Simultaneous estimation of transverse and longitudinal dispersion in unsaturated soils using spatial moments and image processing

Estimation simultanée de la dispersion transversale et longitudinale dans des sols insaturés au moyen de la méthode des moments pour l'analyse des données spatiales et du traitement d'images

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ABSTRACT: A new methodology using spatial moment analysis linked with image processing of a dye tracer behavior in porous media was developed to estimate dispersivities not only in longitudinal but in transverse directions. Laboratory and field tracer experiments using a relatively mobile dye tracer Brilliant Blue FCF were conducted under saturated and unsaturated flow conditions. Dye tracer in field moved through the soils in a preferential path pattern, which induced higher dispersivities in more irregular pore patterns as compared with those in laboratory. Experimental results demonstrated the effectiveness of the developed methodology for simultaneous assessment of transverse and longitudinal dispersion in unsaturated soils.

RÉSUMÉ : Nous avons développé une nouvelle méthodologie utilisant l'analyse par la méthode des moments associée au traitement d'images pour suivre le comportement d'un traceur colorant en milieu poreux afin d'estimer les dispersivités, aussi bien dans la direction longitudinale que transversale. Nous avons mené des expériences de traçage en laboratoire et sur le terrain au moyen d'un traceur colorant bleu brillant FCF relativement mobile dans des conditions de flux saturé et insaturé. Sur le terrain, le traceur colorant s'est déplacé dans les sols suivant un modèle de trajet préférentiel et, comparé à celles constatées en laboratoire, a présenté des dispersivités induites plus élevées dans les profils d'interstices plus irréguliers. Les résultats de ces expériences ont démontré l'efficacité de la méthodologie que nous avons mise au point pour évaluer simultanément la dispersion transversale et longitudinale dans des sols insaturés.

KEYWORDS: longitudinal and transverse dispersion, dye tracer, image processing, spatial moment, unsaturated soils.

1 INTRODUCTION

The movement of groundwater in porous media is subject to convection and dispersion, independently of any material being transported (Vanderborght and Vereecken, 2007). Dispersion results from the irregular movement of water in porous formations where tortuosity of flow paths is induced. The importance of local transverse dispersion is now identified as a key factor in the smoothing of concentration fluctuations and controlling the rate of dilution of conservative and non-conservative solutes. Despite the importance of transverse dispersion, transverse dispersivity is rarely determined due to the lack of data in the experimental and quantitative difficulties associated with such determinations. Recently, a few of the dye tracer experiments combined with image analysis techniques have been conducted to quantify the behavior of dye tracers (McNeil et al., 2006).

The popularity of dyes is attributable to their low detection limits, visualization potential and ease of quantification by chemical analysis (Flury and Flühler, 1995). Application of image analysis has shown that a time series of digitized images reflecting the movement of dye tracer could be successfully used to monitor solute transport in porous media as well as to estimate transport parameters such as dispersion coefficient, dispersivity (Forrer et al., 1999) and retardation factor (Flury and Flühler, 1995). However, there are many aspects of solute dispersion in unsaturated porous media that still be poorly understood. In the present study, a new methodology using spatial moment analysis linked with image processing of a dye tracer behavior was developed to estimate dispersivities not only in longitudinal but in transverse directions. This technique was applied to dye tracer experiments both in laboratory and field.

2 DYE TRACER EXPERIMENTS

2.1 *Materials and method*

Dye tracing has been widely used to characterize water flow and solute transport behavior in porous media. Previous work demonstrated that colored dye tracers could be successfully used to visualize and monitor solute transport in a porous medium confined in a transparent container (Flury and Flühler, 1995). In this study, one of the soluble dyes, Brilliant Blue FCF, was employed as a dye tracer with the initial concentration of 1.0 mg/cm^3. Although the initial concentration of dye tracer was determined to be low enough to avoid density-induced flow effects, there was no denying the effect of gravity on solute transport during the course of movement.

Tracer experiments were carried out in a two-dimensional and vertically placed water tank with the dimensions of 100 cm width, 100 cm height and 3 cm thickness. The water flow tank allowed to contain soils in order to form transparent quasi two-dimensional solute transport phenomena and consisted of two glass plates with 2 cm thickness. Schematic diagram of experimental apparatus is shown in Figure 1.

In dye tracer experiments, silica sand with a low uniformity coefficient was selected in order to simulate a sandy aquifer. In addition, Andisols, which are volcanic ash soils and have unique properties such as a low bulk density, were taken from a maize field, dried at 110ºC and passed through a 2-mm sieve. Grain sizes less than 0.2 mm were excluded to avoid the adsorption of dye onto the surface of silt or clay. Physical properties for both soils (silica sand and Andisol) are listed in Table 1. In Figure 2, the relationship between suction and volumetric water content for the drying process is plotted with fitting curves based on van Genuchten's formula (van Genuchten, 1980).

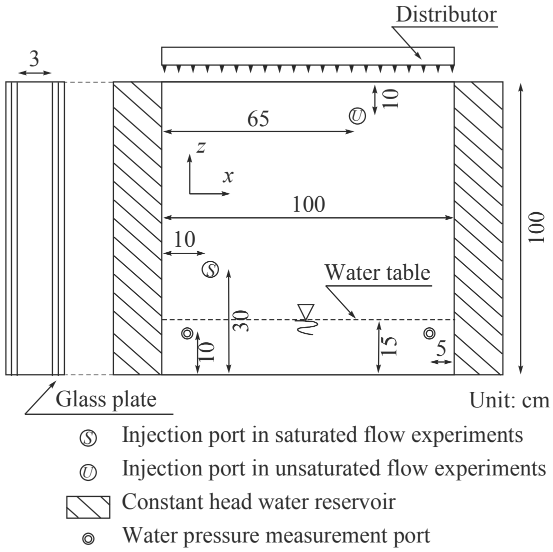

Figure 1. Schematic drawing of an experimental apparatus.

Table 1. Properties of soils.

	Silica sand	Andisol
Particle density (g/cm^3)	2.68	2.40
Mean diameter of particle (cm)	0.085	0.076
Uniformity coefficient (-)	1.80	2.74
Hydraulic conductivity (cm/s)	0.751	0.0341
Porosity (-)	0.42	0.64

2.2 Experimental procedure

Soils were completely washed and saturated before packing to remove organic chemicals attached to the particle surface, to avoid entering air and to conduct experiments under the saturated condition. In the process of creation of flow field, water flow tank was filled with water and soil material of interest from bottom to top in 5 cm layers to achieve uniform packing. In this process, soil was funneled using an extended funnel. Each layer of interest was compacted prior to filling the next layer, resulting in 0.42 and 0.64 of the porosity for silica sand and Andisol, respectively. The porosity of each flow field was able to be estimated indirectly from measurements of the particle density and the dry soil bulk density.

After packing, water was applied to the flow tank under a specific hydraulic gradient controlled by constant head water reservoirs at the upstream and downstream sides, while maintaining saturated condition of porous media. A steady saturated flow field was established when fluctuations in the observed drainage rate, which was effluent from the constant head water reservoir, and piezometer readings could become negligible. After reaching steady state flow conditions, dye tracer with the volume of 25 cm^3, which made flow paths visible, was uniformly injected along the whole thickness of the flow tank. During the experiment, the profiles of tracer migration were periodically recorded using a digital camera.

Dye tracer experiments under unsaturated conditions were conducted in a similar manner. Internal drainage using constant head reservoirs allowed for one day to create an unsaturated flow field after the flow tank was filled with water and sand of concern. After steady state condition was established, water was applied using a distributor placed 10 cm above the soil surface. Three rainfall rates of 0.09, 0.21 and 0.63 mm/min, were set with no rainfall case. Experimental cases are listed in Table 2.

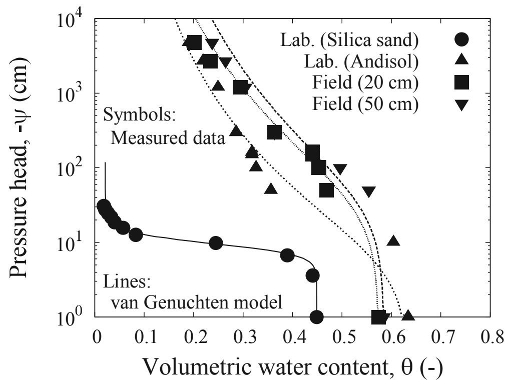

Figure 2. Water characteristic curves for both soils.

Table 2. Experimental cases.

		Rainfall intensity (mm/min)			
Soil type	Saturated	0	0.09	0.21	0.63
Silica sand	S-S	S-1	S-2	S-3	-
Andisol	A-S	A-1	A-2	A-3	A-4

2.3 Image processing and spatial moment approach

Each of the pixels representing an image has a pixel intensity which describes how bright that pixel is. In order to establish the relationship between the pixel intensity of a pixel and dye tracer concentration, a calibration was conducted. Under identical experimental conditions, a known concentration of dye tracer was injected into a corresponding porous formation without a hydraulic gradient. The spread of dye was captured by the digital camera. The same procedure was repeated using different concentrations of dye tracer. Consequently, the concentration of the dye tracer as a function of the pixel intensity varied over the range of 0 mg/cm^3 to 1.0 mg/cm^3.

A commonly used measure of dilution is the spatial moments of aqueous concentrations, which are calculated from snapshots of tracer plume at given times as follows (Tompson and Gelhar, 1990).

$$M_{ij}(t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c(x,z,t)x^i z^j dxdz, \quad i,j=1,2 \qquad (1)$$

where x and z are the Cartesian coordinates, c is the solute concentration, t is the time, M_{ij} is the spatial moments associated with the distribution of tracer plume at a certain time, and i and j are the spatial order in the x and z coordinates, respectively.

The pixel intensity distribution can be converted to a concentration distribution by the calibration, providing an analogy between Eq.(1) and Eq.(2).

$$M_{ij}(t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H(x,z)B(x,z,t)x^i z^j dxdz, \quad i,j=1,2 \qquad (2)$$

where $H(x,z)$ is the area per unit pixel and $B(x,z,t)$ is the intensity at a corresponding pixel. The centroid of plume concentration distribution is calculated as the normalized first order spatial moment by the following equation.

$$x_c=\frac{M_{10}}{M_{00}}, \quad z_c=\frac{M_{01}}{M_{00}} \qquad (3)$$

where x_c and z_c are the centroid locations of plume concentration distribution in the x and z coordinates, respectively. The second order spatial moments are also computed as follows.

$$\sigma_{ij}=\begin{pmatrix}\sigma_{xx} & \sigma_{xz}\\ \sigma_{zx} & \sigma_{zz}\end{pmatrix}=\begin{pmatrix}\dfrac{M_{20}}{M_{00}}-x_c^2 & \dfrac{M_{11}}{M_{00}}-x_cz_c\\ \dfrac{M_{11}}{M_{00}}-z_cx_c & \dfrac{M_{02}}{M_{00}}-z_c^2\end{pmatrix} \quad (4)$$

where σ_{ij} is the second order spatial moments.

Longitudinal and transverse dispersivities from spatial moments of the distributed tracer plume are calculated as the following equations (5) and (6) under saturated and unsaturated conditions, respectively.

$$\alpha_L=\frac{1}{2}\frac{\sigma_{xx}}{\xi_c},\quad \alpha_T=\frac{1}{2}\frac{\sigma_{zz}}{\xi_c} \quad (5)$$

$$\alpha_L=\frac{1}{2}\frac{\sigma_{zz}}{\xi_c},\quad \alpha_T=\frac{1}{2}\frac{\sigma_{xx}}{\xi_c} \quad (6)$$

where α_L is the longitudinal dispersivity, α_T is the transverse dispersivity and ξ_c is the travel distance of the center of tracer plume in the mean flow direction at a given time *t*.

3 DISPERSIVITY IN LABORATORY

3.1 Longitudinal dispersivity

The results of longitudinal and transverse dispersivities as a function of travel distance for silica sand and Andisol are shown in Figures 3 and 4, respectively. Longitudinal dispersivity estimates under unsaturated conditions are larger than those under saturated conditions. The increase of dispersivity may be induced from diversity of solute movement due to the effect of air. Several studies have pointed out the same tendency in unsaturated soils (Vanderborght and Vereecken, 2007).

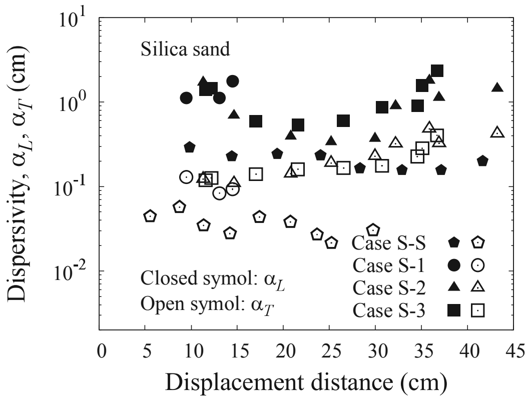

Figure 3. Longitudinal and transverse dispersivity estimates with displacement distance in silica sand.

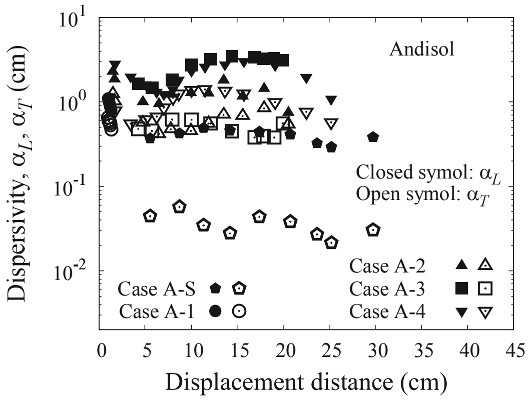

Figure 4. Longitudinal and transverse dispersivity estimates with displacement distance in Andisol.

Table 3. Soil properties in maize field.

	Depth from the ground surface	
	20 cm	50 cm
Dry density (g/cm^3)	0.95	1.13
Mean diameter of particle (cm)	0.021	0.10
Uniformity coefficient (-)	23.7	85.7
Hydraulic conductivity (cm/s)	0.00884	0.0239
Porosity (-)	0.59	0.57

Longitudinal dispersivity estimates also exhibit an increasing and decreasing tendency and show a dependency on infiltration rates. Water applied to the ground surface infiltrates and reaches an upper part of dye tracer. Because flow velocity is larger than solute velocity in unsaturated zone and affects the change of tracer migration, shape of dye tracer distribution is shrunk longitudinally. This duration may correspond to the decreasing process of the longitudinal dispersivity. After water reaches a front of dye tracer, dye tracer migrates with interstitial water. However, part of dye tracer has relatively low velocities due to the effect of air. Therefore, shape of dye tracer extends longitudinally, leading to the increase of longitudinal dispersivity estimates. Similar tendency can be seen in Andisol in Figure 4. A difference of the longitudinal dispersivity estimates between silica sand and Andisol is attributed to the difference of the uniformity coefficient.

3.2 Transverse dispersivity

As for Figures 3 and 4, transverse dispersivity slightly increases under unsaturated conditions or remains constant under saturated conditions with displacement distance. It is inferred that transverse solute displacement depends largely on mixing of water and air whose distribution varies with the depth. The degree of increase of water content induced from rainfall application decreases as rainfall intensity is lower. Since diversity of solute movement pathway in porous media increases at lower saturation this process would make mobile region of water complicated and result in larger values for longitudinal dispersivity. Hence, transverse dispersion phenomenon under unsaturated conditions is clearly different from longitudinal dispersion phenomenon.

4 DISPERSIVITY IN A FIELD

4.1 Experimental setup

Developed methodology for quantifying transverse and longitudinal dispersivities was applied to a maize field soil with approximately 100 cm depth from the ground surface under water-unsaturated conditions. Soil cores were taken at the depth of 20 cm and 40 cm. Physical properties are shown in Table 3 and water characteristic curves are also shown in Figure 2.

Brilliant Blue FCF tracer of 1000 cm^3 with the initial concentration of 2.0 mg/cm^3 was leached into the ground surface from two line sources (referred to as Plot 1 and 2) with 65 cm length and 3cm depth and subsequent tracer distributions were photographically recorded at vertical soil profiles excavated perpendicular to the line source. Artificial rainfall intensity was set to 0.12 mm/min for the application duration of 15 minutes. At a site near Plot 1 and 2, the two relations between the dye concentration and the pixel intensity at the upper and lower zones, which are 3 cm to 40 cm and are 40 cm to 80 cm below the ground surface, respectively, were obtained over the range of 0 mg/cm^3 to 2mg/cm^3. In a similar manner to laboratory tracer experiments, transverse and longitudinal dispersivities were quantified using spatial moments and image processing techniques.

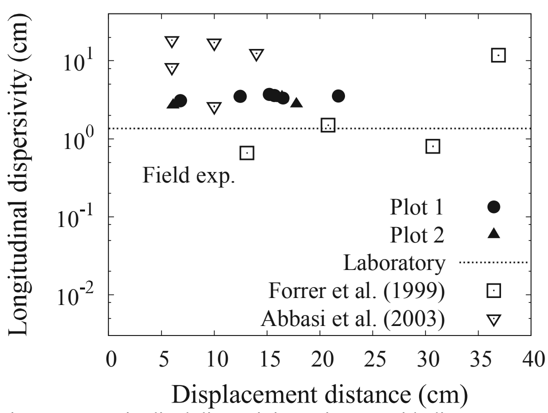

Figure 5. Longitudinal dispersivity estimates with distance.

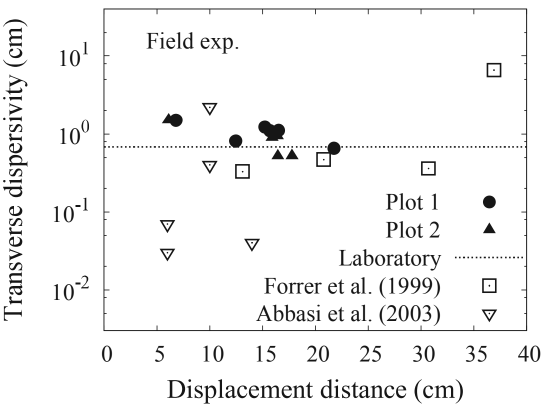

Figure 6. Transverse dispersivity estimates with distance.

4.2 *Longitudinal and transverse dispersivities*

Figures 5 and 6 illustrate the estimates of longitudinal and transverse dispersivities in comparison with the mean value of laboratory experiments and reported values (Forrer et al., 1999; Abbasi et al., 1995). Longitudinal and transverse dispersivity estimates ranged from 1.00 cm to 3.72 cm and from 0.52 cm to 2.35 cm respectively, which are good agreement with reported values, indicating the effectiveness of the developed methodology. Also, the ratio of longitudinal to transverse dispersivities was in the range of 1 to 5.4, which is in the lower range of dispersivity ratios reported in porous media (Persson and Berndtsson, 2002).

As a whole, both dispersivity values are slight larger than those identified in the laboratory. This is attributed to the difference between undisturbed soil in the field and disturbed soil in the laboratory. In addition, dye tracer moved through the soils in a preferential flow pattern, which induced higher dispersivities in more irregular flow patterns as compared with estimates obtained in laboratory tracer experiments. At the experimental site, some macropores were confirmed and influenced seepage and solute pathways in porous media.

Figure 7 shows the relation between the applied rainfall intensity and the dispersivity estimates in the field as well as in the laboratory. Mean values in each experiment are plotted in this figure. Despite of the rainfall intensity, both dispersivities remain constant. This is attributed to a relatively low degree of heterogeneity in the field of concern, while homogeneous packing of soil in the laboratory was reflected as a less variation of dispersivity estimates.

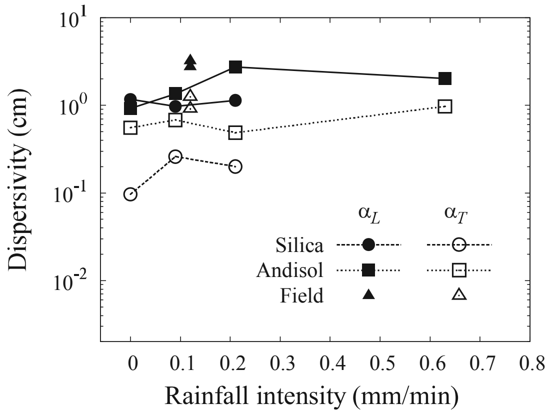

Figure 7. Relation between rainfall intensity and dispersivity estimates.

5 CONCLUSIONS

In the present study, a new methodology using spatial moment analysis linked with image processing of a dye tracer behavior was developed to estimate dispersivities not only in longitudinal but in transverse directions. Laboratory and field tracer experiments under unsaturated flow conditions were conducted with dye, Brilliant Blue FCF. Dispersivities exhibited an increasing and decreasing tendency associated with water content and showed a dependency on infiltration rates. Laboratory and field studies were extended by a literature search to compare the new results with earlier work, demonstrating a good agreement between the experimental and published results. Experimental results indicated the effectiveness of the developed methodology for simultaneous assessment of transverse and longitudinal dispersion in unsaturated soils in a field as well as in a laboratory.

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7 REFERENCES

Vanderborght J. and Vereecken H. 2007. Review of dispersivities for transport modeling in soils, *Vadose Zone J*., 6, 29-52.

McNeil J.D., Oldenborger G.A. and Shincariol R.A. 2006. Quantitative imaging of contaminant distributions in heterogeneous porous media laboratory experiments, *J. Contam. Hydrol*., 84, 36-54.

Flury M. and Flühler H. 1995. Tracer characteristics of Brilliant Blue FCF, *Soil Sci. Soc. Am. J*., 59(1), 22-27.

Forrer I. Kasteel R., Flury M. and Flühler H. 1999. Longitudinal and lateral dispersion in an unsaturated field soil, *Water Resour. Res.*, 35(10), 3049-3060.

van Genuchten M.Th.1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils, *Soil Sci. Soc. Am. J.*, 44, 892-898.

Tompson A.F.B. and Gelher L.W. 1990. Numerical simulation of solute transport in three-dimensional, randomly heterogeneous porous media, *Water Resour. Res*., 26(10), 2541-2562.

Abbasi F., Simunek J., Feyen J., van Genuchten M.Th. and Shouse P.J. 1995. Simultaneous inverse estimation of soil hydraulic and solute transport parameters from transient field experiments: homogeneous soil, *Trans. ASAE*, 46(4), 1085-1095.

Persson M. and Berndtsson R. 2002. Transect scale solute transport measured by time domain reflectometry, *Nord. Hydrol.*, 33, 145-164.