Safety theory in geotechnical design of piled raft

Théorie sur la sécurité pour la réalisation de radier sur pieux.

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ABSTRACT: A significant amount of literature describing and discussing the behavior of piled raft foundations have been produced in the last thirty years; nevertheless, a method to introduce the safety in the design of this kind of foundation has not been established yet. Hence, they have been omitted in most Standards and Codes around the world and have been used in just few cases. Until now, the performance of piled rafts has been analyzed using a global factor-of-safety approach. The application of the Limit State Method to geotechnical designs of this kind of foundation is presented with the purpose of establishing a methodology for the calculation of the bearing capacity of piled rafts. The partial coefficients necessary to describe the safety in this method are defined for the ultimate limit state. The application of the Safety Theory and probabilistic methods in the geotechnical design is presented, as well as the mathematic formulation for its implementation in piled raft foundation. A methodology for the adjustment of partial coefficients (to be used for the method of limit states) based on probabilistic methods is proposed. The expressions for the adjustment to be used in piled raft foundations using the selected design method are finally presented.

RÉSUMÉ : Une importante partie de la littérature décrivant et examinant le comportement de radier sur pieux a été réalisée au cours des trente dernières années. Néanmoins, une méthode pour introduire la sécurité dans la conception de ce type de fondation n'a pas encore été établie. En conséquence, on remarque son oubli dans la plupart des normes et des codes dans quasiment tous les pays et on relève son utilisation dans très peu de cas. Jusqu'à présent, la sécurité du radier sur pieux a été analysée en utilisant un facteur global permettant l'approche sécuritaire. L'application de la méthode des états limites pour la conception géotechnique de ce type de fondation est présentée dans le but d'établir une méthodologie de calcul de la capacité portante du radier sur pieux. Les coefficients partiels nécessaires pour incorporer la sécurité dans cette méthode sont définis pour les états limites ultimes. L'application de la théorie basée sur la sécurité et l'utilisation des méthodes probabilistes dans la conception géotechnique sont explicitées, ainsi que les formules mathématiques pour sa mise en œuvre en ce qui concerne les radiers sur pieux. Une méthodologie pour l'ajustement des coefficients partiels (à utiliser pour la méthode des états limites) basées sur des méthodes probabilistes est proposée. Enfin, les expressions pour l'ajustement à utiliser dans le cas des radiers sur pieux, en utilisant une méthode de conception spécifique, sont obtenues.

KEYWORDS: pile raft, safety theory, limit state, probabilistic method, partial coefficient.

1 INTRODUCTION

After the publication of the Technical Committee Report TC18 of the International Society of Soil Mechanics and Geotechnical Engineering (ISSMGE) in 2001, the use of piles as settlement reducers has increased, with the corresponding design savings. Nevertheless, this kind of foundation is seldom used, because of the complexity generated by the analysis of the load transfer mechanism between the piles and the raft, and also because of the difficulty in determining the *load-settlement* curves. All this mechanisms are influenced by the interaction between the elements (raft-piles-soil) of the system. (Sales, 2000).

It´s worth adding that, despite the TC18, the lack of Design Codes for piled raft foundation (PR) in many countries has had a great influence in the slow pace incorporation of this kind of foundation in engineering projects (Ahner, Soukhov, & König, 1997).
Based on the concepts of Limit States, most of the engineering schools (and Standards) use *partial coefficients* as safety approach. Notwithstanding, this is not applied in foundation design (Quevedo 2002, Eurocode 7 2005) and let alone in deep foundations in which the *global safety factor* approach is still used in most codes and standards. Some of the disadvantages of this approach are: a) it does not explicitly take into account the material´s variability y; b) it uses nominal loads, while the structural design uses design loads, i.e. characteristic loads affected by weighting coefficients (Caneiro 2003).
The definition of a global safety factor for a PR is very difficult, because it has to involve both the raft bearing capacity and the corresponding pile group capacity. A rigorous safety analysis has to take into account the interaction of these distinct elements (raft and piles) with the incorporation of the soil.

2 CALCULATION METHODS FOR THE ULTIMATE BEARING CAPACITY

The design of any foundation has two main stages: first, the bearing capacity analysis (ultimate state) and second, the design settlement estimation (service state). This article focuses in safety studies related to the ultimate (state) stage which, according to Mandolini (2003), is the stage that generally governs the design of PRs with a raft breath between 6 to 14m.
Methods for evaluating the bearing capacity of PR are based in empirical correlations or in numerical analyses calibrated by field tests. These are very simple methods, but they can only be used in similar conditions to those where the in situ tests or instrumentation were carried out.
One of the more recent methodologies for the PR design is the one developed by Sanctis & Mandolini (2006). The authors

carried out a series of numerical analyses and proposed the following equation:

$$Q_{PR} = \alpha_{UR} \cdot Q_{UR,ult} + Q_{PG,ult} \quad (1)$$

Q_{PR} , Q_{UR} and Q_{PG} are the ultimate bearing capacities of the PR, the isolated raft and the isolated pile respectively; α_{UR} is a coefficient introduced to affect the raft bearing capacity when it is considered to be working in the pile-raft system.

Numerical models were generated inducing settlements of 10% of the raft width, which are considered to be capable of mobilizing all the strength capacity of the system. The results were used to make out a numerical correlation to obtain the relation between α_{UR} and the geometry of the PR, which led to the following expression:

$$\alpha_{UR} = 1 - 3\left(\frac{A_G}{A} \Big/ \frac{s}{d}\right) \quad (2)$$

where A_G is the area enclosed by the piles; A is the raft area; s and d are the pile spacing and diameter respectively.

3 APPLICATION OF THE LIMITE STATE METHOD (LEM) TO THE PILED RAFT FOUNDATION DESIGN

The application of the Limit State Method (LEM) to the PR design to be described in this article is based on a design in which the piles behave as floating piles under ultimate or limiting capacity. It is one of the options that generate the greatest efficiency of the system, because it considers the piles working at their ultimate creep stages (Randolph 1994). This forces an analysis of the PR rather as a system than as independent elements, since the piles will be working at their ultimate stages and will consequently not satisfy any of the safety criteria established in pile design codes. Off course this obliges the contribution of the raft into design. Also, the safety coefficient of the mechanical characteristics of the soil can be applied to the mean values, as suggested by Quevedo (2002).
Figure 1 shows how LEM concepts are introduced in the PR design. The resisting loads function Y_2 is determined as the sum of the raft average bearing capacity plus the respective one of the pile group, as shown by the following equation:

$$Y_2 = Y_{2,PG} + Y_{2,R} \quad (3)$$

where $Y_{2,R}$ and $Y_{2,PG}$ are the raft and piles bearing capacity, respectively, calculated with average values of soil properties.
The design load function of each element of the system can be obtained from the following equations:

$$Y^*_{2,R} = \frac{Y_{2,R}}{\gamma_{g,R}} \text{ , } Y^*_{2,PG} = \frac{Y_{2,PG}}{\gamma_{g,PG}} \quad (4)$$

where $Y^*_{2,R}$ and $Y^*_{2,PG}$ are raft and the pile´s resistant loads, respectively. They can be calculated from the design values of the soil mechanical properties.
By mean of the former definitions and Figure 1 it is possible to obtain the design equation for the ultimate limit state of a PR.

$$P^* \le \frac{Q^*_{PR}}{\gamma_S} \quad (5)$$

where P^* is the overall vertical design load; γ_S is a partial coefficient that considers the quality of the construction and the type of failure; Q^*_{PR} is the design load capacity of the PR obtained from equation 6.

$$Q^*_{PR} = \alpha_{UR} \cdot Q^*_{UR} + Q^*_{PG} \quad (6)$$

where Q^*_{UR} and Q^*_{PG} are the raft and the piles design bearing capacity respectively, calculated separately from the soil design values.
The overall vertical design load can be obtained as a sum of the individual characteristic load weighted by a particular coefficient, as shown in the following equation:

$$P^* = \sum \left(P_{ki} \cdot \gamma_{fi} \right) \quad (7)$$

where P_{ki} is the characteristic loads and γ_{fi} are the weighting coefficients for the loads.

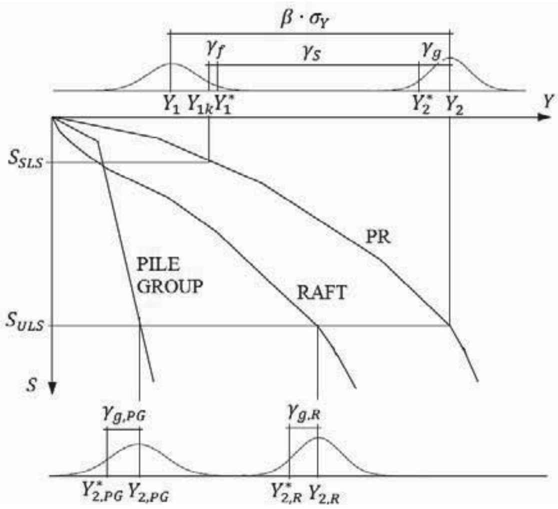

Figure 1. Introduction of safety by the LEM in PR (Lorenzo 2010)

In this article, the resistant load function and its design values are obtained in a manner that differs from the way they are normally obtained in structural analyses. As suggested by Quevedo (1987), Becker 1996), Gonzalez-Cueto (2000) and Lima (2006), the soil properties coefficients are applied directly to the mean values, while in structural design they are applied to the characteristics values.
The design soil properties are calculated from values obtained from the statistical processing of laboratory tests results, weighting them by partial coefficients, as shown in equations 8, 9 and 10.

$$\phi^* = \tan^{-1}\left(\frac{\tan\phi}{\gamma_{gtg\phi}}\right) \quad (8)$$

$$C^* = \frac{C}{\gamma_{gC}} \quad (9)$$

$$\gamma^* = \frac{\gamma}{\gamma_{g\gamma}} \quad (10)$$

where ϕ, C and γ are the mean values of the soil internal friction angle, cohesion and specific weigh, respectively; and $\gamma_{gtg\phi}$, γ_{gC} , $\gamma_{g\gamma}$ are the partial safety coefficients that affect each soil property.

4 APLICATION OF SAFETY THEORY TO THE GEOTECHNICAL DESIGN OF PRF

By means of probabilistic methods it is possible to calibrate the partial safety coefficients to be used in the LEM. This approach has not yet been introduced in the design codes. Consequently, the loads and resistance partial coefficients have not been establish as well. Nevertheless, the necessary expressions for the calibration of partial coefficients to use in the LEM can be developed by means of the general procedure described in Quevedo (1987). This author applied the procedure to the design of shallow foundations in cohesive soils and

subsequently, Gonzalez-Cueto (2000) applied it to the design of shallow foundations in frictional soils. Lima (2006) used the approach in the design of drilled caisson's foundation and anchorages.
The procedure to calibrate the weighting coefficients by this methodology requires the calculation of the global safety factor ($F_{s,d}$) and the optimal safety factor. The global safety factor is associated with the partial coefficients introduced in the design by the LEM. The optimal safety factor is determined with the application of the probabilistic methods.

1.1 *Global Design Safety Factor*

The weighting coefficient that are applied to adjust the soil specific weight and the mechanical properties of the soil can be respectively calculated from equations 12 and 13.

$$\gamma_{gi} = \frac{1}{1 - \frac{t_\alpha v_i}{\sqrt{n_i}}} \quad (12)$$

$$\gamma_{gi} = \frac{1}{1 - t_\alpha v_i} \quad (13)$$

Where t_α is the t-student function value for α = 0.95 at n-1 degrees of freedom; v_i is the coefficient of variation of the *i* soil property; n_i is the amount of repetitions of the test where the *i* soil property was obtained.
The partial safety coefficient of the resistant loads can be obtained from the following equation:

$$\gamma_g = \frac{Q_{PR,ult}}{Q_{PR}^{*}} \quad (14)$$

Where γ_g is the aforementioned coefficient.

In order to calculate $Q_{PR,ult}$ and Q_{PR}^{*} Equations 1 and 6 can be used, respectively.

The weighting coefficients of the loads can be calculated from equation 15.

$$\gamma_f = \frac{\sum P_{ki} \cdot \gamma_{fi}}{\sum P_{ki}\left(1 - t_\alpha \cdot v_i\right)} \quad (15)$$

Finally, the global design safety factor can be obtained as:

$$F_{s,d} = \gamma_f \cdot \gamma_g \cdot \gamma_s \quad (16)$$

1.2 *Optimal Safety Factor*

In order to obtain the optimal safety factor $\left(F_{s,o}\right)$ it is necessary to deal with equations that are more complex than those used to obtain the global safety factor. The load function (Y_1) and the resistant capacity function $\left(Y_2\right)$ can be defined by the following equations:

$$Y_1 = P \quad (17)$$

$$Y_2 = Q_{PR,ult} \quad (18)$$

Where *P* is the mean value of the loads.
In the design of a PR, the required safety level $\left(H_{req}\right)$ is related to the $F_{s,o}$ by means of the equation 19.

$$H_{req} = 0.5 + \phi_n\left[\frac{F_s - 1}{\sqrt{v_P^2 + F_S^2 \cdot v_{QPR,ult}^2}}\right] \quad (19)$$

Where ϕ_n is the error function, v_P and $v_{Q_{PR,ult}}$ the coefficient of variation of the loads and the ultimate bearing capacity, respectively.

As v_P and $v_{Q_{PR,ult}}$ are unknown, they can be expressed as:

$$v_P = \frac{\sigma_P}{P} \quad (20)$$

$$v_{Q_{PR,ult}} = \frac{\sigma_{PR,ult}}{Q_{PR,ult}} \quad (21)$$

where σ_P and $\sigma_{Q_{PR,ult}}$ are the standard deviation of the total vertical load and the ultimate bearing capacity of the PR, respectively.
Taking into account that the total load is the result of the sum of independent loads, which are values that represent random variables, the standard deviation of the total vertical load can be obtained as the sum of the standard deviation of each load, as shown by the following equation:

$$\sigma_P^2 = \sum \sigma_{Pi}^2 \quad (22)$$

where σ_{Pi} is the standard deviation of the *i* vertical load.
The bearing capacity of the PR is a function that depends on several random variables. This increases the complexity of the calculation of the standard deviation. As shown in Lorenzo (2010), it is possible to apply the Taylor`s series method where the linearization of the function is combined with application of the general theorem of standard deviation. With this, the standard deviation can be expressed by:

$$\sigma_{Q_{PR,ult}}^2 = \left(\frac{\partial Q_{PR,ult}}{\partial \tan\phi}\right)^2 \sigma_{\tan\phi}^2 + \left(\frac{\partial Q_{PR,ult}}{\partial C}\right)^2 \sigma_\gamma - 2\left(\frac{\partial Q_{PR,ult}}{\partial \tan\phi}\right)\left(\frac{\partial Q_{PR,ult}}{\partial C}\right)\sigma_{\tan\phi}\sigma_C r_{C\cdot\tan\phi} \quad (23)$$

where $r_{C\cdot\tan\phi}$ is a correlation coefficient between the cohesion and the tangent of the friction angle.
The derivatives on Equation 23 can be obtained from the bearing capacity equations presented in (CTE 2006). This leads to equations 24 to 29. In order to simplify the expressions, the depth coefficients of the bearing capacity equations are considered to be 1. This does not introduce a significant error, because the contribution of the soil above the foundation is a small percent of the total load. Equations 24 to 26 can be used in a drained type analysis. These expressions become more simple for cohesive soils in an undrained analysis, leading to equations 27 to 29, as follows.

$$\frac{\partial Q_{PR,ult}}{\partial \tan\phi} = \alpha_{UR} A\left(C \cdot S_C \frac{dN_c}{d\tan_\phi} + \gamma_1 \cdot d \cdot S_q \cdot \frac{dN_q}{d\tan\phi} + \frac{1}{2} B \cdot \gamma_2 \cdot S_\gamma \cdot \frac{dN_\gamma}{d\tan\phi}\right) + n\eta\left[A_P \cdot \sigma'_{Vp} \cdot \frac{dN_q}{d\tan\phi} + A_F \cdot \sigma'_V \cdot k \cdot f\right] \quad (24)$$

$$\frac{\partial Q_{PR,ult}}{\partial c} = \alpha_{UR} \cdot A\left[S_C \cdot N_C\right] \quad (25)$$

$$\frac{\partial Q_{PR,ult}}{\partial \gamma} = \alpha_{UR} \cdot A\left[d \cdot N_q \cdot S_q + \frac{1}{2} \cdot B \cdot S_\gamma \cdot N_\gamma\right] + n\eta\left(A_p \cdot N_q \cdot L + A_F \cdot L \cdot k \cdot f\right) \quad (26)$$

$$\frac{\partial Q_{PR,ult}}{\partial \tan\phi} = 0 \quad (27)$$

$$\frac{\partial Q_{PR,ult}}{\partial c} = 5{,}14 \cdot \alpha_{UR} \cdot A + n\eta\left[A_p \cdot F_E + A_F \cdot \frac{100 C_U\left(C_U + 200\right)}{\left(C_U + 100\right)^2}\right] \quad (28)$$

$$\frac{\partial Q_{PR,ult}}{\partial \gamma} = 0 \quad (29)$$

The standard deviation of the soils properties can be expressed by equations 30 to 32.

$$\sigma_{\tan\phi} = v_{\tan\phi} . \tan\phi \quad (30)$$

$$\sigma_c = v_c . C \quad (31)$$

$$\sigma_\gamma = \nu_\gamma . \gamma \qquad (32)$$

Some reference guiding values for the coefficient of variation of the geotechnical parameters are presented in Table 1.

Table 1. Reference values of geotechnical properties of de soil.

Parameter	Coefficient of variation
Specific weigh	0.05
Tangent of friction angle	0.07
Cohesion	0.1
Shear undrained resistance	0.15

In this way, it is possible to obtain the coefficient of variation of the load (ν_p) and the ultimate bearing capacity of the PR $(\nu_{Q_{PR,ult}})$. With this and a defined safety level, the $F_{s,o}$ can be calculated. Quevedo (2002) recommended to use a safety level $H = 0,98$, in the geotechnical designs by the ultimate limits states. It means a failure probability of 0.02.
In order to know if the partial safety coefficients used in a specific design are appropriated, a comparison between $F_{s,o}$ and $F_{s,d}$ has to be done. Generally, the coefficients used in regular designs are conservative and they have to be calibrated to find one or more combinations that make $F_{s,o}$ equal to $F_{s,d}$.
In practice, because the variability of the load is lesser known, or measured, than the geotechnical parameters, it is better to reduce the weighting coefficient of the parameters of the soil. Also, it is easier for engineers to use the same weighted load for structures and foundations projects alike. Figure 2 shows the algorithm that resumes the method of calibration.

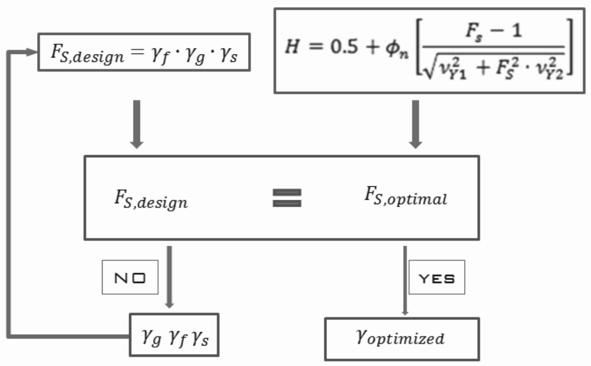

Figure 2. Algorithm for obtaining safety factors by LEM (Lorenzo 2010)

5 CONCLUSIONS

The cost of the (more) rational approach for the foundation design methods is the increase of the complexity level. Nevertheless, this cost is compensated by a proportional advantageous decrease of the execution costs. Besides, it will enhance the understanding of the design and yield a better assurance of its related variables. Thus, the use of the safety theory is the first step to obtain a more economical and rational design.
The PRF analysis approach for a foundation system leads to an effective and optimized use of its components (Cunha et al. 2001), as it allows the raft-soil contact contribution both in the overall stiffness and load capacity of the system. This analysis is a generalization of the calculation methods for determining the bearing capacities of raft and pile groups separately.
Based on the method proposed by Sanctis & Mandolini (2006), a methodology for the application of the LEM in the design of PR was established. The use of three sets of partial coefficients allow to separately consider the uncertainties introduced in the design of the materials, the loads and the working conditions.
By means of the methodology described by Quevedo (2002) it is possible to calibrate the partial safety coefficients which are necessary for the application of the LEM in the design of PRF systems. This makes it possible to better define and understand the safety level to be achieved in design.

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