A practical method for the non-linear analysis of piled rafts

Une méthode d'analyse pratique pour déterminer la réponse non linéaire des fondations mixtes de type radier sur pieux

Basile F.
Geomarc Ltd, London, United Kingdom

ABSTRACT: The paper describes a practical analysis method for determining the response of piled rafts. The key feature of the method lies in its capability to provide a non-linear complete boundary element solution of the soil continuum, while retaining a computationally efficient code. The validity of the proposed analysis is demonstrated through comparison with alternative numerical solutions and field measurements. Examples are given to demonstrate the importance of considering soil nonlinearity effects in piled rafts (given the relatively high load level at which the piles operate), thereby leading to more realistic predictions of the raft and pile response. The negligible computational costs make the analysis suitable not only for the design of piled rafts supporting high rise buildings (generally based on complex and expensive 3D FEM or FDM analyses) but also for that of bridges and ordinary buildings.

RÉSUMÉ: Cet article décrit une méthode d'analyse pratique pour déterminer la réponse des fondations mixtes de type radier sur pieux. La principale caractéristique de la méthode réside dans sa capacité à fournir une solution non linéaire de type « Boundary Element » du continuum sol, tout en conservant un code de calcul efficace. La validité de l'analyse proposée est démontrée par comparaison avec d'autres solutions numériques et des mesures *in situ*. Des exemples sont donnés pour démontrer l'importance de la prise en compte de la non-linéarité du sol dans l'analyse des radiers sur pieux, ce qui conduit à des prévisions plus réalistes de réponse du radier et pieu. Les coûts négligeables de calcul rendent l'analyse appropriée non seulement pour la conception des radiers sur pieux supportant des immeubles de grande hauteur (basé sur des analyses 3D en éléments ou différences finies, complexes et coûteuses), mais aussi pour celle des bâtiments ordinaires et des ponts.

KEYWORDS: CPRF, piled raft, pile group, non-linear, numerical analysis

1 INTRODUCTION

In conventional foundation design, it is assumed that the applied load is carried either by the raft or by the piles, considering the safety factors in each case. In recent years, an increasing number of structures have been founded on Combined Pile-Raft Foundations (CPRFs), an attractive foundation system which allows the load to be shared between the raft and the piles, thereby offering a more economical solution. In the design of piled rafts, a sufficient safety against geotechnical failure of the *overall* pile-raft system has to be achieved, while the piles may potentially be used up to their ultimate geotechnical capacity. Contrary to traditional pile foundation design, no proof for the ultimate capacity of each individual pile is necessary (Katzenbach 2012). Given the high load level at which the piles operate, consideration of soil nonlinearity effects is essential, and ignoring this aspect can lead to inaccurate predictions of the deformations and structural actions within the system.

Due to the 3D nature of the problem and the complexity of soil-structure interaction effects, calculation procedures for piled rafts are based on numerical analyses, ranging from simplified Winkler approaches (e.g. “plate on springs” methods) to rigorous 3D finite element (FEM) or finite difference (FDM) solutions using available packages. While Winkler models suffer from some restrictions mainly related to their semi-empirical nature and fundamental limitations (e.g. disregard of soil continuity), finite element and finite difference solutions retain the essential aspects of interaction through the soil continuum, thereby providing a more realistic representation of the problem. However, even though 3D FEM and FDM analyses are powerful numerical tools which allow complex geometries and soil behaviour to be modelled, such analyses are burdened by the high computational cost and specialist expertise needed for their execution, particularly if non-linear soil behaviour is to be considered. This aspect restricts their practical application in routine design, where multiple load cases need to be examined and where the pile number, properties and location may have to be altered several times in order to obtain an optimized solution.

In an attempt to provide a practical tool for the designer, the paper describes an efficient analysis method for determining the response of piled rafts. The main feature of the approach lies in its capability to provide a non-linear complete boundary element (BEM) solution of the soil continuum (i.e. the simultaneous influence of all the pile and raft elements is considered), while retaining a computationally efficient code. Validity of the proposed analysis is assessed through comparison with alternative numerical solutions and a published case history. Examples are given to highlight the significance of considering soil nonlinearity effects, thereby leading to more realistic predictions of the raft and pile response.

2 METHOD OF ANALYSIS

The safe and economic design of piled rafts requires non-linear methods of analysis which have the capacity of simulating all relevant interactions between the fondation elements and the subsoil, specifically (1) pile-soil-interaction (i.e. single pile response including shaft-base interaction), (2) pile-pile-interaction (i.e. group effects), (3) raft-soil-interaction, and (4) pile-raft interaction (Katzenbach 2012).

The proposed method is an extension of the BEM formulation employed in the pile-group program PGROUPN (Basile 2003) and widely used in pile design through the software Repute (Bond and Basile 2010). The originality of the approach lies in its ability to provide a complete BEM analysis of the soil continuum (in which all four of the above interactions are modelled), while incurring negligible computational costs. Indeed, compared to FEM or FDM analyses, BEM provides a complete problem solution in terms of boundary values only, specifically at the raft-pile-soil interface. This leads to a drastic reduction in unknowns to be solved for, thereby resulting in substantial savings in computing time and data preparation effort. This feature is particularly significant for three-dimensional problems such as piled rafts and makes the analysis suitable not only for the design of piled rafts supporting high rise buildings (generally based on complex and expensive 3D FEM or FDM analyses) but also for that of bridges and ordinary buildings.

A description of the BEM formulation adopted in PGROUPN for the case of pile groups has been presented by Basile (2003). In a similar fashion, the approach has been extended to include the raft analysis (including its reciprocal interaction with the piles) by discretizing the raft-soil interface into a number of rectangular elements (Fig. 1), whose behaviour is evaluated using the traditional Mindlin solution. Completely general loading conditions (axial, lateral and moments) on the piled raft can be examined, even though only the bearing contribution of the raft is considered (i.e. the raft-soil interface is assumed to be smooth). Similarly to the pile analysis, non-linear soil response is modelled, in an approximate manner, by adopting a hyperbolic stress-strain model within a stepwise incremental procedure which ensures that the specified limiting stresses at the raft-soil interface are not exceeded. Limiting values of raft-soil contact pressure (based on the traditional bearing capacity theory) are set for both compression and tension in order to allow for local bearing failure or lift-off of the raft from the soil.

The proposed PGROUPN analysis is currently restricted to the assumption of perfectly rigid raft. In practice, this assumption makes the analysis strictly applicable to "small" piled rafts (Viggiani et al. 2012), i.e. those rafts in which the bearing capacity of the unpiled raft is usually not sufficient to carry the applied load with a suitable safety margin, and hence the primary reason for adding piles is to increase the factor of safety. This generally involves rafts in which the width (B_r) amounts to a few meters (typically B_r<15m) and is small in comparison to the length (L) of the piles (B_r/L<1). Within this range (whose limits should however be regarded as tentative and indicative only), the raft response may be considered as truly rigid and hence the design should aim at limiting the maximum settlement (being the differential settlements negligible). In practical applications, a simple check on the validity of the assumption of rigid raft may be performed by calculating the raft-soil stiffness ratio (K_{rs}) as defined by Horikoshi and Randolph (1997):

$$K_{rs} = 5.57\frac{E_r}{E_s}\frac{1-\nu_s^2}{1-\nu_r^2}\left(\frac{B_r}{L_r}\right)^{0.5}\left(\frac{t_r}{L_r}\right)^3 \qquad (1)$$

where the subscripts r and s denote the raft and soil properties, respectively, E is the Young's modulus, ν is the Poisson's ratio, B_r is the raft breadth, L_r is the raft length (with $B_r \leq L_r$), and t_r is the raft thickness. For values of K_{rs} > 5-10 the raft can be considered as rigid while a lower limit K_{rs} > 1.5 may be assumed for practical purposes (Randoph 2003). It is however observed that the above definition of K_{rs} does not include the additional stiffening contribution provided by the piles and by the superstructure which in effect increases the raft rigidity. Clearly, for "large" flexible rafts (in which B_r/L > 1 according to the definition by Viggiani), the assumption of rigid raft is no longer valid and the limitation of differential settlement becomes one of the design requirements. It is interesting to note that Poulos (2001) has shown that, except for thin rafts, the maximum settlement and the load sharing between the raft and the piles are little affected by the raft rigidity.

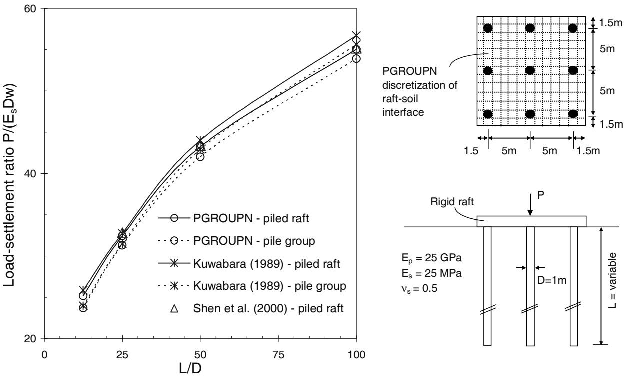

Figure 1. Load-settlement ratio and piled raft analysed

3 NUMERICAL RESULTS

3.1 *Comparison with Kuwabara (1989)*

The accuracy of PGROUPN is initially assessed in the linear elastic range for the piled raft (3x3 group) sketched in Fig. 1. The figure shows the dimensionless load-settlement ratio (P/E_sDw, where P is the total applied load and w is the settlement) of the piled raft for a wide range of pile length-diameter ratios (L/D). For comparison, results from the corresponding free-standing pile group are also reported and show the small influence of the raft contribution to the resulting settlement. However, the load distribution is considerably affected by consideration of the ground-contacting raft, as illustrated in Figure 2 which shows the percentage of the total load carried by the raft and by the corner pile as a function of the L/D ratio. For comparison, the load taken by the corner pile of the pile group is also reported, demonstrating a significant reduction of corner load in the piled raft as compared to the pile group. Both figures show a favourable agreement of PGROUPN with the boundary element solution of Kuwabara (1989) and the variational approach of Shen et al. (2000).

3.2 *Comparison with Poulos (2001)*

The effects of soil nonlinearity are examined in the piled raft (3x3 group) shown in Fig. 3, as reported by Poulos (2001). The non-linear load-settlement response predicted by PGROUPN agrees well with the corresponding settlement value obtained by Poulos using the program GARP (employing a FEM analysis for the raft and a BEM analysis for the piles), under the assumption of rigid raft (i.e. a raft thickness t_r = 1m giving K_{rs} = 6.1), and for a typical design load P = 12 MN (equivalent to an overall factor of safety of 2.15 against ultimate capacity). For consistency with the Poulos analysis, an elastic-perfectly plastic soil model has been adopted in PGROUPN with an assumed raft bearing capacity of 300 kPa and a pile load capacity of

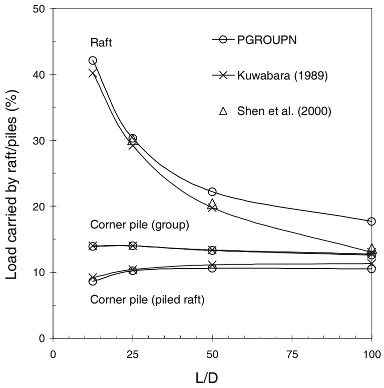

Figure 2. Load sharing between raft and piles

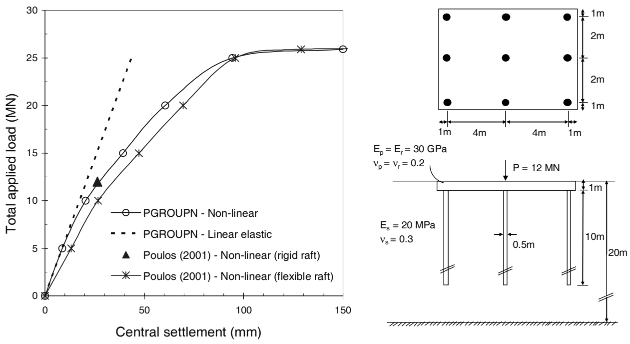

Figure 3. Load-settlement response and piled raft analysed

873 kN in compression and 786 kN in tension. The figure also shows a fair agreement with the load-settlement curve obtained by Poulos for a flexible raft (i.e. t_r = 0.5m giving K_{rs} = 0.8), as previously reported. It is noted that, as the load capacity of the piles becomes nearly fully utilized at a load of about P = 10-12 MN, the load-settlement behaviour reflects that of the raft, which is significantly less stiff than the overall pile-raft system, while the load carried by the raft starts to increase significantly (Fig. 4). As previously observed, the fact that some of the piles (usually the stiffer piles located around the perimeter of the group) are close to their ultimate capacity is not an issue for a piled raft and is actually inevitable for an efficient design.

The load sharing between the raft and the piles as a function of the total applied load reported in Fig. 4 shows a significant reduction of the total load carried by the piles with increasing load level. Under a total load P = 12 MN, the figure shows a good agreement with the load carried by the piles predicted by Poulos for the rigid raft and a slightly less agreement with that obtained for the flexible raft. Overall, the comparison shown in Figs. 3-4 demonstrates the importance of considering non-linear behaviour of the pile-raft system in order to obtain realistic predictions of the settlement and the load sharing between the raft and the piles. Assumption of linear elastic behaviour beyond a load of about 10 MN would lead to an under-estimation of the settlement and an over-estimation of the amount of load carried by the piles, with a consequent over-design of the requirements for structural strength of the piles. As emphasized by Poulos (2001), an analysis which accounts for soil non-linearity, even though in an approximate manner, is preferable to a complex analysis in which linear behaviour is assumed.

3.3 *Design example*

The hypothetical design example shown in Fig. 5 is described in order to demonstrate that, in suitable ground conditions, a significant reduction of the piling requirements can be achieved with the use of a piled raft as compared to a conventional pile foundation. Two foundation systems are evaluated:

1. A 4x4 pile group (i.e. with no raft contribution) designed according to a traditional approach in which an overall (geotechnical) factor of safety FS = 2 is assumed to apply to the maximum axial force of the single pile;
2. A piled raft (3x3 group) in which FS = 2 is assumed to apply to the total force acting on the whole pile-raft system.

A total force E_k = 25 MN is acting on the foundation and a maximum allowable settlement of 25mm has been prescribed. The analyses have been carried out using PGROUPN (non-linear soil model) with the parameters indicated in Fig. 5 (the raft may be considered as fully rigid being K_{rs} = 10.5). The initial solution of an unpiled raft (11m x 11m) has been discarded due to both bearing capacity and settlement requirements, given that the raft bearing capacity is equal to 54.5 MN (based on $q_u = 6C_u$) and the raft settlement results in 38mm. Thus, a pile-group solution is considered and is found that a group of 4x4 piles (30.5m long) at a spacing of $3D$ = 3m is required in order to achieve FS = 2 on the maximum axial force (V_{max}) of the corner pile, (i.e. Q_{all} = 2421 kN > V_{max} = 2390 kN). It is noted that the calculated pile-group settlement is equal to 14mm, i.e. below the allowable value of 25mm, thereby indicating that a design optimization may be achieved.

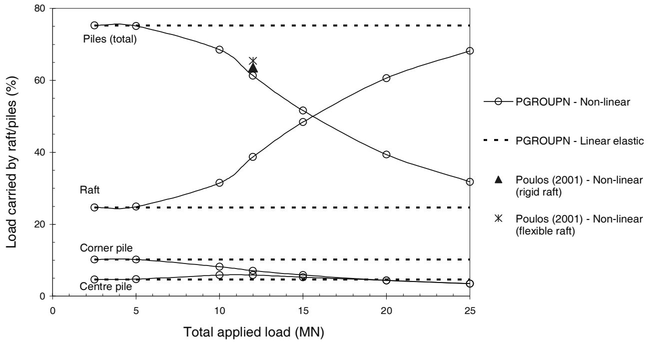

Figure 4. Load sharing between raft and piles

A piled raft solution (3x3 group with pile spacing of $4D$ = 4m and pile length of 20m) is then evaluated following the methodology outlined in the International CPRF Guideline (Katzenbach 2012). According to the guideline, a sufficient safety against failure of the overall pile-raft system is achieved by fulfilling the following inequation:

$$E_d \leq R_d \;\rightarrow\; E_k \cdot \gamma_F \leq \frac{R_{tot,k}}{\gamma_R} \;\rightarrow\; E_k \cdot \gamma_F \cdot \gamma_R \leq R_{tot,k} \qquad (2)$$

where E_k is the characteristic total force acting on the CPRF, γ_F and γ_R are the partial safety factors on actions and resistance, respectively, and the characteristic value of the total resistance $R_{tot,k}$ has to be derived from the load-settlement response of the CPRF and is equal to the load at which the increase of the settlement becomes increasingly superproportional, as determined from a "numerical" load test. In order to allow a direct comparison with the above pile-group solution, it is assumed that an overall FS = 2 applies to the force E_k (this assumption is equivalent to consider a value of $\gamma_F \cdot \gamma_R$ = 2). This implies that Equ. (2) is fulfilled by proving that $R_{tot,k} \geq 2E_k$ = 2·25 = 50 MN. Thus, using PGROUPN, a numerical load test has been performed to generate the typical relationship between the settlement and the total load (i.e. the CPRF overall resistance), as illustrated in Fig. 5. From this figure, it can be seen that, up to the loading of 50 MN, the increase of the settlement is not yet superproportional (i.e. $R_{tot,k}$ > 50 MN), implying that no significant failure of the CPRF has occurred. Thus, the ultimate bearing capacity (ULS) of the piled raft has been proved. It is noted that the maximum pile axial load is equal to V_{max} = 2210 kN, which would give FS = 1.5 (being the pile capacity Q_{ult} = 3358 kN); however, in contrast to conventional pile foundations, the proof of the bearing capacity

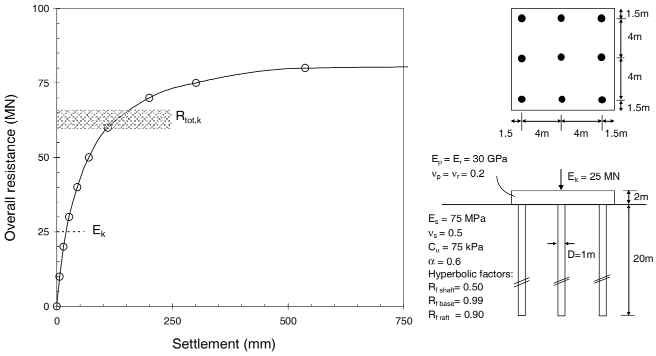

Figure 5. Load-settlement response and piled raft analysed

for single piles is unnecessary because this proof is inconsistent with the concept of piled rafts. Within the same numerical load test, proof of the serviceability limit state (SLS) for the piled raft can be performed and, under the total load of 25 MN, a settlement of 20mm is calculated, i.e. below the allowable value of 25mm. It may be observed that at this load level the raft carries 39% of the total load. Finally, it should be emphasized that the piled raft solution leads to a significant reduction in the required number and length (L) of the piles as compared to the conventional pile group, resulting in a saving of 63% in total pile length, i.e. from 488m for the 4x4 pile group (L = 30.5m) to 180m for the 3x3 piled raft (L = 20m).

4 CASE HISTORY

The case history for the Messe-Torhaus building in Frankfurt is presented (Sommer et al 1985). The building is supported by two separate piled rafts, each with 42 bored piles with a length of 20m and a diameter of 0.9m. The piles under each raft are arranged in a 6x7 rectangular configuration with a centre-to-centre spacing of 2.9m and 3.5m along the shorter and the larger side of the raft, respectively. Each raft is 17.5m x 24.5m in plan, 2.5m thick and is founded at 3m below ground surface.

The piled raft is embedded in the Frankfurt clay and, within PGROUPN, it is assumed that C_u increases linearly with depth from 100 kPa at the foundation level to 200 kPa at the pile base, with a correlation E_s/C_u = 600 and ν_s = 0.5. The same soil parameters were adopted in the variational approach by Chow et al (2001) so that a direct comparison between analyses may be made. For consistency with the non-linear Chow analysis, an elastic-perfectly plastic soil model has been adopted, while a total load of 181 MN is assumed to act on the piled raft (as only approximately 75% of the total structural load of 241 MN was applied at the time of the measurements reported herein). In addition, the following parameters have been assumed (as these were not reported by Chow): an adhesion factor (α) of 0.7 (in order to achieve an ultimate pile load of about 7 MN, given that the measurements showed that piles were carrying at least this amount of load), and a Young's modulus of 23.5 GPa for the piles and of 34 GPa for the raft. The latter value results in K_{rs} = 2.2 and hence the PGROUPN assumption of rigid raft is valid, as confirmed by the field measurements which showed that the raft actually behaved as fully rigid.

The settlement of the piled raft and the proportion of load carried by the raft are reported in Table 1 showing a good agreement between analyses and measurements. In this case, soil nonlinearity appears to have only a relatively small effect on the computed response (at least in terms of settlement and load carried by the raft). The rather low value of the measured load carried by the raft (20%) suggests that the effect normally intended by a piled raft was not realised, thereby indicating a quite conservative design. Indeed, the contact pressures between raft and soil are scarcely larger than those due to the dead weight of the raft (i.e. about 25 MN, resulting in a load proportion of 14%), so that almost the complete load of the superstructure is carried by the piles. It is also noted that, while the aim of reducing settlements of the foundation in comparison to a shallow foundation has been reached (resulting in a reduction of about 50%), a more efficient design could have been achieved using fewer piles of greater length. Indeed, PGROUPN shows that an identical value of settlement can be attained with a significantly smaller total pile length, specifically with 25.5m long piles in a 4x5 group configuration (at a spacing of 5.0m and 5.5m along the shorter and the larger side of the raft, respectively). In this case, a better ratio of the raft-pile load sharing could have been achieved (i.e. 23%) with a saving of 39% in total pile length, i.e. from 840m for the original 6x7 group (L = 20m) to 510m for the 4x5 group (L = 25.5m). Finally, it is noted that PGROUPN non-linear analyses for the 6x7 and 4x5 group configurations run in 3 and 1 min, respectively, on an ordinary computer (Intel Core i7 2.7 GHz), thereby resulting in negligible computing costs for design.

Table 1. Settlement and load proportion carried by raft

	Settlement (mm)	Load carried by raft (%)
Measured (Sommer et al 1985)	45	20
Chow et al (2001)	45	26
PGROUPN	44	21
PGROUPN (linear elastic)	43	21
PGROUPN (4x5 group, L= 25.5m)	44	23

5 CONCLUSIONS

The paper has described a practical analysis method, based on a complete BEM solution and implemented in the code PGROUPN, for determining the non-linear response of piled rafts. The method has been successfully validated against alternative numerical analyses and field measurements.

It has been shown that the concept of piled raft, generally adopted for "large" flexible piled rafts, can also be applied effectively to "small" rigid piled rafts (and to any larger piled raft in which the assumption of rigid raft is valid), making PGROUPN suitable to a wide range of foundations such as bridges, viaducts, wind turbines and ordinary buildings. In such cases, if the raft can be founded in reasonable competent ground (which can provide reliable long-term resistance), then the extra raft component of capacity can be used to significantly reduce the piling requirements which are necessary to achieve the design criteria (e.g. ultimate bearing capacity, settlement).

Given the relatively high load level at which the piles operate within a pile-raft system, the influence of soil nonlinearity can be significant, and ignoring this aspect can lead to inaccurate predictions of the deformations and the load sharing between the raft and the piles. Consideration of soil nonlinearity would also be required if PGROUPN is used to perform a numerical load test following the methodology outlined in the International CPRF Guideline. Due to the negligible costs (both in terms of data preparation and computer execution times), a large number of cases can be analysed efficiently, enabling parametric studies to be readily performed. This offers the prospect of more effective design techniques and worthwhile savings in construction costs.

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