waste rock inclusions on tailings impoundment, James (2013) also included earthquake analysis. Wanatowski et al. (2013) considered the possibility of earthquake induced liquefaction across a wide spectrum of soils. Siddiquee and Islam (2013) modeled a visco-elasto-plastic material under cyclic loading, and its application is clearly related to seismic excitation. Chang et al. (2013) considered the seismic responses of piles subjected to earthquake excitations.

2.5.2 *Dynamic loadings*

There at least 9 papers address the dynamic effects in their analyses. These include Breugnot et al. (2013), Bennani et al. (2013), Hoshina and Isobe (2013), Kwon et al. (2013), Yamada and Noda (2013), Hamann et al. (2013), Lyngs et al. (2013), Yerro et al. (2013), and Stickle et al. (2013).

All other studies mainly deal with static problems.

2.6 *Applications to geo-structures*

2.6.1 *Pile analyses*

There are 14 papers out of 52 papers considered the deformations in piles. They include Balakumar et al. (2013), Breugnot et al. (2013), Elkady (2013), Tan et al. (2013), Kwon et al. (2013), Hamann and Grabe (2013), Lyngs et al. (2013), Wardani et al. (2013), Everaars et al. (2013), Smith et al. (2013), Sturm (2013), Shahin (2013), Chaudhary et al. (2013), and Chang et al. (2013). Therefore, behavoir of piles remains an area of active research. It probably is due to soil nonlinearity and both static and dynamic soil-pile interactions. The interaction is particularly important in the case of earthquake excitations. Although equivalent springs with stiffnesses depending on the frequency and damping was commonly calibrated by geotechnical engineers and used by structural engineers, nonlinearity is seldom incorporated. The study by Chau and Yang (2001) provided the first step forward in this direction. Koo et al. (2003) proposed a fully coupled continuum model for pile-soil-structure interaction and the resonant frequency and the amplification factor of the coupled system can differ from those of the soil, pile or structure alone. A 3-D version of this analysis is given by Chau and Yang (2005). Shaking table tests by Chau et al. (2009) illustrated that nonlinear pounding between piles and soil (separation of soil and pile following by impacts) will cause spikes in the dynamic responses at the pile cap level. Special contact element needed to be installed at the interface between soil and piles (Chau et al., 2009).

In short, we expect pile analysis, especially under seismic loads, remains an area of active research.

2.6.2 *Tunnels*

There are 7 papers considered the construction and design for tunnels. They include Kholmyansky and Sheynin (2013), Dong and Anagnostou (2013), Rezaei et al. (2013), Tan et al. (2013). Mayoral et al. (2013), Everaars and Peters (2013), Smith et al. (2013). We want to emphasize here that some of these tunnels were proposed in urban areas with existing structures. Such problems are now becoming more and more common and important in geotechnical engineering practice as interaction between the old existing structures and the newly installed geo-structures will lead to problems that did not exist in the past. It is a result of rapid urbanization. For example, Rezaei et al. (2013) reported that urban development and increasingly growth of population have been accompanied by a considerable growth in mechanized Shield tunnelling. For example, Mayoral et al. (2013) considered numerical analysis of the static behavior of the intersection of two major metro lines located in a soft lacustrine clay deposit overlaid by a very dense clayed sand deposit, in Mexico City. The intersection consists of a new tunnel excavated under an existing metro station-tunnel system, using the earth pressure balance, EPB, construction technique. This required the construction of a support structure for the station foundation. Actually, these kinds of problems exist in many developing cities in the world. For example, the MTR project of Shatin to Central link in Hong Kong also faced similar problems. Building new railway in developed and densely populated urban areas is a very challenging task. Everaars and Peters (2013) considered the case studies of a large infrastructural railway project through the historical city centre of Delft, The Netherlands and of an underground expansion project of the Drents Museum in Assen, The Netherlands.

2.6.3. Retaining walls

There are 4 papers considering the application of numerical methods to retaining walls. These are Sadrekarimi and Monfared (2013), Smith et al. (2013), Mirmoradi and Ehrlich (2013), and Bui et al. (2013). Retaining wall design is probably one of the oldest geotechnical problems that was dealt with rigor and mathematics. However, today it remains one of the most difficult problems in geomechanics.

2.6.4 Slopes

There are 5 papers investigating the deformation of slopes (both natural and cut slopes), including Yerro et al. (2013), Hoshina and Isobe (2013), Kamalzare et al. (2013), Bryson and El Naggar (2013), and Lu et al. (2013). Progressive failure of slopes remains an elusive problem as it is a highly nonlinear problem. The propagation of shear crack in slopes may play a crucial role in such process (Palmer and Rice, 1973). Unfortunately none of the submitted papers summarized in this report addresses such problem. Numerical simulation of such progressive failure will be highly sensitive to the constitutive models. If bifurcation type of instability of slope sliding occurs (Chau 1995, 1999), numerical modeling will be very difficult.

2.6.5 Levees

Three studies addressed the erosion and instability of levees. They are Fujisawa and Murakami (2013), Kamalzare et al. (2013), and Smith et al. (2013). This topic becomes extremely since the levee failure occurred at New Orleans during the attack of the South Asian Tsunami in 2004 and the Hurricane Katrina in 2005. Recently, tsunami disasters in Japan after 2011 Tohoku earthquake further reinforce the importance of this topic.

2.6.6 Breakwaters

Stickle et al. (2013) considered wave-induced nonlinear dynamic soil response in vertical breakwaters foundation using Biot's (1941) theory extended to include dynamic terms. Note that Biot's (1941) theory allows for the coupling between soil deformations and pore water pressure fluctuations.

2.7 *Experimental validation*

2.7.1 *Centrifuge tests*

Kamalzare et al. (2013) used the experimental results of a 150 g-ton geotechnical centrifuge to calibrate their models for modeling levee's erosion. Kwon et al. (2013) compared their results of dynamic analysis based on the finite difference method under seismic loading to the observations in dynamic centrifuge tests. Although centrifuge test can provide the required high stress level typically observed in the field, the size of particles in soil samples may also present an unwanted scale effect in soil deformation mechanism.

2.7.2 *1-g experiments*

As expected, 1-g experiments in laboratory have been the most commonly used tool in calibrating and validating numerical models. Various experiments (including direct shear box and triaxial test) have been reported in the following papers: Wardani et al. (2013), Stirling and Davie (2013), Ramon and Alonso (2013), Tanchaisawat et al. (2013), Kamalzare et al. (2013), Hashash et al. (2013), Sokolić and Szavits-Nossan (2013), Pinkert and Klar (2013), and James et al. (2013).

2.8 *Constitutive modeling*

The constitutive modeling of responses of soils and rocks were considered by Biru and Benz (2013), Biru et al. (2013), Dong and Anagnostou (2013), Ebrahimian and Noorzad (2013), Pereira et al. (2013), Siddiquee and Islam (2013), and Yao et al. (2013). To avoid singularity at the yield vertex of the Mohr-Coulomb failure surface, Dong and Anagnostou (2013) replaced the original Mohr-Coulomb yield surface by the Matsuoka-Nakai criterion. However, this leads to the issue of whether yield surface vertex is real. Rudnicki and Rice (1975) and Rudnicki (1984) argued based on the mechanics of sliding microcrack that yield vertex should exist in rocks. However, as summarized in Chau (2013) the existence of such yield surface vertex is inconclusive.

3 VALIDATION OF FEM MODELS

As remarked by Brinkgreve and Engin (2013), the use of the Finite Element Method for geotechnical analysis and design has become quite popular. It is often the younger generation of engineers who operate easy-to-use finite element programs and produce colourful results, whilst the responsible senior engineers find it difficult to validate the outcome. Brinkgreve and Engin (2013) presented the recent finding by the NAFEMS Geotechnical Committee of the Netherlands on the validation of geotechnical finite element analysis. This is an extremely important topic that has largely been ignored in the past. The NAFEMS Geotechnical Committee has concluded that there is a need for guidelines on validation of geotechnical finite element calculations. This section summarizes the essential findings reported in Brinkgreve and Engin (2013).

3.1 *Sources of discrepancies*

Brinkgreve and Engin (2013) classified the sources of discrepancies into 6 categories: Simplifications, Modelling errors, Constitutive modeling, Uncertainties, Software and hardware issues, Misinterpretation of results. Examples of these are:

3.1.1 *Examples of simplifications*

Simplifications are normally made on the geometries of the problem, on the selection of model boundaries, on material behaviour, and on the presumed construction process.

3.1.2 *Examples of modelling errors*

Modelling errors can include input errors, discretisation errors (meshing), boundary conditions, time integration, tolerances (tolerated numerical errors), and limitations in theories and methods (e.g. the use of small deformation theory for problems with large deflections).

3.1.3 *Typical issues related to constitutive modelling*

In real soils and rocks, their constitutive behaviors may be non-unique arising from non-associated plasticity and strain-softening, undrained and unsaturated behaviour.

3.1.4 *Examples of uncertainties*

Assumptions are normally made in numerical modeling because of the lack of soil data, the lack of spatial variation of soil properties, the lack of information of shaking information during an earthquake, and the lack of information of future developments around the project to be designed. The actual construction may also deviate from the original design.

3.1.5 *Specific software and hardware issues*

Users are of no control of the specific implementations made by the developers of the software. The software may depend on operating system and configuration of computer. 'Bugs' (programming flaws in the application software) may exist and they will only appear when certain constraints or limitations of the software are encountered. Issues may also come from the specific implementations of models (for example rounding-off the corners of the Mohr-Coulomb failure criterion), the iterative solvers and their numerical solution tolerances, and the parallel solvers (solution differences depending on the number of threads or cores being used).

3.1.6 *Examples of misinterpretation of results*

Some users of FEM may miss-use the safety factors, misinterpret structural behaviour (if the structure is too much simplified), overlook essential details (in particular complex 3D models), and possess insufficient knowledge and understanding of the modelling software being used.

3.2 *Methods of validation*

Brinkgreve and Engin (2013) gave the following examples of validating FEM models and methods: (i) analytical solutions of elasticity problems, plasticity problems, constitutive models, dynamic problems, bearing capacity solutions, solutions of flow and coupled problems, (ii) limit equilibrium solutions for global safety factors or bearing capacities, (iii) upper and lower bound solutions (limit analysis), and (iv) benchmarks. The following aspects of validation were discussed in full details by Brinkgreve and Engin (2013): (i) Validation of constitutive models and parameters, (ii) Validation of model boundaries, (iii) Validation of initial conditions, (iv) Validation of (the accuracy of) results, (v) Benchmarking, and (vi) Checklists.

This paper is a very useful reference and is a must read for serious practitioners of numerical analysis and young engineers.

4 CONCLUSION

In this brief report, we have summarized a total of 52 papers submitted to the area of TC103 "Numerical Methods". Instead of summarized each paper, we have provided an overall view of where these papers were from. A master table is given for all 52 papers in terms of the types of numerical methods empolyed by different authors together with the full references given in the end of the paper (paper number in alphabetic order). The numerical methods used include finite element method (FEM), finite difference method (FDM), material point method (MPM), smoothed particle hydrodynamics (SPH), neural network (NN), genetic algorithm (AG), and finite volume method (FVM). The failure models used in studies include Mohr-Coulomb failure criterion, Drucker-Prager plastic potential, Cam clay model, Matsuoka-Nakai failure model, and Hoek-Brown failure criterion. In terms applications, these numerical analyses have been applied to model piles, tunnels, retaining walls, slopes, levees, tailings impoundment, and breakwaters. Experiments used in calibrating these numerical models can be classified as centrifuge tests and 1-g laboratory tests. Finally, the interesting report on the vailidation of finite element method by Brinkgreve and Engin (2013) was summarized briefly.

In terms of future challenges, we expect more research work considering the coupling between FEM and FDM (this was also considered by Javadi et al., 2013). It is because FEM is good for far field linear response whereas FDM (as such FLAC) better suits the near field large deformation and failure. In terms of multi-physics problems, we expect future developments in numerical methods for coupling between thermal, electrical, fluid, solids and chemical problems (like petroleum or thermal energy extraction problems). Multi-physics problems are commonly encountered in petroleum mining and geothermal energy extraction problems. In fact, Yoneda (2013) was motivated by the extract of methane hydrate (a new source of energy fuel in Japan). In terms of multi-scale approach, we expect more refined model for coupling of microscopic behavior (such as grain rotations and sliding) to macroscopic behavior (such as plastic yielding) in soils and coupling of microscopic cracking process to macroscopic damage (at continuum scale) and material degradation in rocks (Borja, 2011). This is sometimes referred as micro-meso-macro modeling. For fluid-solid coupling problems, lattice Boltzman model can be useful in fluid flow simulation modeling. In rock blasting or fragmentation, computational simulations of dynamic fragmentation of rocks is another area of active research.

A typical example of multi-physics and multi-scale problems in geomechanics is the so-called carbon capture and storage (CCS). Carbon capture and storage (CCS) (or carbon capture and sequestration), is the process of capturing waste carbon dioxide (CO_2) from large point sources, such as fossil fuel power plants, transporting it to a storage site, and depositing it where it will not enter the atmosphere, normally an underground geological formation (like man-made rock caverns). The aim is to prevent the release of large quantities of CO_2 into the atmosphere (from fossil fuel use in power generation and other industries). Another related problem is nuclear power waste storage in underground rock layers. Coupling effect of the thermo-hydro-mechanical responses in rock becomes very important.

Another area that needs the use of accurate numerical modeling is related to geohazards and geo-disasters. These disasters include ground and basin amplification of earthquake shaking, slope failure and landslides, and tsunami and storm surge-induced failure of levees and breakwaters.

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