Fabric and critical state of granular materials

La structure et l'état critique des matériaux granulaires

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ABSTRACT: Critical state of particulate materials traditionally refers to a state where the material undergoes continued distortion at constant volume and constant stresses. By saying so the internal state of the material at the critical state is described solely by an isotropic (scalar-valued) parameter – the void ratio. Advances in modern laboratory tests have initiated the discussion of the effect of fabric on critical state and thus its uniqueness. More recently Li and Dafalias (2012) have shed light on the uniqueness of critical state from a thermodynamics perspective. This study uses the discrete element approach to investigate the fabric evolution of idealized two-dimensional assemblages having different initial fabrics subject to numerical biaxial shearing. The current paper focuses on the orientation of particles and void spaces at very large strains. It is shown that a unique fabric of particle orientation and void space is achieved at very large strains where the granular assemblage distorts continuously at constant density and stresses.

RÉSUMÉ : L'état critique des matériaux particulaires réfère normalement à un état où le matériau est soumis à un cisaillement continu sous volume et contraintes constants. Cela signifie que l'état interne des matériaux a l'état critique est décrit seulement par un paramètre (scalaire) isotrope – l'indice des vides. Avec les avancées en essais en laboratoire modernes a commencé la discussion de l'effet de la structure sur l'état critique et son unicité. Li & Dafalias (2012) ont étudié récemment l'unicité de l'état critique par la thermodynamique. Cette étude utilise une approche aux éléments discrets pour examiner l'évolution de la structure d'assemblées idéalisées en deux dimensions ayant des structures initiales différentes et soumises à un cisaillement bi-axial numérique. L'objet de cet article est l'orientation des particules et espaces des vides aux grandes déformations. On montre qu'aux grandes déformations une structure unique faite de l'orientation des particules et des vides est obtenue quand l'assemblée granulaire est cisaillée de façon continue sous densité et contraintes constantes.

KEYWORDS: anisotropy; critical state; discrete element analysis, fabric; microstructure.

1 INTRODUCTION

The concept of critical state is important to soil mechanics. It defines the existence of a unique state where the particulate material exhibits constant volume shearing at constant effective stresses under continuous distortion. The state is often obtained phenomenlogically. Over decades many constitutive models are formulated in accordance with this concept since the pioneering work by Roscoe and Schofield (1963) and Roscoe and Burland (1968). On the one hand, those models successfully capture the key mechanical behavior of many geomaterials subject to compression and shear. On the other hand, with the advances in modern laboratory testing techniques the influence of initial fabric on the material's stress-strain-strength responses have received much attention and the uniqueness of critical state has been great challenged (Vaid et al. 1985, Negussey and Islam 1994, Mooney et al. 1998, Finno and Rechenmacher 2003, etc). Herein fabric is a collective term to describe the geometric arrangement of grains and the associated voids, and the distribution of inter-particle contact forces. Material anisotropy has often believed to be the prime reason for the observation of a non-unique critical state. The critical state is often represented by two projection lines (critical state line, CSL) in the *q-p'* and *e-p'* (or *v-p'*) plane, in which $q=\sqrt{3\mathbf{s}:\mathbf{s}/2}$, $p'=\mathrm{tr}(\boldsymbol{\sigma}')/3$ where $\mathbf{s}$ is the deviatoric part of the effective stress tensor $\boldsymbol{\sigma}'$; e is the void ratio and $v=1+e$ is the specific volume. Internal state of the soil at the critical state is solely described by a scalar quantity of density (e or v) which implicitly shows that any anisotropic information of the material cannot be properly addressed. However, one may doubt why the material state remains (or becomes) isotropic at the critical state where the imposed stress is anisotropic ($q/p'=M$ where M is the critical stress ratio). Besides, while density and stress are uniquely related, should there be a unique particulate fabric at the critical state?

More recently, Li and Dafalias (2012) revisited the critical state concept and proposed an anisotropic critical state theory (ACST) by considering the role of the particulate fabric. From a thermodynamics perspective and based on the Gibbs stability requirement, uniqueness of the critical state line (CSL) has been proved. Furthermore, they also concluded that a unique fabric as a function of the loading direction must exist.

This paper investigates the evolution of the fabric of a two-dimensional idealized granular assemblage subject to numerical biaxial shearing. It aims to shed light on ACST proposed by Li and Dafalias (2012) from a numerical perspective using the discrete element approach.

2 NUMERICAL SPECIMEN AND TEST

Two-dimensional mono-sized non-crushable pill shape rigid particles with length-to-width ratio of 1.5 (width = 1 mm) are generated with the built-in clump function in PFC2D (Itasca 2008). The linear contact model between particles is adopted. The particles are then rained into a model container under gravity fields of different directions (see Figure 1). By doing so, assemblages formed by particles having different average preferential orientations could be expected. Generally speaking, by adopting this approach particles will have their long axis roughly perpendicular to the gravity field direction (Yan and Zhang 2013). Another approach is used to create assemblies of particles having random orientations. In this case, particles with random orientations are generated inside the model container and numerical iterations are allowed to achieve an equilibrium of the assemblage.

A square assemblage having 80 mm in each side is then extracted from the model container as shown schematically in Figure 1. An isotropic confining pressure is applied to the boundary walls of the specimen right after the body force field is turned off. Isotropic pressure of three different magnitudes is modeled: 50, 100 and 200 kPa. Attempts are made by controlling the inter-particle friction coefficient and selecting appropriate location of the square such that specimens confined at the same pressure show essentially identical void ratios but different initial fabrics prior to shear. The difference in void ratio in all cases is less than 0.0006.

A specimen is labeled by its initial stress, details of specimen generation method and void ratio prior to shearing. For instance, I100D90_219 denotes a specimen confined at 100 kPa, formed by depositing particles from a direction perpendicular to (i.e., 90°) to the major principal stress direction and had an initial void ratio of 0.219 before shearing. By the same token, I50Ran_219 represents a specimen confined at 50 kPa, having an initial void ratio of 0.219 with particles generated with random orientations.

To simulate a constant lateral stress biaxial compression, the top and bottom walls are moved simultaneously and slowly inwards while the horizontal position of the left and right lateral walls is continuously adjusted according to a servo-controlled mechanism to keep the lateral confining pressure at the initial value. It is to ensure that a quasi-static equilibrium is maintained throughout the analysis. A compression strain rate of 2.5×10^{-5}% per computational step is used and all the specimens are sheared to 80% axial strain.

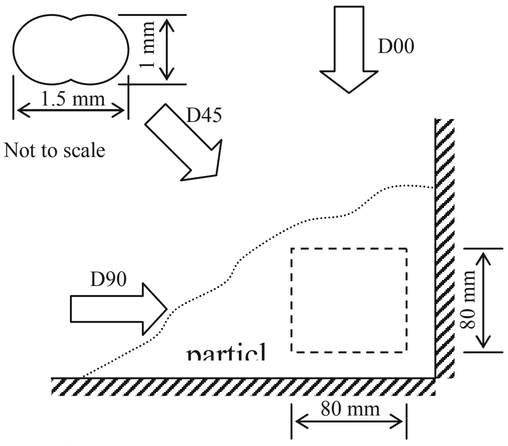

Figure 1. Specimen generation.

3 PARTICULATE FABRIC

Following the pioneer work presented by Oda et al. (1985), three major sources of fabric are considered in the study. They are (i) particle orientation, (ii) contact normal; and (iii) void distribution which are denoted by superscripts p, fn, and v respectively. Directional information of each fabric parameter is represented statistically by a rose diagram (Yan and Lin 2013). Two scalar quantities, a and θ_a are used to describe the distribution density $f(\mathbf{n})$ of the fabric (Rothenburg 1981, see Equation (1)).

$$f(\mathbf{n}) \approx \left[1 + a\cos 2(\theta - \theta_a)\right] / 2\pi \tag{1}$$

where $\mathbf{n}$ is a unit vector, a is called the coefficient of anisotropy that characterizes the degree of anisotropy and $0° \le \theta_a < 180°$ describes the preferred direction of the anisotropy given that $\oint f(\mathbf{n})\mathrm{d}\mathbf{n} = 1$. Clearly $a = 0$ indicates an isotropic distribution and the degree of anisotropy increases with a. In this study, $\theta_a = 0°$ denotes a direction perpendicular to the major principal stress. Details of the fabric description could be found in Yan and Zhang (2013).

4 RESULTS

4.1 *Influence of initial fabric*

This paper focuses on the evolution of fabric, in particular the particle orientation and void space, during biaxial shearing. The stress-strain-strength behavior of the assemblages is not shown due to the page limitation. It is found that the stresses and volumetric strain become essentially steady beyond 60% axial strain. Figure 2 and 3 show the evolution of the fabric of specimens having identical initial density but different initial fabric. As shown in Figure 1, specimens having different initial particle orientations have been created by different methods of generation. Specimens generated by the particle deposition method generally show an average particle orientation highly correlated to the deposition direction. For instance specimen I100D90_219 shows $\theta_a^p \approx 70°$ before shear, which indicates that the particles tend to align with its long axis perpendicular to the deposited direction (D90). Specimens generated by the deposition method exhibit a higher initial degree of anisotropy as compared to the one with particles randomly generated. Upon shearing, particles gradually rearrange in a way that their long axis becomes perpendicular to the principal stress direction (i.e., the shearing direction). Furthermore, a unique fabric of very similar pair of a^p and θ_a^p is shown at large strains ($a^p \approx 0.55$ and $\theta_a^p \approx 0°$).

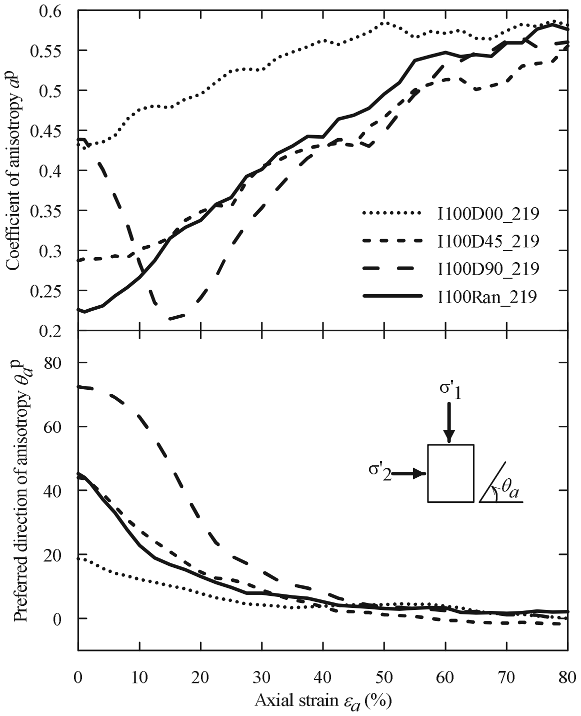

Figure 2. Evolution of particle orientation fabric – the influence of initial fabric.

A void space is defined following Li and Li (2009). Figure 3 shows its evolution with shear. It can be seen from the figure that the void space evolutes from a highly isotropic distribution prior to shear commencement to an anisotropic one at large strains with a preferential orientation in the direction of the major principal stress ($\theta_a^{\mathrm{v}} \approx 90^\circ$). Substantial amount of anisotropy is developed within 10% axial strain. Initial fabric seems to have very little influence on the evolution of void fabric. Like particle orientation, a unique void fabric is exhibited at large strain ($a^{\mathrm{v}} \approx 0.3$ and $\theta_a^{\mathrm{v}} \approx 90^\circ$).

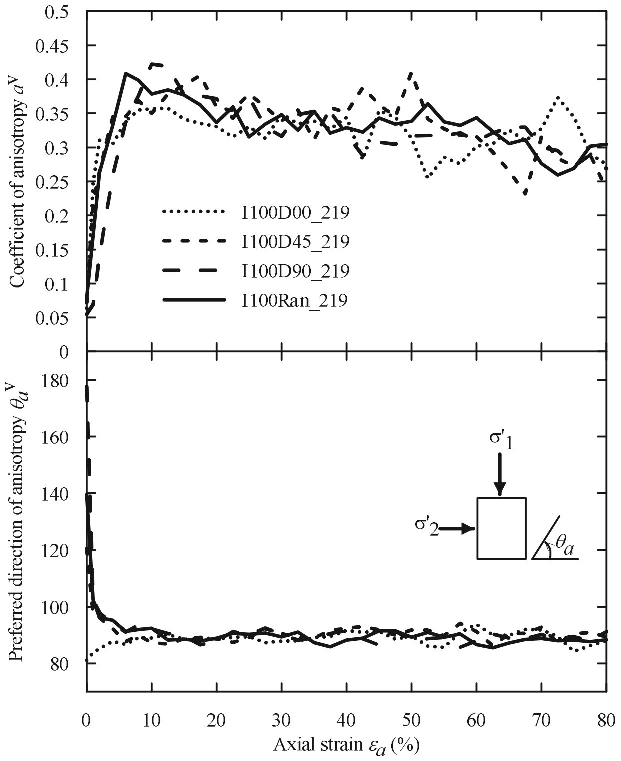

Figure 3. Evolution of void space fabric – the influence of initial fabric.

4.2 *Influence of initial density*

Figure 4 shows the effect of initial density on the evolution of particle orientation fabric. Deposited assemblages (D45) confined at 100 kPa having three different initial densities (initial void ratio from loose to dense = 0.237, 0.219 and 0.212, respectively) are investigated. Firstly, the particle orientation fabric changes gradually with increasing axial strain. The coefficient of anisotropy a^{p} increases gradually with axial strain while θ^{p} changes from its initial value (45°) to 0°. It means that the particles align with their long axis perpendicular to the loading direction when subject to prolonged shearing. A noticeably anisotropic and unique fabric can be observed at large strains. The effect of initial density is minimal. The evolution of void space fabric exhibits very similar behavior as the one shown in Figure 3, regardless the initial density of the assemblage.

4.3 *Critical state*

Figure 5 shows the initial and critical state of the assemblages in an *e*-*s*' space where $s' = (\sigma'_1 + \sigma'_2)/2$. Initial state of the specimen includes various void ratios and fabrics confined at different pressures. It can be seen that a unique line can be used to describe the critical state. Together with the findings as revealed from Figure 2-4, it is known that a critical state can be achieved at very large strains where not only the density and mean stress reaches a steady value but also a unique fabric is obtained. Furthermore, the fabric at critical state is correlated to the loading direction. The results echo the theory proposed by Li and Dafalias (2012).

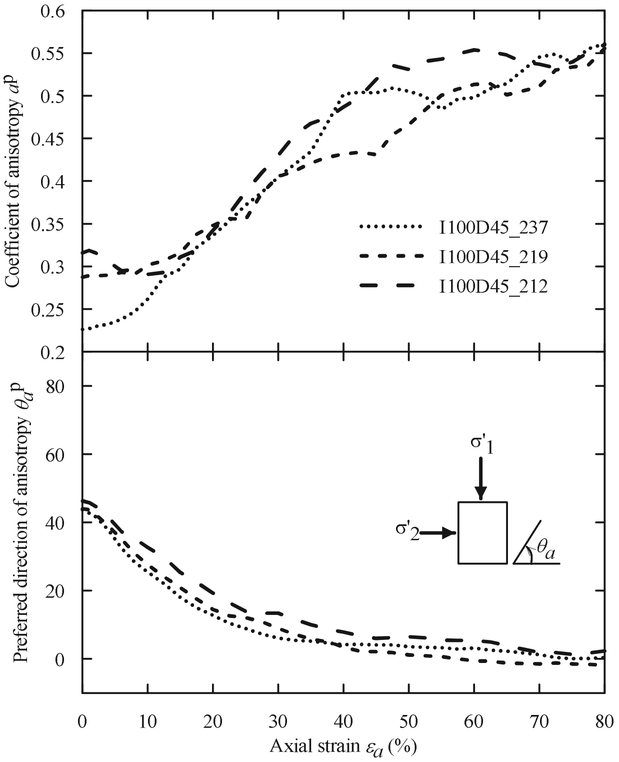

Figure 4. Evolution of particle orientation fabric – the influence of initial density.

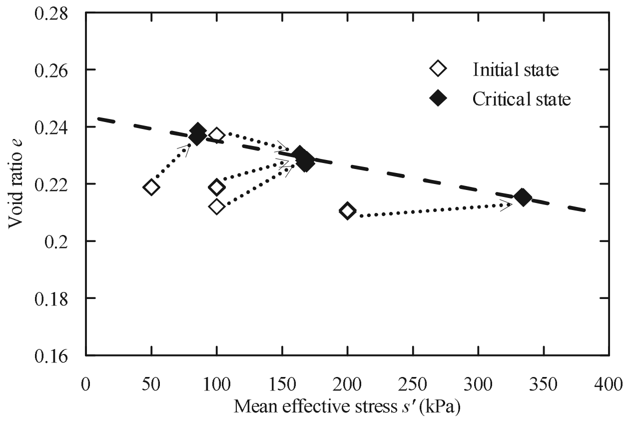

Figure 5. Initial and critical states of the study assemblages.

4.4 *Limitations*

The findings of unique fabric at critical state are obtained by conducting numerical tests limited to monotonic biaxial shearing of specimens composed of idealized mono-sized pill shape particles. Tests with continued changing of loading direction are not considered herein.

5 CONCLUSIONS

A series of numerical biaxial compression tests is undertaken on idealized two-dimensional granular specimen having various initial particulate fabrics in terms of particle and void space spatial arrangement to investigate the uniqueness of the critical state. It is found that a unique fabric is obtained at large strains where the stresses and volume of the assemblages are essentially constant. Fabric at critical state is correlated to the loading direction. This study is only limited to the case whereas the loading direction remains the same.

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