Creep and long-term bearing capacity of a long pile in clay

Fluage et capacité portante à long terme d'un long pieu dans de l'argile

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ABSTRACT: A rheological equation is proposed to describe shear deformations in party saturated hardening-softening clay soil, based on modification of Maxwell rheological model. It is shown that the proposed equation describes creep, relaxation and kinematic shear with identical parameters, including decaying, stable and progressive creep, depending on shear stress intensity.

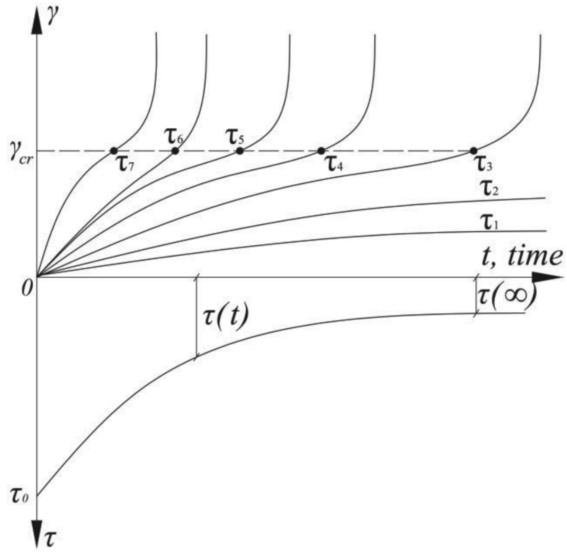
RÉSUMÉ : Une équation rhéologique est proposée pour décrire les distorsions au sein d'une argile partiellement saturée, avec durcissement et radoucissement, basée sur une modification du modèle rhéologique de Maxwell. Il est montré que l'équation proposée décrit le fluage, la relaxation et le cisaillement cinématique avec des paramètres identiques, y compris la diminution stable et progressive du fluage, en fonction de l'intensité de la contrainte de cisaillement.

KEYWORDS:long-time bearing capacity, hardening, softening, kinematic shear,stress relaxation, progressive creep.

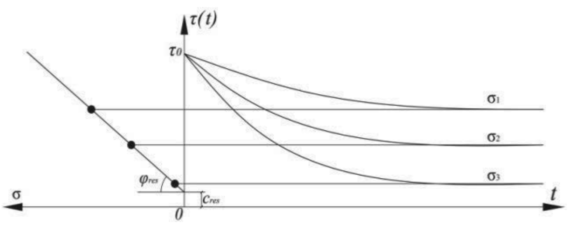
1 INTRODUCTION

1.1 *Investigation of clay soils in shear [1…16] shows that rheological curves can be summarized for static loading ($\tau = const$) as creep curves (Fig.1, a); for kinematic loading ($\dot{\gamma} = const$, $\dot{\varepsilon}_1 = const$) as curves $\tau(t) = f(\dot{\gamma}, \sigma)$ (Fig. 1, b); for given fixed strain value ($\gamma(0) = const$, $\tau(t) \neq const$) as relaxation curves $\tau(t) = f(\gamma_0, \sigma)$ (Fig. 1, c), with τ and σ as shear and compression stresses, γ and $\dot{\gamma}$ as shear strain and its rate, ν – as shear strain rate, t - time.*

In each case the curves are quantified as empirical dependencies, based on rheological models [1…16].



(a)



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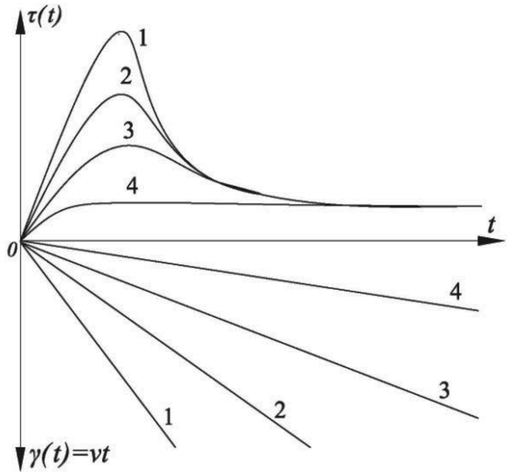


(c)

Fig.1. Rheological curves:

a) creep and long-term strength, ($\tau_1 < \tau_2 \ldots < \tau_7$ – critical values of τ in case of γ_{cr} = const);
b) shear stresses τ(t) for different shear rates $\dot{\gamma} = const$, and $\dot{\gamma}_1 > \dot{\gamma}_2 > \dot{\gamma}_3 > \dot{\gamma}_4$;
c) stress relaxation for different values of σ (right) and limit straight line of residual (long-term) shear strength (left)

The paper presents a rheological equation to describe shear deformations of clay soil with strong rheological properties and Maxwell rheological creep threshold τ^* for strengthening and softening, depending on accumulated shear strain (Fig. 2).

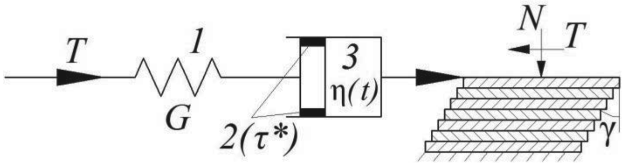


Fig. 2. Maxwell rheological model, having strengthening, softening and shear structural strength properties:
1 – elastic element, 2 – structural strength, 3 – viscous element.

The concept of simultaneous strengthening and softening of deforming clay soil was repeatedly stressed by S.S.Vyalov, M.N.Goldstein, G.I.Ter-Stepanayan, and it was also proved by experiments [1…5].

While summarizing these investigations S.S. Vyalov [1] emphasized that soil creep is accompanied by mutually opposite events of soil hardening and softening. If hardening dominates, then it leads to decreasing of deformations, if softening then it leads to failure. And he developed a kinematic theory of soil strength and creep, based on Ya.I.Frekel molecular theory of soil flow.

The equation below relates to the flow theory, in which strain rate isthe sum of elastic $\dot{\gamma}^{e}$ and visco-plastic $\dot{\gamma}^{vp}$ strain rates i.e., $\dot{\gamma}=\dot{\gamma}^{e}+\dot{\gamma}^{vp}$ where viscosity and cohesion variation rates versus time are taken into account:

$$\dot{\gamma}=\frac{\tau-\tau^{*}}{\eta(t)}\cdot\left(\frac{e^{\alpha\gamma}}{a}+\frac{e^{\beta\gamma}}{b}\right)+\frac{\dot{\tau}}{G} \quad (1)$$

1.2 *witha ,b , α , β as strengthening and softening parameters, G as shear modulus; τ^{*} as creep threshold:*

$$\tau^{*}=\sigma`tg\varphi+c(t) \quad (2)$$

with σ` as effective stress, c(t) as time dependent cohesion.

For triaxial compression eq. (1) looks similar if index i is added to all parameters that means transfer to strain ratesγ_i due to shear stresses τ_i , τ_i^{*} andσ`. Consider rheological processes on the basis of eq. (1) below.

2 CREEP AND LONG-TERM STRENGTH

Analysis of eq. (1) with constant cohesion ratio ($c(t)=const$) and volume deformation showed that at flexure points of creep curves (see Fig.1 a, top portion) the critical values of γ_{cr} , based on condition $\ddot{\gamma}=0$, are constant and are described by equations as

$$\gamma cr=\frac{1}{\alpha+\beta}\ln\frac{a\beta}{\alpha b}=const \quad (3)$$

with respective stresses $\tau_{cr}(\gamma_{cr})$ depend on applied τ и γ_{cr}, i.e., $\tau_{cr}=f(\tau,\gamma_{cr})$, $t_{cr}=f(\tau,\gamma_{cr})$.

The creep curve flexure time point t_n can be determined from the curve (see Fig. 1) i.e., from the crossing points of lines $\gamma(t)$ and γ_{cr}=const. Hence, each τ corresponds to τ_{cr} and t_{cr}.Thus, (1) and (3), based on parameters of creep curves, yield long-term strength curve $\tau_n(t_n)$, using parameters τ_0 and τ_∞ (see Fig. 1a, bottom part).

Eq. (1) can be used for analyzing laboratory test data. In order to describe creep in soil mass asin (1) the following equation can be applied:

$$\dot{\gamma}=\frac{\tau-\tau^{*}}{\eta}\left(\frac{e^{\alpha_1 t}}{a_1}+\frac{e^{-\beta_1 t}}{b_1}\right)+\frac{\dot{\tau}}{G} \quad (4)$$

If $\tau=const$:

$$\dot{\gamma}=\frac{\tau-\tau^{*}}{\eta}\left(\frac{e^{\alpha_1 t}}{a_1}+\frac{e^{-\beta_1 t}}{b_1}\right) \quad (5)$$

Solution (5) can be expressed as follows:

$$\gamma(t)=\frac{\tau-\tau^{*}}{\eta}\left(\frac{e^{\alpha_1 t}}{\alpha_1 a_1}-\frac{e^{-\beta_1 t}}{\beta_1 b_1}\right) \quad (6)$$

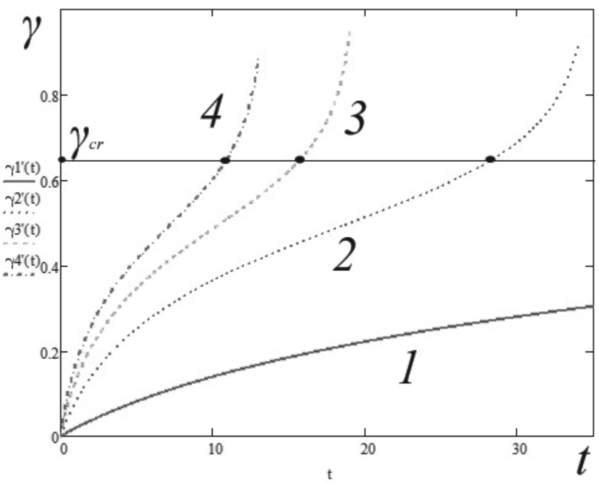


Fig. 3. Curves γ (in decimals) and$t$ (in hours) for clay soil with different values of tangential stresses in simple shear conditions, according to eq. (1) with known parameters α,β, a, b и η and $\tau>\tau^{*}$, $\tau_1<\tau_2<\tau_3<\tau_4$

Calculation as per (5) demonstrates that dependence $\gamma(t)$ features double curvature same as in case (1) i.e., depending on the level of stress τ , and parameters a_1 ,b_1 , α_1 , β_1 that depict decaying, non-decaying and progressive creep (Fig. 3). Such result is due to the difference of exponential functions in brackets in eq. (4), the first of which describes strengthening while the second relates to softening.

Eqs. (1) and(5) are identical, as they give the same results.In order to apply eq. (5) for solving boundary problems it is necessary to determine parameters a_1 ,b_1, α_1, β_1 from experiments that can differ from parameters in Eq.(1).

3 KINEMATIC SHEAR

Soil sample deviator loading is a broadly applied triaxial test, following hydrostatic compression with constant axial deformation rate $\dot{\varepsilon}_1=const$. In simple shear (distortion) under kinematic loading ($\dot{\gamma}=const$) eq. (1) with $\dot{\gamma}=const$ looks, as follows:

$$\dot{\gamma}=\frac{\tau-\tau^{*}}{\eta}\left(\frac{e^{\alpha vt}}{a}+\frac{e^{-\beta vt}}{b}\right)+\frac{\dot{\tau}}{G} \quad (7)$$

with$v$ as angular strain rate $\dot{\gamma}=v=const$

We obtain from eq. (7)

$$\dot{\tau}+\frac{\tau G}{\eta}\left(\frac{e^{\alpha vt}}{a}+\frac{e^{-\beta vt}}{b}\right)=v\eta G+\frac{\tau^{*}G}{\eta}\left(\frac{e^{\alpha vt}}{a}+\frac{e^{-\beta vt}}{b}\right) \quad (8)$$

Solution of this differential equation, obtained numerically with the help of MathCad software for various shear strain values $\dot{\gamma}_1,\dot{\gamma}_2...\dot{\gamma}_n$, enablesplotting a family of curves $\tau(t)$ - γ (Fig. 4). The calculations showed that they haveextreme points at characteristic time t^{cr} =const and acommon asymptote. It is obvious thatfrom those curves wecan plotcurves $\tau_{max}(\sigma)$ и $\tau_{min}(\sigma)$ in case of $\dot{\gamma}=const$.

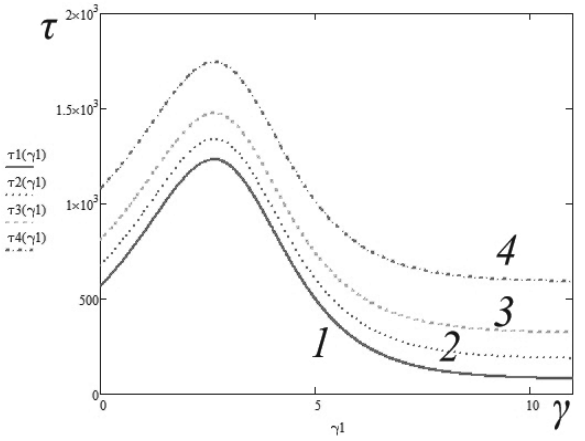


Fig. 4. Curves of τ (kPa), depending on γ *(%)*, in kinematic loading $\dot{\gamma} = const$ at various values of compacting loading σ (7) and $\sigma_1 > \sigma_2 > \sigma_3 > \sigma_4$

4 STRESS RELAXATION

Equation (1) demonstrates a stress relaxation process for $\dot{\gamma} = 0$ i.e., with $\gamma(t) = \gamma(0) = const$ and with initial $\tau(0) = \tau_0 > \tau^*$ and $\tau(t) \neq const$. Solution (1) in this case looks, as follows:

$$\tau(t) = \tau_{res}\left(1 - e^{-At}\right) + \tau_0 e^{-At} \qquad (9)$$

with $$A = \frac{G}{\eta}\left(\frac{e^{\alpha\gamma_0}}{a} + \frac{e^{-\beta\gamma_0}}{b}\right), \tau_{res} = f(\sigma) \qquad (10)$$

τ_{res} as residual strength

Let us determine the limit curve of residual strength from relaxation curves for different values of compressive stresses σ (see Fig.1, c, on the left side).

5 SOME PROBLEMS OF APPLIED SOIL MECHANICS

The problem of a pile interaction with rheological soil can be reduced to determining regularities of constant force N distribution between side resistance and bottom resistance (fig.5) and $N = R(t) + T(t)$

with $N = \pi a_0^2 p_i$, $T = 2\pi a_0^2 l$, $R = \pi a_0^2 p_2$, a_0 , b_0 as pileradius and pile influence area; l as pile length, p_1, p_2 as stresses at pile head and under its tip respectively.

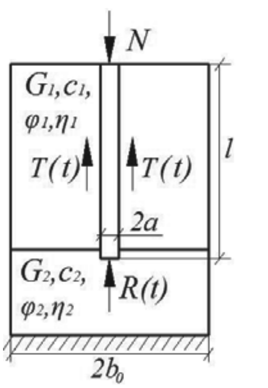


Fig.5 Principal schematic of interaction between pile and 2-layer soil massive, where G, φ, c и η are parameters of deformation, strength and viscosity respectively

In order to solve this problem the pile settlements, caused by forces $T(t)$ and $R(t)$, shall be calculated and then related to the pile deformation modulus E_p that is much greater than the surrounding soil modulus E_s i.e., $E_p >> E_s$. Consider various cases of bi-layer soil with upper layer, having viscoelastic properties as in eq. (4) while the lower one being elastic, viscoelastic, elastic-plastic and viscous.

5.1 *Linear deforming soil under pile tip*

Let us determine pile settlement rate due to friction $T(t)$ from solution, based on the assumption for the shear mechanism of soil displacement around pile with volume deformations being neglected [11]. For $\tau^* = 0$

$$\dot{S}_T = \frac{a\tau_a}{\eta_1(t)}\ln(b_0/a_0) + \frac{a_0\dot{\tau}_a}{G_1(t)}\ln(b_0/a_0) \qquad (12)$$

with $\tau_a = T/2\pi a l$ and $\dot{S}_T$ as pile settlement rate. $\dot{\tau}_a$ - rate of changing τ_a

$$\eta_1(t) = \eta_1 \Big/ \left(\frac{e^{\alpha_1 t}}{a_1} + \frac{e^{\beta_1 t}}{b_1}\right) \qquad (13)$$

The rate of settlement, generated by force $R(t)$ is also determined from solution for a circular stiff plate, pressed in elastic medium

$$\dot{S}_T = \dot{p}_2 \frac{\pi a_0(1-\nu_2)K_1}{4G_2} \qquad (14)$$

With $K(l) \leq 1$ as coefficient, accounting for the depth of load application to the plate; p_2 и $\dot{p}_2$ - applied stress and rate of its changing.

By comparing eq. (12) and eq. (14) with the account of eq. (11) we obtain:

$$\frac{a_0^2(p_1 - p_2)}{2l\eta_1(t)}\ln(b_0/a_0) - \dot{p}_2\frac{a_0^2\ln(b_0/a_0)}{2lG_1} = \dot{p}_2\frac{\pi a\left(1-\nu^2\right)K_1}{4G_2} \qquad (15)$$

After some transformations we get the following differential equation:

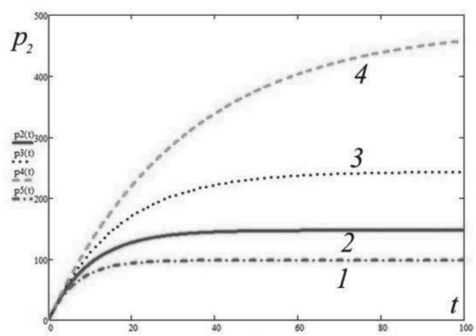
$$\dot{p}_2 + p_2 P(t) = p_1 Q(t) \qquad (16)$$

with

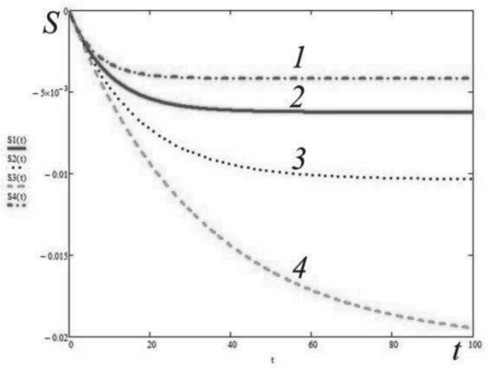
$$P(t) = \frac{B(t)}{A}, Q(t) = \frac{D(t)}{A}; A = \frac{\pi\left(1-\nu^2\right)K_1}{4G_2} + \frac{a_0}{2l}\frac{\ln(b_0/a_0)}{G_1};$$

$$B(t) = \frac{a_0}{2l}\frac{\ln(b_0/a_0)}{\eta_1(t)}; D(t) = \frac{a_0}{2l}\frac{p_1\ln(b_0/a_0)}{\eta_1(t)} \qquad (17)$$

Solution (16) for initial condition $p_2(0) = 0$, obtained with the help of MathCad software, yielded that p_2 varies versus time with different rates and tends to constant values (Fig. 6). The pile settlement is also determined from eq, (14), by introducing $p_2(t)$ instead of $\dot{p}_2(t)$.



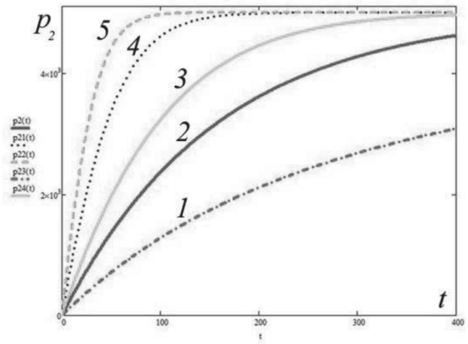
(a)



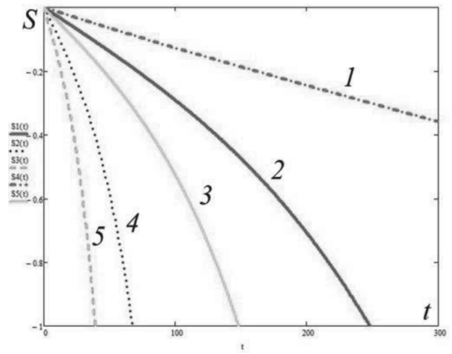
(b)

Fig. 6. Here $p_2(t)$ (kPa) - t (hours)(a) and *s(*in meters*)* (b) curves from (16) and (14) respectively with input parameters from (15)

5.2*Elasto-plastic bed under pile tip*



(a)



(b)

Fig. 7.p_2(kPa) - t (hours) (a) according (19) and *s(meters) - t(hours)* (b) according (18) plots for different viscosity and elasticity parameters of soil around pile and different elasto-plastic parameters of soil under pile tip.

The settlement rate of soils under pile tip is roughly approximated as:

$$\dot{S}_r = \dot{p}_2 \frac{\pi a(1-\nu_2)}{4G_2} \frac{p_2^*}{p_2^* - p_2} \quad (18)$$

With p_2^* as limit load on soil bed, determined from known solutions [11].

Eq. (1) yields that $p_2 \to p_2^*$ if $\dot{S} \to \infty$

Comparison ofeqs. (4) and (18) according (11) yields a differential equation versus p_2:

$$\frac{a_0^2(p_1 - p_2)}{2l\eta_1(t)} \ln(b_0/a_0) - \dot{p}_2 \frac{a_0^2 \ln(b_0/a_0)}{2lG_1} = \dot{p}_2 \frac{\pi a(1-\nu^2)}{4G_2} \frac{p_2^*}{p_2^* - p_2} \quad (19)$$

Analysis of solution (19) showed that p_2 decays versus time at different rates and tends to constant values (Fig. 7) while the settlement can decay or not decay, depending on the intensity of applied load $p_1 = N/\pi a^2$ (Fig. 7b).

6 CONCLUSIONS

1. A rheological equation is proposed to describe soil shear, based on the modified Maxwell model, having clay strengthening and softening parameters.
2. Analysis of the equations has shown that for the case of constant loading it describes decaying, non-decaying and progressing soil creep as well as stress and shear strain relaxation processes in kinematic loading mode.
3. In the pile-soil interaction problem solution the distribution of applied force between the side surface and the lower tip is time-related, and it can result either in decaying or in non-decaying pile settlements, depending on the parameters of soil around the pile and under its tip.

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