An undrained upper bound solution for the face stability of tunnels reinforced by micropiles

Une solution en limite supérieure non drainée pour la stabilité du front de tunnels renforcés par micropieux

Pinyol N.M.
Centre Internacional de Mètodes Numèrics en Enginyeria (CIMNE).
Department of Geotechnical Engineering and Geo-Sciences. Universitat Politècnica de Catalunya (UPC)

Alonso E.E.
Department of Geotechnical Engineering and Geo-Sciences. Universitat Politècnica de Catalunya (UPC)

ABSTRACT: Tunnel in difficult soils may require procedures to prevent tunnel face failures. Face stabilization can be achieved by the installation of some structural elements. This paper presents an analysis of face stability of shallow tunnels in undrained soils reinforced by an umbrella of subhorizontal micropiles. Upper bound solutions for two dimensional plane strain conditions are given including the effect of micropiles. The micropile umbrella is embedded in the soil and supported on the tunnel lining. The kinematically admissible collapse mechanism defined to calculate the upper bound solution includes the action provided by a subhorizontal micropile at limiting conditions. The solutions are given in practical dimensionless charts which are useful to quantify easily the effect of the umbrella of micropiles. The plots provide a simple procedure to design the umbrella. The most relevant properties defining the umbrella are grouped into a single dimensionless coefficient which includes the yielding conditions and the geometry of the micropiles as well as the distance between them.

RÉSUMÉ : Les tunnels dans les sols difficiles peuvent nécessiter des procédés pour prévenir les ruptures du front du tunnel. La stabilisation du front peut être réalisée par l'installation de certains éléments structurels. Cet article présente une analyse de la stabilité du front dans des tunnels peu profonds en conditions non drainées renforcés par un parapluie de micropieux subhorizontaux. Des solutions de la limite supérieure pour des conditions bidimensionnelles de déformation plane sont présentées, y compris l'effet des micropieux. Le parapluie de micropieux est intégré dans le sol et soutenu sur le revêtement du tunnel. Le mécanisme de rupture cinématiquement admissible défini pour calculer la solution de la limite supérieure comprend la réponse prévue par un micropieu subhorizontal dans des conditions limites. Les solutions sont données dans des graphiques pratiques et sans dimensions qui fournissent une procédure simple de concevoir le parapluie. Les propriétés les plus pertinents qui définissent le parapluie sont regroupées en un seul coefficient sans dimension qui inclut les conditions de plastification et de la géométrie des micropieux, ainsi que la distance qui les sépare.

KEYWORDS: Tunnel, face stability, micropiles, upper bound, plasticity, undrained strength.

1 INTRODUCTION

Tunnel in difficult soils may require procedures to prevent tunnel face failures. In tunnel excavated by means of boring machines, a pressure can be applied against the face to counteract water and earth pressure. Several publications provide procedures to calculate the pressure required for stability. Well known solutions given initially by Davis et al. (1980) offer practical dimensionless charts for shallow tunnels in cohesive materials based on plasticity theorems (upper and lower bound solutions). This contribution was followed by several authors that presented similar solutions for frictional materials (Leca and Dormieux 1990) or improved solutions by using limit equilibrium, finite difference and finite element methods (Lyamin and Sloan 2002a,b, Augarde et al. 2003, Vermeer et al. 2002, Klar et al. 2007, among others).

Another calculation approach is to use Limit Equilibrium techniques (Anagnostou and Kovari 1996). They provide their results in terms of "bearing capacity" expressions. Finite Element and Distinct Element methods have been used extensively to examine face stability, in most cases under three dimensional conditions (Vermeer *et al*. 2002, Galli *et al*. 2003, Melis and Medina 2005). Among them, Vermeer *et al.* (2002) determined failure conditions of the face by means of a "c, ϕ reduction method" and provided three dimensional solutions for the drained case.

Face stabilization can also be achieved by the installation of some structural elements (bolts distributed in the front, concrete prevaults and umbrellas of micropiles). Several analysis of tunnel face stability taking into account the effect of a prevault and a reinforcement by bolts have been published (Peila et al. 1996, Wong et al. 2000, Yoo and Shin 2003, Lignola et al. 2008, 2010). However, limited attention has been paid to the reinforcement of tunnel faces by micropiles.

This paper presents a stability analysis of tunnel faces including an umbrella of sub-horizontal micropiles. The micropiles are considered as beams subjected to the kinematic motion imposed by the assumed failure mechanism. The limiting resistance of the supporting beams is first addressed. The failure mechanism imposes a displacement pattern on the beam, which reacts applying a critical combination of normal and shear forces on the boundary of the sliding body. These limiting supporting forces are calculated by assuming a Von Mises yield criterion for the micropile material. Then, they are introduced into the general minimization process associated with the upper bound formulation. Stability conditions are described in terms of dimensionless parameters and plotted in ready to use design charts. In particular, a dimensionless Micropile Coefficient, which includes all the relevant design parameters of the umbrella, could be isolated and plotted in terms of undrained soil strength and tunnel geometry.

2 UPPER BOUND SOLUTION INCLUDING SUBHORIZONTAL MICROPILES

2.1 *Collapse mechanism*

Consider the plane strain shallow circular tunnel of diameter D, having a cover depth C, represented in Figure 1. The soil around the tunnel is characterized by its unit weight (γ) and its undrained strength (c_u). A vertical stress, σ_S, is applied on the soil surface. In order to prevent a potential failure of the front, a pressure σ_T is applied on the tunnel face. In addition, a micropile supported on the already-built tunnel support is embedded in the soil. The micropile inclination with respect to a horizontal is defined by means of an angle η.

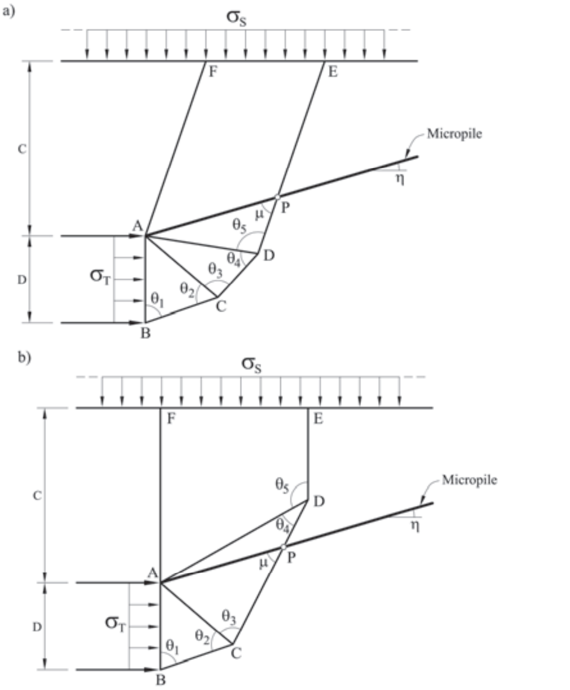

Figure 1. Collapse mechanism for upper bound calculation. (a) Micropile crossing DE side (upper wedge); (b) Micropile crossing CD side (lower wedge).

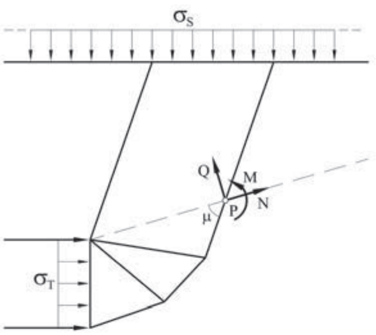

Figure 2. Micropile action on failure mechanism for upper bound calculation.

Under these conditions an upper bound solution for tunnel face failure is analyzed by means of a kinematically admissible collapse mechanism, defined by means of five degrees of freedom (the five angles θ_1, θ_2, θ_3, θ_4 and θ_5 in Fig. 1). Two possible collapse mechanisms are considered regarding the relative position between the micropile and the resulting collapse mechanism. In Figure 1a the micropile crossed the upper wedge on the DE side. This condition can be expressed by the following restriction: $3\pi/2-\theta_1-\theta_2-\theta_3-\theta_4-\eta<0$. Otherwise, the micropile will cross the side CD (Fig. 1b).

In the mechanism described, the micropile will react against the expected displacement imposed by the soil wedge. This effect will be included in the upper bound solution adding the work performed by the external forces transmitted by the micropile on the mobilized wedge. At point P (Fig. 2) he micropile action on the wedge will be characterized by a normal force *N*, a shear force *Q* and a bending moment, *M*. Notice that only *N* and *Q* will contribute directly to increase safety because the moment developed at point P will not produce any external stabilizing work.

2.2 Upper bound theorem

The upper bound theorem of plasticity is applied to the kinematically admissible failure mechanism shown in Figure 1. External work per unit of length performed by the external forces (weight, σ_S, σ_T and shear and tensile forces applied by the micropiles) due to a relative virtual rate of displacement is made equal to the internal dissipation of shearing work. The resulting equation is:

$$\begin{aligned}
&\frac{(\sigma_s-\sigma_T)}{c_u}-\frac{2C}{D}\frac{\sin\theta_2\sin\theta_4}{\sin\theta_1\sin\theta_3\sin\theta_5\cos(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5)}-\\
&\frac{1}{\tan\theta_1}-\frac{2}{\tan\theta_2}-\frac{2}{\tan\theta_3}-\frac{2}{\tan\theta_4}-\frac{1}{\tan\theta_5}- \qquad (1)\\
&\frac{\cos(\theta_1+\theta_2+\theta_3+\theta_4)}{\sin\theta_5\cos(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5)}+\frac{\gamma D}{c_u}\left(\frac{C}{D}+\frac{1}{2}\right)-\\
&\frac{\sin\theta_2\sin\theta_4}{\sin\theta_1\sin\theta_3\sin\theta_5 D c_u s}\left[N\cos\mu+Q\sin\mu\right]\Gamma=0
\end{aligned}$$

where *s* is the distance between two micropiles (assumed to be parallel) of the umbrella; μ is the relative angle between the micropile and the upper sliding wedge (Fig. 1) defined as:

$$\mu=5\pi/2-\theta_1-\theta_2-\theta_3-\theta_4-\theta_5-\eta \qquad (2a)$$

when the micropile crosses DE; and

$$\mu=3\pi/2-\theta_1-\theta_2-\theta_3-\theta_4-\theta_5-\eta \qquad (2b)$$

when the micropile crosses CD.

The parameter *A* in Eq. (1) is an auxiliary coefficient which also depends on the relative position between the micropile and the collapse mechanism:

$\Gamma=1$ when the micropile crosses DE (3a)

$\Gamma=\dfrac{\sin\theta_5}{\sin\theta_4}$ when the micropile crosses CD (3b)

Any combination of external forces that verifies Eq. (1) will be either greater than, or equal to, the forces causing collapse. Notice that the first terms (except the last one) of Equation (1) identify the upper bound expression in the absence of micropiles (Augarde et al. 2003).

Forces exerted by the micropile on the critical wedge will be determined by an independent analysis in the following section. The micropile will be considered as a beam and subjected to the kinematic motion imposed by the assumed failure mechanism.

2.3 Micropile behaviour. Limiting conditions

The micropile is idealized as a beam subjected to a uniform imposed displacement δ due to the moving wedge of the expected collapse mechanism. Figure 3 shows the micropile isolated from the surrounding soil. In order to simplify the calculation of the beam, the effective embedded length *b* (distance between the crossing point P and a fixed, fully clamped, point X) of the micropile into the stationary soil will be assumed to be known. An estimation of the value of the clamping distance *b* may be obtained from the theory of piles embedded in an elastic half-space, subjected to a horizontal load and a moment at its head. This problem is described in Poulos and Davis (1980). According to this, *b* has been estimated in the range 0.2 to 0.1.

The displacement δ defines the type of deformation and the stresses of the beam. Its actual value will be found through the assumption that the micropile section will be taken to limiting conditions. Yielding conditions of the steel of the micropile will be assumed to follow a Von Mises criterion. (The grouting contribution is very small and it will be disregarded). The Von Mises criterion in plane strain can be expressed as follows:

$$\sigma^2+3\tau^2=\sigma_e^2 \qquad (4)$$

where σ and τ are the normal and shear stress acting on a fiber of a cross section of the micropile and σ_e is the tensile strength of the steel. σ and τ will be expressed in terms of the normal force (N), shear force (Q) and bending moment (M).

The conditions leading to the maximum support provided by the micropile will be defined by those leading to the yielding of the most stressed fiber within the critically loaded steel cross section of the micropile. This section is point P in Figure 1.

Forces N and Q and moment M at point P, due to an imposed displacement δ, can be calculated if the mechanical and geometrical parameters of the micropile are known:

$$M=\frac{6EI_x}{b^2}\delta_v;\ Q=\frac{12EI_x}{b^3}\delta_v;\ N=\frac{AE}{b}\delta_h \qquad (5a;b;c)$$

where E is the steel elastic modulus, I_x is the moment of inertia with respect to the horizontal axis of the section and A is the cross-sectional area of the micropile (a steel tubular section has been choosen having a diameter d and thickness t). δ_h=δcos μ and δ_v=δcos μ are the horizontal and vertical components of the imposed displacement, δ, expressed in terms of the angle μ (Eq.(2)).

Under these conditions, normal and shear stresses due to the normal (N) and shear (Q) forces and moment (M) are calculated:

$$\sigma=\frac{N}{A}+\frac{M\,z}{I_x} \qquad (6a)$$

$$\tau=-\frac{QS_x}{2I_x t}=\frac{2Q\sqrt{d^2-4z^2}}{\pi d^2 t} \qquad (6b)$$

where z is the distance from the beam axis (x direction) to a particular point of the section and S_x is the static moment of the cross-sectional area above coordinate z.

Substituting N, Q and M from Equations (5) into Equations (6) and the resulting expressions for σ and τ into Equation (4), the Von Mises criterion can be written.

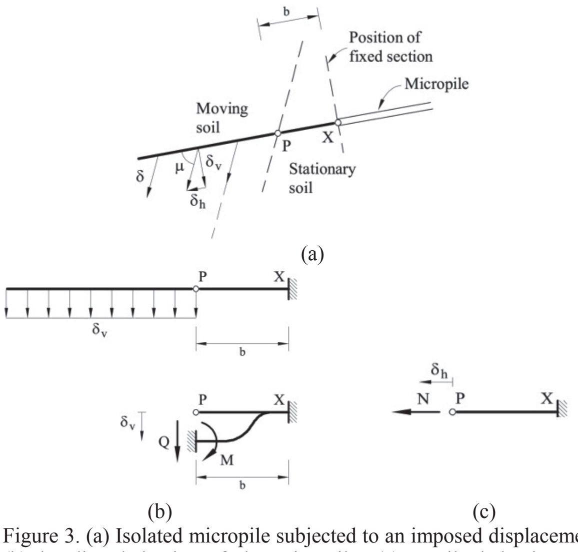

Figure 3. (a) Isolated micropile subjected to an imposed displacement δ; (b) bending behavior of the micropile; (c) tensile behavior of the micropile.

A conservative assumption is now introduced in the calculation. The available strength provided by the micropile is calculated as the value associated with the state in which the section starts to yield at some fiber. Therefore, the stress provided by the micropile beyond this point, due to the yielding of the rest of the section, is not considered here.

The shear stress τ reaches a maximum in the center of the section. On contrary, the stress σ due to N and M reaches a maximum at z = -R. Bending dominates the tensile stressing of the micropile for the particular problem we are considering due to the particular cross-section of the micropiles and the imposed loading mechanism. It turns out that the critical stress is located at the outer part of the cross section.

Applying Von Mises' criterion (Eq. 4) to the fiber characterized by z = -R the following expression for the displacement, δ, leading to the first fiber yielding in the micropile cross section at point P is derived:

$$\delta=\frac{\sigma_t b}{E}\frac{1}{\sqrt{f\left(d/b,\mu\right)}} \qquad (7)$$

where $f(d/b,\mu)$ is a function of the ratio between the diameter of the micropile (d) and the equivalent length of the beam (b) and the relative orientation between the micropile and the upper sliding wedge of the failure mechanism (μ) (Eq. (2)):

$$f\left(d/b,\mu\right)=6\cos\mu\sin\mu\left(d/b\right)+9\sin^2\mu\left(d/b\right)^2+\cos^2\mu \qquad (8)$$

Finally, when the value of δ given in Equation (7) is substituted into equation (5b and c), the following shear and tensile forces applied by the micropile on the sliding mechanism, at point P, are found:

$$N=\sigma_e t d\pi\frac{\cos\mu}{\sqrt{f\left(d/b,\mu\right)}} \qquad (9a)$$

$$Q=\frac{3}{2}\sigma_e t d\pi\left(d/b\right)^2\frac{\cos\mu}{\sqrt{f\left(d/b,\mu\right)}} \qquad (9b)$$

These expressions for N and Q are now introduced into Eq. (1) to find the external loads that leads to the defined failure mechanism. The resulting equation is:

$$\frac{\left(\sigma_s-\sigma_T\right)}{c_u}-\frac{2C}{D}\frac{\sin\theta_2\sin\theta_4}{\sin\theta_1\sin\theta_3\sin\theta_5\cos\left(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5\right)}-$$
$$\frac{1}{\tan\theta_1}-\frac{2}{\tan\theta_2}-\frac{2}{\tan\theta_3}-\frac{2}{\tan\theta_4}-\frac{1}{\tan\theta_5}-$$
$$\frac{\cos\left(\theta_1+\theta_2+\theta_3+\theta_4\right)}{\sin\theta_5\cos\left(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5\right)}+\frac{\gamma D}{c_u}\left(\frac{C}{D}+\frac{1}{2}\right)-$$
$$\frac{\sigma_e t d}{D c_u s}\frac{\pi\left(\cos^2\mu+1.5\sin^2\mu\left(d/b\right)^2\right)}{\sqrt{f\left(d/b,\mu\right)}}\frac{\sin\theta_2\sin\theta_4}{\sin\theta_1\sin\theta_3\sin\theta_5}\Gamma=0$$
$$(10)$$

Notice that the fist term identifies the external forces without including the micropile. This term will be referred to as the "External Stress Coefficient". The reinforcement is identified by the dimensionless parameter $\sigma_e td/Dc_u s$ which combines in a simple expression the mechanical properties of the tubular reinforcement (σ_e, t and d), the undrained soil strength (c_u) and the spacing between micropiles axis (s). This ratio will be named the "Micropile Coefficient".

The most critical collapse mechanisms will be calculated optimizing the energy conservation equation with respect to the five angles describing the geometry.

2.4 *Upper bound solution for the External Stress Coefficient*

The coefficient (σ_s-σ_T)/c_u has been isolated from Equation (10) and minimized with respect to the angles in order to find the smallest upper bound solution linked to the mechanism proposed. The upper bound solution obtained depends on $\gamma D/c_u$, on the Micropile Coefficient and on the cover ratio C/D.

The set of parameters defining the problem have been collected in Table 1. The table indicates also the range of values typically encountered in practice. Three values of the Micropile Coefficient (0, 20 and 50) have been selected to plot the minimized values of the External Stress Coefficient (with respect to the five angles) against the cover ratio C/D for different values of the strength ratio $\gamma D/c_u$ (Fig. 4). The adopted values of b, that defines the clamped length of the micropiles (Fig. 2), is five times the micropiles diameter (d).

To visualize better the effect of the micropile umbrella, the unreinforced case is plotted in Figure 4. Note that the reinforcement leads to a reduction of the required pressure applied to the tunnel face of even to make it unnecessary.

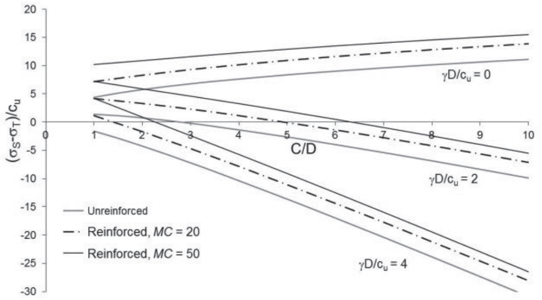

Figure 4. Upper bound solution of the External Stress Coefficient for the cases of Micropile Coefficient equal to 0 (reinforced case), 20 and 50. $d/b = 0.1$.

Table 1. Typical range of parameters.

Parameter	*Range of values*
Beam diameter d (m)	0.04-0.12
Beam thickness t (m)	0.0003-0.015
Distance between micropiles s (m)	0.1-1
Steel strength σ_e (MPa)	200-400
Soil undrained strength c_u	0.03-0.5
Tunnel diameter D (m)	2-12

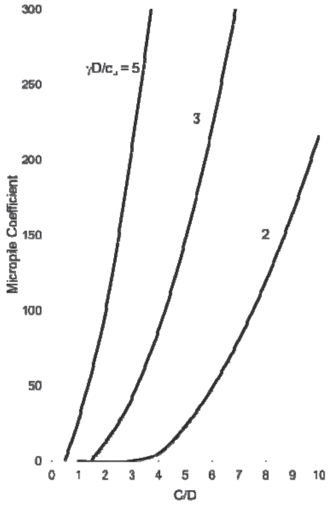

Figure 5. Upper bound solution of the Micropile Coefficient for the case of $(\sigma_s-\sigma_T)/c_u = 0$ and $d/b = 0.20$.

2.5 *Upper bound solution of Micropile Coefficient*

An upper bound solution is calculated now for the Micropile Coefficient assuming that the remaining external loads are known. The Micropile Coefficient is isolated in Equation (10). This expression provides values of the Micropile Coefficient leading to collapse. It is then interesting to find the maximum value of the Micropile Coefficient by mean of its optimization with respect of the angles and to find the critical failure mechanism.

The calculated critical value of the Micropile Coefficient has been plotted in Figure 5 in terms of *C/D* and $\gamma D/c_u$ and for the special case of =0. This is an interesting case in practice because it describes a conventional tunnel excavation procedure. In general, when boring machines are used micropiles reinforcement of the front is seldom used.

3 CONCLUSION

Tunnel face instability is a risk associated with open front construction methods. This paper presents an analysis of the face stability of tunnels reinforced with an umbrella of micropiles. Non-dimensional solutions based on the upper bound classical theorem of plasticity have been developed. The procedure is based on two aspects: a) defining the limiting resisting conditions of the individual micropiles and b) including the micropile forces within the formulation of the upper bound theorem. Micropiles limiting resisting forces have been calculated starting from a basic yield criterion (Von Mises) for tubular steel reinforcements. Also included in the analysis was the stabilisation of the tunnel head by a pressure applied on the tunnel face.

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