Aspects on the modelling of smear zones around vertical drains

Aspects de la modélisation de la zone remaniée autour des drains verticaux

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ABSTRACT: The analytical design of vertical drains in soft clay requires knowledge of the coefficient of consolidation and also of the disturbance effects induced during the installation of the drains. Several analytical models describing the disturbance effects in different ways are proposed in the literature. The earliest and simplest models describe the disturbance effect in terms of concentric cylinders around a drain where a reduced and constant permeability is assumed, while more recent models attempt to describe the disturbance more realistically via more complex mathematical formulations. Although these new models describe the real in situ behaviour more realistically than the early ones, they may not always be suitable for practical use as many of the required variables are difficult to assess by standard investigation methods. This study investigates and discusses the difference between some of the available models and evaluates the influences on the results of the variables incorporated in the models.

RÉSUMÉ: L'étude analytique des drains verticaux dans les argiles molles nécessite la connaissance du coefficient de consolidation et des effets du remaniement produit par l'installation des drains. Ces effets peuvent être modélisés de plusieurs façons. Les modèles les plus anciens et les plus simples décrivent le remaniement à l'aide de cylindres concentriques autour d'un drain en supposant que la perméabilité est réduite et constante, tandis que des modèles plus récents s'efforcent à décrire le remaniement de façon plus réaliste à l'aide de formulations mathématiques avancées. Bien que ces modèles décrivent le comportement in situ de manière plus réaliste que leurs prédécesseurs, leur utilisation pratique est souvent limitée car plusieurs des paramètres requis sont souvent difficiles à évaluer à l'aide de sondages, forages et essais classiques. Cette étude s'intéresse aux différences entre certains des modèles existants et évalue l'influence des divers paramètres sur les résultats.

KEYWORDS: Vertical Drains, Design, Modelling

1 INTRODUCTION

During the installation of prefabricated vertical drains (PVDs) in soft clay, the original soil fabric is disturbed. The disturbance occurs when the installation device, the mandrel, is pushed through the clay displacing the soil material. According to e.g. Hird and Moseley (2000) this results in a disruption of the initial soil fabric, e.g. the destruction of any permeability anisotropy (the ratio of horizontal to vertical permeability k_h/k_v), and causes excess pore pressures that trigger a subsequent reconsolidation of the clay and an associated decrease in void ratio that in turn decreases the permeability (e.g. Tavenas et al. 1983). The nature of the disturbance is highly complex and depends on many factors such as the characteristics of the soil material, the shape, surface roughness and size of the mandrel, the installation rate and the soil movement after the mandrel has been removed (e.g. Onoue et al. 1991, Hird and Moseley 2000). Laboratory studies investigating the spatial characteristics of the disturbed zone show that the degree of disturbance (i.e. the reduction in k_h) is most pronounced in the vicinity of the drain where k_h approaches k_v and decreases with increasing radial distance from the drain (Onoue et al. 1991, Bergado et al. 1991, Madhav et al. 1993, Indraratna and Redana 1998, Hird and Moseley 2000, Sharma and Xiao 2000, Sathananthan and Indraratna 2006).

For the design of PVDs and the assessment of the average degree of consolidation (U), several theoretical models describing the characteristics of the disturbed zone have been proposed over the years. The early rather simple models (Barron 1948, Hansbo 1979) assumed a unit cell soil cylinder dewatered by one centric drain and a disturbed (smear) zone with a constant and reduced horizontal permeability (Figure 1). According to Basu et al. (2006), previous studies based on this model suggest that the extent (diameter) of the smear zone (d_s) is 2 to 4 times larger than the equivalent diameter of the PVD (d_w) and that the reduced horizontal permeability (k_{hs}) is 2 to 10 times lower than the undisturbed permeability (k_{h0}), i.e. $s = d_s/d_w \approx$2-4 and $\kappa = k_{h0}/k_{hs} \approx$2-10. However, the cited laboratory studies have indicated that the extent of the disturbed zone can be as large as d_s/d_m =9 (where d_m is the equivalent diameter of the mandrel).

More recent models attempt to capture the nature of the smear zone more realistically, describing the variation of k_h within the disturbed zone (e.g. Walker and Indraratna 2006, Basu et al. 2006, Chung et al. 2009). In addition, temporal effects, such as the reconsolidation of the clay after drain installation, affecting the characteristics of the disturbed zone have been incorporated in the models presented by Indraratna et al. (2005) and Walker et al. (2012).

To a practising engineer creating a design involving PVDs, the choice of model and the widely varying suggestions regarding the values of s and κ may be confusing. This paper investigates the differences between six of the analytical models available in the literature and the influences of the involved variables on the assessment of U. All the models investigated can be written on the form:

$$U = 1 - e^{\frac{-8 \times T_h}{F}} \quad (1)$$

where $T_h = c_h \times t/d^2$ is the time factor for horizontal consolidation, $c_h = k_h \times M_v/\gamma_w$ is the undisturbed horizontal coefficient of consolidation in the clay (where M_v is the vertical

Table 1. Characteristics and formulations of F in the investigated models (valid for $n > 10$ [A] and neglecting well resistance)

no.	*Characteristics*	*Formulation* [A]	*Reference and comments*
I	No smear zone, c_v is used instead of c_h [B]	$F_{\mathrm{I}} = \ln(n) - 0.75$	Kjellman (1949), smear effects accounted for by adopting c_v instead of c_h
II	$k_h = k_{hs}$ and constant in the smear zone	$F_{\mathrm{II}} = \ln(n/s) - 0.75 + \kappa\ln(s)$	Hansbo (1979), equal to model no. I for s =1
III	Equal to no. II, k_h dependent on the void ratio	$F_{\mathrm{III}} = \frac{2F_{II}}{1 + (1 + \Delta p/\sigma_i)^{1 - C_c/C_k}}$	Indraratna et al. (2005), valid for normally consolidated clays, equal to model no. II for $C_c/C_k = 1$
IV	Parabolic variation of k_h in the smear zone	$F_{\mathrm{IV}} = \ln(n/s) - 0.75 + \frac{\kappa(s-1)^2}{(s^2 - 2\kappa s + \kappa)}\ln\left(\frac{s}{\sqrt{\kappa}}\right) - \frac{s(s-1)\sqrt{\kappa(\kappa-1)}}{2(s^2 - 2\kappa s + \kappa)}\ln\left(\frac{\sqrt{\kappa} + \sqrt{\kappa - 1}}{\sqrt{\kappa} - \sqrt{\kappa - 1}}\right)$	Walker and Indraratna (2006)
V	$k_h = k_{hs}$ in the inner smear zone thereafter linear variation	$F_{\mathrm{V}} = \ln(n/s) - 0.75 + \kappa\ln(m) + \frac{s - m}{s/\kappa - m}\ln\left(\frac{s}{\kappa m}\right)$	Basu et al. (2006), case b, equal to model no. VI for m =1
VI	Linear variation	$F_{\mathrm{VI}} = \ln(n/s) - 0.75 + \frac{s - 1}{s/\kappa - 1}\ln\left(\frac{s}{\kappa}\right)$	Basu et al. (2006), case d

[A] $n = d/d_w$; *σ_i & Δp=initial stress & stress from the applied load; C_c & C_k=compression & permeability indices;* $m = d_i/d_w$

[B] $c_v = c_h/1.5$ was used based on suggestions in Tavenas et al. (1983) for the anisotropy in permeability in homogeneous clays.

compression modulus and γ_w is the unit weight of water), t is the consolidation time, d is the diameter of the assumed unit cell dewatered by a single drain (cf. Figure 1) and the expression F is dependent on the model.

1 METHODS

The characteristics and formulations of the expression F in the six investigated models are presented in Table 1 and Figure 1b.

Denoting the variables in Eq. 1 and in the formulations of F (i.e. $T_h, n, s, \kappa, \Delta p/\sigma_i, C_c/C_k, m$) as $x_1, x_2, \ldots, x_n$, the partial derivative of U with respect to the variable x_i, i.e. $\partial U/\partial x_i$, can be obtained and the influence of each variable on U can be assessed:

$$\alpha_i = \frac{\partial U/\partial x_i}{\sqrt{\sum_{i=1}^{n}(\partial U/\partial x_i)^2}} \tag{2}$$

This was done for all of the aforementioned models, assigning $d = (1.1, 1.6, 2.1)$ metres and for values of t resulting in assessments of U ranging from 0 to 1. In addition, the uncertainties in the assessments of U (expressed as the variance, Var_U) were evaluated. In these analyses, the variables c_h, s, κ and C_c/C_k were treated stochastically, while the other variables were assumed to be deterministic, and the variances in the four variables were propagated through Eq. 1 via second order Taylor series approximations (e.g. Fenton and Griffiths, 2008 pp. 30-31). The contribution to Var_U from each variable was then assessed as (e.g. Christian et al. 1994):

$$dVar_{U,i} = \frac{(\partial U/\partial x_i)^2 Var_i}{\sum_{i=1}^{n}[(\partial U/\partial x_i)^2 Var_i]} \tag{3}$$

Values assigned to the variables adopted in the analyses are presented in Table 2.

2 RESULTS

2.1 *Assessments of U from the six models*

In Figure 2, the degrees of consolidation U assessed from the six models are presented as a function of t for the three values

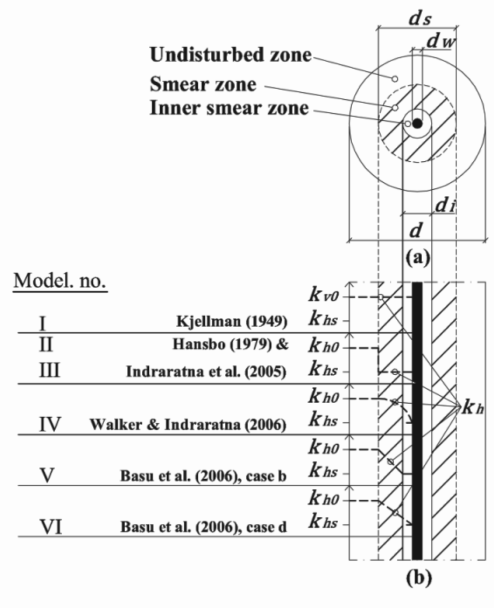

Figure 1. a) Plan view of the unit cell; b) Vertical section of the unit cell and illustration of the analytical models investigated.

of d. In the figure, a span representing two standard deviations (SD), i.e. $2 \times \sqrt{Var_U}$, is presented for d =1.1 m. The appearance is similar for the other two values of d. The curves plot at a close distance and well within the span of 2xSD for the respective values of d, i.e the uncertainties in the variables had a greater impact on the assessed value of U than the choice of model.

Table 2. Values assigned to the variables in the analyses, μ is the average value and COV is the coefficient of variation

Variable	μ_i	COV_i	*Comment*
c_h	5x10^{-8} m^2/s ^{A}	0.35	μ considered representative for soft clays and COV chosen based on Lumb (1974)
d_w	0.066 m ^{B}	Det.	Rectangular PVD 0.003 m x 0.1 m
d_m/d_w	1.7 ^{B}	Det.	Rectangular mandrel 0.06 m x 0.12 m
d_s/d_m	4.7	0.34	^{C}
s	8	0.34	$s = d_m/d_w \times d_s/d_m$
κ	1.6	0.34	^{C}
$\Delta p/\sigma_i$	2	Det.	Arbitrary chosen
C_c/C_k	0.75	0.34^D	μ arbitrary chosen
m	2	Det.	^{C}

^{A} *5x10^{-8}/1.5=3.3x10^{-8} m^2/s for model I*

^{B} *Equivalent diameter evaluated as proposed by Hansbo (1979)*

^{C} *μ and COV evaluated from the cited laboratory tests*

^{D} $COV_{Cc/Ck} = \sqrt{COV_{Cc}^2 + COV_{Ck}^2}$ *where COV_{Cc} =0.3 (Lumb 1974) and COV_{Ck} =0.15 (from compilation in Müller and Larsson 2012)*

2.2 *The influences of the variables on the assessments of U*

The influences of the variables T_h and κ (Eq. 2) are shown vs. assessed values of U in Figure 3 for d =1.6 metres. The appearance is similar for the other two values of d. In models I, II, IV, V and VI, the influences of the other variables were <0.045 for all values on U. However, formodel III, the influences of $\Delta p/\sigma_i$ and C_c/C_k were equal to that of κ, so that the curves for $\alpha_{\Delta p/\sigma i}$ and $\alpha_{Cc/Ck}$ coincide with the curve for α_κ (the short-dashed curve). Model I was excluded from this figure, as α_{Th} was equal to 1 for all values of U. In the figure, it can be seen that α_{Th} >0.8 for U <0.8, whereafter α_{Th} decreases rapidly and α_κ (and in case III also $\alpha_{\Delta p/\sigma i}$ and $\alpha_{Cc/Ck}$) become progressively more influential.

2.3 *The variables' contribution to Var_U*

In Figure 4, the relative influences of the four variables treated stochastically on Var_U are shown for d =1.6 metres. The appearance is similar for the other two values of d. It can be seen that $Var_{U,Th}$ contributes more than 50% to Var_U in all the analyses, that $Var_{U,\kappa}$ accounts for most of the remainder and that the contributions from s and C_c/C_k are smaller.

3 DISCUSSION

3.1 *Values on the variables*

The values assigned to the variables in the analyses were chosen by the present authors based on suggestions in the literature and are considered to be representative for soft clays. In the framework of this study (results not presented), μ for the variables were varied within reasonable ranges one at a time rendering a similar appearance in the results to that presented. Other combinations of the variables might render results that deviate from the results presented here, but it is the authors' belief that the appearance of the results is typical for most cases.

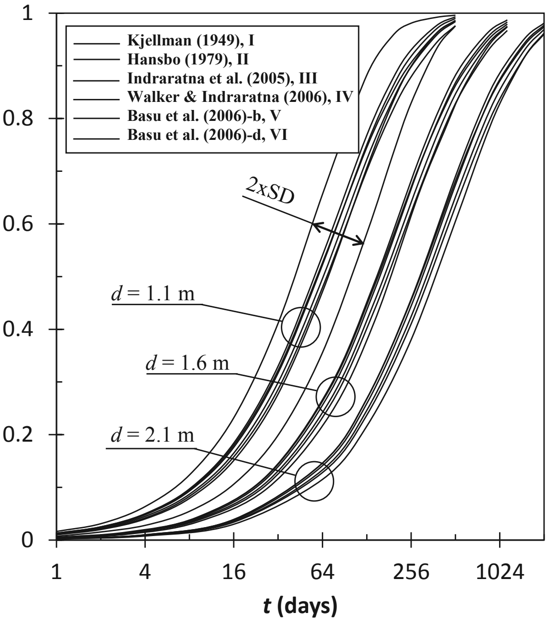

Figure 2. U assessed via the six models for different values of d.

3.2 *The assessed U and the influences of the variables*

As seen in Figure 2, model I followed by model II were the most conservative, predicting the slowest consolidation rate. Comparing the formulations for F in model II with those in models IV-VI (Figure 1b and Table 1), this is obvious since model II assigns a constant value of k_h over d_s whereas k_h is successively increased in the other three models. In this context, it should be noted that model III gives lower values of U than model II at corresponding t for C_c/C_k >1 (0.75 in this study). The finding that model I was the most conservative emphasises the relative importance of c_h compared to the modelling of the smear zone. Model I does not take the smear zone into account but adopts c_v instead of c_h (c_v was assumed to be 1.5 times less than c_h in this study). The relative importance of c_h is also shown in Figure 3 where α_{Th} predominates in the assessment of U for all but the last parts of the consolidation sequences.

The significance of (re)consolidation effects and the associated decrease in k_h (incorporated in model III) is confirmed by the results of laboratory oedometer tests presented by Indraratna and Redana (1998), Sharma and Xiao (2000) and Sathananthan and Indraratna (2006). The results presented in their studies suggest that the resulting decrease in void ratio when the consolidation stresses are increased by 25-50 kPa lead to a more pronounced decrease in k_h than the disturbance induced by the installation process. Hence, in most cases it is more important to consider the change in k_h that occurs due to the decrease in void ratio during consolidation than the disturbance effects.

3.3 *The uncertainty in U*

To reduce the uncertainty in the assessment of U via any of the investigated models, it is obvious that attention should be directed primarily towards c_h, since the uncertainty in T_h is dependent on Var_{ch} via $Var_{Th} = Var_{ch} \times t^2/d^4$, and secondarily towards κ (Figure 4). Hence, site investigations intended for the design of PVDs should focus on reducing the level of uncertainty in c_h and possibly the degree of disturbance in the smear zone (i.e. κ).

In ordinary engineering projects involving clay, investigations of c_v (e.g. via oedometer tests) are far more frequent than investigations of c_h and it might therefore be worth considering model I. However, if model I is used for design purposes, care must be taken as c_v is used instead of c_h

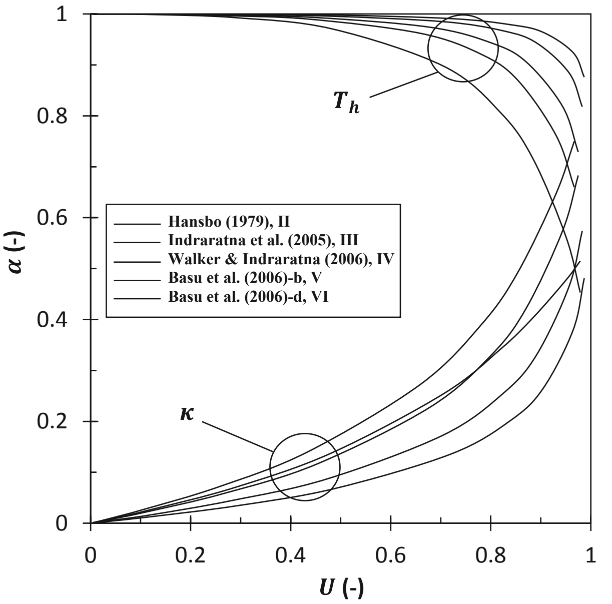

Figure 3. The influence on U of T_h and κ for d =1.6 metres.

and the results are therefore highly dependent on the permeability anisotropy in the clay of interest. For instance, if $k_h \approx k_v$, the consolidation rate might be overestimated.

4 CONCLUSION

Although they may capture the nature of the smear zone more realistically, the impacts on the assessment of U of the more complex models (III-VI) rather than model II are insignificant under the assumptions made in this study and, as argued by Onoue et al. (1991) and Hird and Moseley (2000), model II (Hansbo 1979) is still useful for practical engineering purposes due to its simplicity. This study shows that the even more simple model suggested by Kjellman (1949), neglecting the smear zone but adopting c_v instead of c_h, might give satisfactory results. Care should however be taken, as assessments using this model are dependent on the permeability anisotropy in the clay of interest.

It is the authors' opinion that it is more important to put an effort into reducing the uncertainty in c_h (or c_v for use in model I) than trying to investigate s and m in ordinary engineering projects. It is also important to consider the change in c_h that occurs as a result of the decrease in void ratio as consolidation of the clay proceeds (e.g. via model III).

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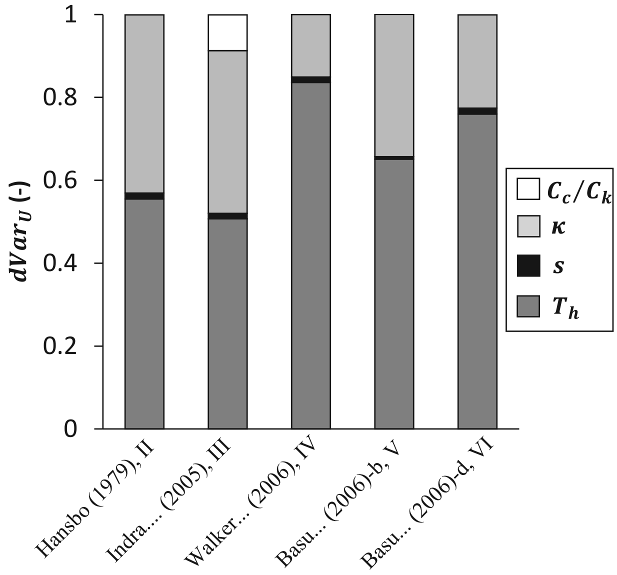

Figure 4. The contribution to Var_U of the variances in T_h, s, κ and C_c/C_k.

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