Constitutive model and simulation of non-segregation freezing and thawing in soils

Modèle de comportement et simulation du gel et le dégel des sols sans ségrégation

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ABSTRACT: An elasto-plastic constitutive model for non-segregation freezing and thawing soils is developed to capture the deformation behavior and strength evolution of the soil subjected to arbitrary loading as well as temperature changes. The model is based on the critical state framework, with the yield condition as in the modified Cam clay model. It uses the pore ice ratio as a scalar parameter to describe the evolution of the yield condition due to freezing and thawing. The model has been implemented in the finite element system ABAQUS. A thermal and mechanical process in a soil column was simulated to illustrate the response of the model.

RÉSUMÉ : On présente un modèle élasto-plastique de comportement pour les sols soumis à des cycles de gel-dégel, ségrégation exclue. Ce modèle représente la réponse en déformation et l'évolution de la résistance du sol sous n'importe quel chargement mécanique, avec variations de température. Il s'inscrit dans le cadre de la théorie de l'état critique, avec une fonction de charge du type Cam Clay modifié. Un paramètre scalaire appelé pore ice ratio est défini, qui régit l'évolution de la fonction de charge sous l'effet du gel et du dégel. Le modèle a été implémenté dans le code de calcul aux Eléments Finis ABAQUS. La réponse du modèle est illustrée par la simulation d'une colonne de sol soumise à un chargement thermique et mécanique.

KEYWORDS: non-segregation soil freezing, pore ice, soil thawing, constitutive model

1 INTRODUCTION

Various soils experiencing freezing and thawing may exhibit dramatically different behaviors. For a non-frost-susceptible soil, such as medium sand, no ice segregation will take place during freezing. The pore water and pore ice will co-exist while the temperature drops below freezing point. An increase in strength occurs as the soil freezes; weakening is expected during the melting process. The bulk volume of the soil expands and contracts little due to the phase change. The same may occur in frost susceptible soils subjected to quick freezing. In this paper, both of these conditions will be considered as non-segregation cases in which no ice lenses are generated during freezing.

Elasto-plastic models and models accounting for viscous properties of ice (creep), as well as ice melting, have been developed (Lai *et al.*, 2009, Wei *et al.*, 2011). Most of these models are based on continuum approach, and are suited for one type of the frozen soil. Changes of the soil components upon freezing and thawing, and the corresponding changes in strength, are not addressed by these models. Some effort was made to include the governing parameters of soil freezing into the constitutive model (Shastri and Sanchez, 2012), but its applicability is yet to be assessed.

This paper focuses on developing an elasto-plastic constitutive model including freezing and thawing, to capture the deformation behavior and strength evolution of the soil subjected to loading and temperature changes. The model is based on the critical state framework and it is formulated by introducing the influence of ice ratio into the modified Cam clay model. The model developed is suited for non-segregation freezing and thawing soils (no ice lens formation). In non-segregation soils, the strength of the soil upon thawing will return to the strength prior to freezing, i.e, if the soil only experiences freeze-thaw cycle, but no change in loading, no thaw-settlement will take place. This is because the soil volume variations during freezing/thawing are only due to the phase change of pore water. Creep effect of frozen soil is not considered in this paper.

2 THE MODEL

The model developed is temperature-dependent, with the *ice ratio* e_i being the key parameter that describes the influence of the ice content. The ice ratio is defined as

$$e_i = \frac{V_i}{V_s} \tag{1}$$

where V_i is the volume of ice and V_s is the volume of the solid constituent (skeleton). e_i is related to the unfrozen water content in the frozen soil.

2.1 *Constitutive model*

Compression tests on frozen soils indicate that the behavior of frozen soil in the v,p-plane (specific volume, isotropic stress) can be represented by normal compression line (NCL) and unloading-reloading line (URL) (Qi *et al.*, 2010, Lee *et al.*, 2002). The slopes of these lines vary, however, depending on the extent of freezing. The slopes for the two lines are defined by λ and κ for unfrozen soil, and λ_f and κ_f for frozen soil, respectively. The specific volume upon isotropic compression of frozen soil is given by

$$v = v_f - \lambda_f \ln p \tag{2}$$

and elastic behavior in unloading-reloading regime is given by

$$\delta v^e = -\kappa_f \frac{\delta p}{p} \tag{3}$$

Both λ_f and κ_f are functions of ice ratio e_i. Soils become stronger upon freezing, which is characterized by higher preconsolidation stress p_o, and the slopes NCL and URL lines become flatter (Qi *et al.*, 2010). A reasonable relative position of normal compression lines for a soil in both unfrozen state and frozen state is shown in Figure 1(a). The relationship between preconsolidation stress and ice ratio is illustrated in Figure 1(b). The yield surface has elliptical shape in *q,p*-plane (deviatoric, isotropic stress), and its evolution, as function of e_i, is illustrated in Figure 1(b). p_o is the preconsolidation stress for unfrozen soil, and p_0^f is the preconsolidation stress for the same soil in frozen state with the ice ratio of $e_i = e_{iC}$ (p_0^r is the reference stress).

Consider freezing and loading path B-C-D as illustrated in Figure 1a. A virgin compressed unfrozen soil (saturated) at point B is subjected to freezing under a constant isotropic stress state. The specific volume increases due to the volumetric expansion upon phase change. Then, isotropic load is added at constant temperature until point D is reached on the isotropic stress yield line for the frozen soil. During the isotropic loading process the ice ratio remains constant, $e_i = e_{iC}$. This is because the ice ratio follows the unfrozen water content curve in non-segregation freezing process, thus having a unique relationship to the temperature. The unfrozen water content will be discussed in the next section.

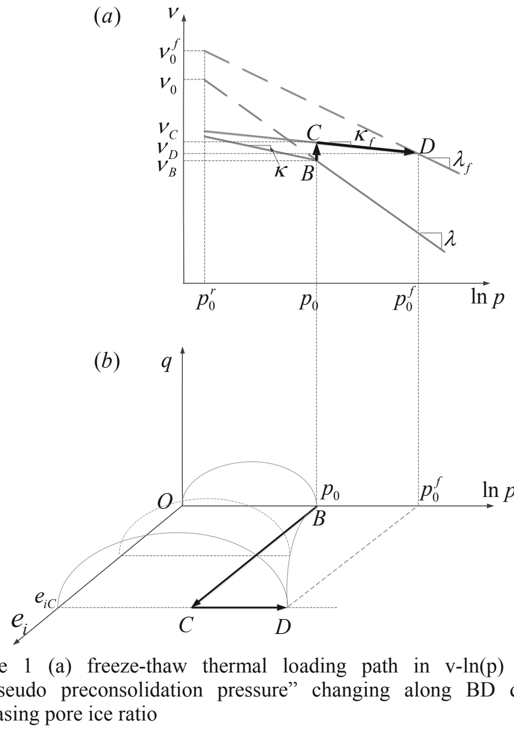

Figure 1 (a) freeze-thaw thermal loading path in v-ln(p) plane; (b)"pseudo preconsolidation pressure" changing along BD due to increasing pore ice ratio

Points B and D belong to the same yield curve on p, e_i- plane, and yield stresses p_0^f for frozen states with different ice ratio e_i during freezing from point B to C are located on curve BD (Figure 1(b)).

The specific volume at the final point D can be described as

$$v_D = v_B + \Delta v_{BC} - \Delta v_{CD} \tag{4}$$

where Δv_{BC} and Δv_{CD} are the increments due to initial freezing and due to subsequent loading.

The expansion upon freezing is related to phase change and is treated as reversible, because it reversed during thawing. This volume change can then be calculated as

$$\Delta v_f = 0.09\, e_i \tag{5}$$

Substituting equations (2), (3), and (5) into (4) renders the following equation

$$v_0^f - \lambda_f \ln\frac{p_0^f}{p_0^r} = (v_0 - \lambda \ln\frac{p_0}{p_0^r}) + 0.09\, e_i - \kappa_f \ln\frac{p_0^f}{p_0} \tag{6}$$

where p_0^r is the reference pressure to locate the virgin compression lines for both soil states. Consequently, the increase in isotropic yield stress caused by the ice ratio increase is found as

$$\frac{p_0^f}{p_0^r} = e^{\frac{v_0^f - v_0 - 0.09\, e_i}{\lambda_f - \kappa_f}} \left(\frac{p_0}{p_0^r}\right)^{\frac{\lambda - \kappa_f}{\lambda_f - \kappa_f}} \tag{7}$$

The slopes of the NCL and URL of frozen soil are both functions of e_i. This path of derivation was followed earlier by Alonso *et al*. (1990) in the context of unsaturated soils. Because the frozen soil has limited tendency for compression (pores are filled with ice), slopes λ_f and κ_f drop dramatically compared to those for the unfrozen soil. The following relationships are postulated

$$\lambda_f = \lambda \exp(-\alpha_1 e_i) \tag{8}$$

$$\kappa_f = \kappa \exp(-\alpha_2 e_i) \tag{9}$$

where α_1 and α_2 are soil constants.

In order for equation (7) to yield p_0 for unfrozen soil when $e_i = 0$, we postulate the following linear law defining the shift of v_0 to v_f as function of e_i

$$v_0^f = v_0 - (\beta - 0.09)e_i \tag{10}$$

where β is a soil constant. Equation (7) can, therefore, be written as

$$\frac{p_0^f}{p_0^r} = e^{\frac{-\beta e_i}{\lambda_f - \kappa_f}} \left(\frac{p_0}{p_0^r}\right)^{\frac{\lambda - \kappa_f}{\lambda_f - \kappa_f}} \tag{11}$$

An illustration of how the preconsolidation stress p_0^f changes in soil with progressively increasing ice ratio e_i is shown in Figure 2, for three cases, each starting from a different preconsolidation stress p_0 for unfrozen soil (the following parameters were used: $\alpha_1 = \alpha_2 = 0.2$, $\beta = 0$, $\lambda = 0.35$, $\kappa = 0.07$)

The plastic volumetric strain due to mechanical load and thermal process can be calculated as

$$d\varepsilon_v^{pl} = \frac{\lambda_f - \kappa_f}{v}\frac{dp_0^f}{p_0^f} = \frac{\lambda - \kappa_f}{v}\frac{dp_0}{p_0} \tag{12}$$

The yield function in a *p,q*-plane is

$$f = q^2 - M^2 p(p_0 - p) = 0 \tag{13}$$

An associate flow rule is assumed, i.e., and the plastic potential $g = f = 0$.

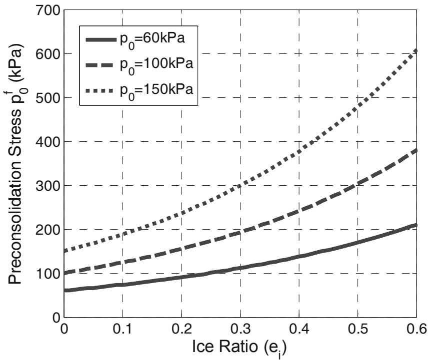

Figure 2 Yield stress for frozen soil with different ice ratio e_i

Thawing of a frozen soil will decrease the amount of ice in its pores, reducing "ice cementation" and consequently, reducing strength. For non-segregation frozen soils, the strength after complete thawing tends to move back to the same envelope as before freezing. There will be no further weakening since no thermally induced mass migration occurred during freezing, and the change of the soil fabric during the thermal process is not significant.

2.2 *Unfrozen water content*

The unfrozen water curve can be used to identify the amount of pore ice formed during freezing. In most of the soils other than coarse granular ones, not all water freezes at the freezing point. A 3-parameter function (Michalowski and Zhu, 2006) was developed to describe the unfrozen water content for most soils (see Figure 3). This function has the following form:

$$w = w^* + (\overline{w} - w^*)\, e^{a(T-T_0)} \tag{14}$$

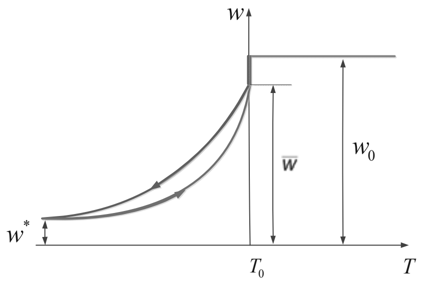

Figure 3 Unfrozen water content curve

At freezing point T_0, a portion of the water freezes and the rest remains in the liquid state ($\overline{w}$). The liquid moisture content reduces with the decrease in the temperature to reach w^* at some low reference temperature. Parameter a describes this reduction rate.

The ice ratio e_i can be calculated from the unfrozen water content as

$$e_i = (w_0 - w) \cdot \frac{\rho_s}{\rho_w} 1.09 \tag{15}$$

3 THERMAL-MECHANICAL LOADING PROCESS

To better illustrate the model, a freeze-thaw cycle along with an external load applied is shown in Figure 4. The thermal process is depicted with the blue (freezing) and red (thawing) lines, whereas the loading-unloading is depicted by the green lines.

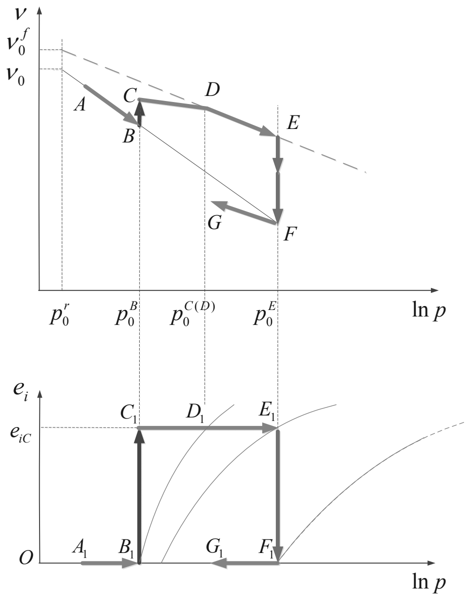

Figure 4 Freeze-thaw cycle and load path with corresponding yield curves

A saturated soil specimen is preconsolidated under isotropic compression from point A to B. Then, under constant stress, the freezing process takes place (B-C), and the ice ratio at point C is e_{iC}. In this process, the isotropic yield stress (apparent preconsolidation stress for corresponding frozen soil) increases from B_1 to D_1 following equation (7), and it has a value of p_0^D at point D_1 (the soil has the same apparent preconsolidation stress after it had been frozen at point C_1). The below-freezing temperature is then maintained, and isotropic compression is increased to reach the normal compression line for frozen soil at pressure p_0^D; the load is then continued along the NCL to reach p_0^E. During this loading, the void ratio e is changing while the ice ratio remains constant. This is based on an assumption that ice and soil skeleton are both incompressible. From C to D, the frozen soil experiences elastic behavior, whereas from D to E it behaves plastically.

Thereafter, the soil is thawed at constant stress p_0^E. Once ice starts melting, the soil can no longer sustain load $p = p_0^E$, and the process of consolidation will start, moving the soil to the normal compression line for the unfrozen soil (point F). Unloading from F results in an elastic rebound along the URL for unfrozen soil to G.

4 APPLICATION AND FINAL REMARKS

The constitutive model was implemented into the FE system ABAQUS using subroutine UMAT and UEXPAN to solve boundary value problems. A soil column subjected to both the mechanical load and a thermal process was simulated.

The parameters and initial values used in the simulation are listed in Table 1. λ and κ are the slopes for NCL and URL in compression plane. M is the slope of the critical state line in *p-q* space. p_0 and e_0 are the initial values for the pre-consolidation pressure and the void ratio. w*, $\overline{w}$, and a are parameters for unfrozen water content function; this function was calibrated with the data for Fairbanks silt, Figure 5. The curve fitting function is represented in Figure 5 by the continuous curve (red). The unfrozen water curve is quite steep, and it is assumed that the functions for freezing and thawing process are the same.

Table 1. Parameters and initial values in simulation

λ	κ	Initial p_0	M	α_1	α_2
0.35	0.1	60 (kPa)	1.0	0.2	0.2

e_0	w*	$\overline{w} = w_0$	a	μ	β
0.85	0.08	0.325	6.0	0.3	0

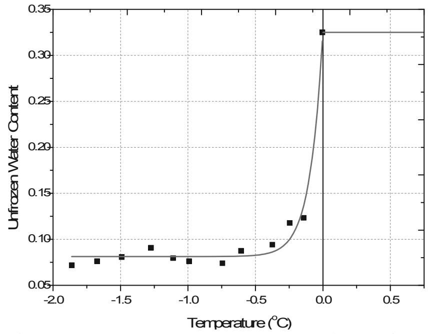

Figure 5 Unfrozen water content curve (experimental points from Huang *et al*., 2004))

The soil column is 1 m tall and 0.05 m in width. The walls of the column are adiabatic and rigid. The initial uniform temperature is 1°C, and the initial vertical and horizontal compressive stresses are 20 kPa and 10 kPa, respectively.

Initial and boundary conditions in terms of load and temperature at the top of the column are shown in Figure 6. The bottom of the column is fixed and the temperature is maintained at 1°C throughout the process.

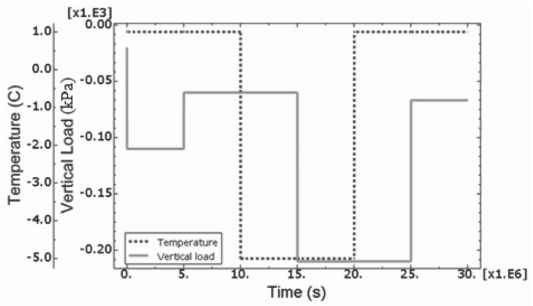

Figure 6 Boundary condition on top of the soil column

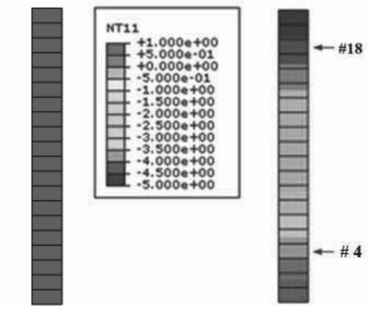

Figure 7 Temperature distribution at t=0 and t=1.3e7s

The relationship between void ratio and the vertical stress is shown in Figure 8 for element #18 and #4. For element #18, the temperature is about -4.5°C when the column reaches steady state after freezing, having more pore ice and being stronger than element #4 whose steady state temperature is around -0.3°C. There is a substantial difference in the behavior of the two elements during the loading segment from 60 to 210 kPa: while element #18 behaves elastically, element #4 is elastic only until the load reaches 160 kPa, and becomes elasto-plastic afterward. Both elements experience additional settlement due to thawing.

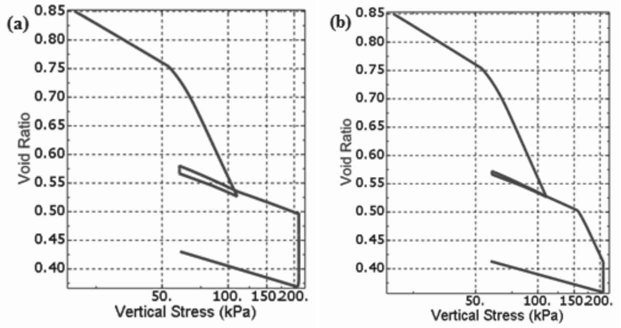

Figure 8 Compression for element (a) #18 and (b) #4

The model using the pore ice ratio as model parameter captures freezing and thawing process for non-segregation soils well. The parameter is related to the unfrozen water content curve and is easy to obtain from tests. This constitutive model is convenient to use, and it has been implemented in the FE system. The model will be calibrated based on the test data available, and it will be applied to solve practical boundary value problems.

The model can be used as a tool to predict the behavior of soil subjected to freezing and thawing as long as no ice lenses are formed. Such problems include, for instance, construction using artificial ground freezing. The model will be extended to include ice lens formation in frost-susceptible soils.

5 ACKNOWLEDGEMENTS

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