Analysis of Displacements of GPA in Normally Consolidated Soft Soil

L'analyse des déplacements des GPA dans le sol mou Normalement consolidé

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ABSTRACT: Granular piles (GP) offer most effective and economical solution for ground improvement due to their drainage, densification and reinforcement actions, GPs mitigate liquefaction induced damages. An anchor placed at the base of the granular pile and attached to the footing by a cable or rod transfers the applied pullout force to the bottom of the GP termed as Granular Pile Anchor (GPA). The effective stresses in a normally consolidated saturated soil increase linearly with depth. Consequently, the undrained strength and the deformation modulus of the soil increase linearly with depth. Analysis of the displacements of granular pile anchor is presented considering the influence of the linearly increasing undrained modulus of soil and of the GPA with depth on the load – displacement response of the GPA. A parametric study quantifies effects of the length to diameter ratio of GPA, and the relative stiffness of the GP with respect to that of in situ soil at ground level, on the variations of tip and top displacements of GPA with applied load, variation of shear stresses and pullout load with depth, etc.

RÉSUMÉ : Pieux granulaires (GP) offrent une solution plus efficace et économique pour amélioration des sols en raison de leur drainage, la densification et des actions de renforcement, les médecins généralistes atténuer les dommages induits par liquéfaction. Un point d'ancrage placé à la base du pieu granulaire et fixée à la semelle par un câble ou une tige transfère la force de traction appliquée sur le fond de la GP qualifié de mouillage pieu granulaire (GPA). Les contraintes effectives dans un sol normalement consolidé saturé augmente linéairement avec la profondeur. Par conséquent, la résistance non drainée et le module de déformation du sol augmente linéairement avec la profondeur. L'analyse des déplacements de l'ancre empilement granulaire est présentée compte tenu de l'influence de plus en plus le module linéaire non drainée du sol et de l'GPA avec la profondeur de la réponse force - déplacement du GPA. Une étude paramétrique de quantifier les effets de la longueur par rapport au diamètre de l'GPA, et la rigidité relative du GP par rapport à celle du sol in situ au niveau du sol, sur les variations de pointe et des déplacements supérieurs de GPA avec la charge appliquée, la variation de cisaillement contraintes et la charge de retrait avec la profondeur, etc

KEYWORDS: Granular pile anchor, modulus of deformation, homogenous ground, displacements, load transfer.

1 INTRODUCTION.

Granular piles (GP) offer most effective and economical solutions in soft marine clays to counter-act low undrained shear strength and stiffness of the deposits. GPs improve the performance of ground by reinforcement, densification, increasing bearing capacity and resistance to liquefaction by increasing strength and stiffness of ground. GPs are ideally suited as they form elements of low compressibility and high shear strength. The effective stresses in a normally consolidated saturated soil increase linearly with depth. As a result the undrained shear strength and deformation modulus of the soil also increase linearly with depth. The increase in modulus of soil and granular material with depth result in reduced load-displacements response and increased confinement pressure.

The functional utility of the GP in compression is extended by placing an anchor at its base to transfer the pullout load or uplift forces to the base and the assembly is termed as Granular Pile Anchor (GPA). Granular pile treated expansive soil adjusts itself to changes in moisture better than an untreated-soil (Phani Kumar *et al.,* 2004). White *et al.* (2001) studied the application of reinforced geopiers for resisting tensile loads and settlement control. Lillis *et al.* (2004) reported results from in situ tests on pullout response of GPA. Kumar *et al.* (2004) present results from laboratory and field tests on pullout response of GPA in cohesive and cohesionless soils. A linear analysis of displacements of GPA is presented by Madhav *et al.* (2008).

2 PROBLEM DEFINITION

A granular pile of length, L, and diameter, d, with the soil and pile material characterized by moduli of deformation E_s and E_{gp}, and unit weights of γ_s and γ_{gp}, respectively is considered (Fig.1). A force, P_o, applied at the base of GPA is resisted by the shear stress, τ, acting along the periphery of the pile. The force and the stresses acting on the GPA are depicted in Figure 2a. The stresses transferred to the in situ soil are shown in Figure. 2b. The non-homogeneities of deformation moduli of soil, E_s and granular material, E_{gp} (Fig. 3 & 4)are defined by the parameters, α_s & α_{gp}, to quantify the rate of increase of deformation moduli of in-situ soil and granular material with depth. The Poisson's ratio of the soil is ν_s. In order to evaluate the upward displacements of the elements of the soil adjacent to the GPA

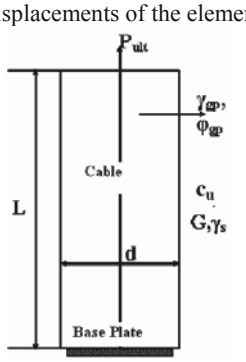

Fig. 1 GPA under Pullout

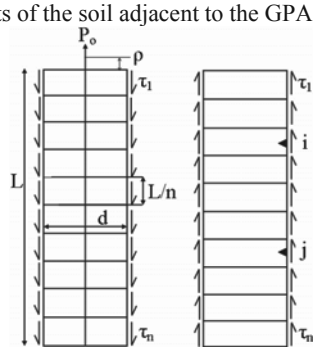

Fig. 2 Forces and Stresses acting on GPA and Soil.

due to the boundary stresses, τ, the GPA surface is divided in to 'n' elements of length, ΔL (=L/n). The stress acting on a typical

element, j, is τ_j. The displacement at the centre of an element, i, due to stresses acting on element, j, are obtained by the method described by Poulos and Davis (1980).

Integrating numerically, the Mindlin's equation for a point load in the interior of a semi-infinite elastic continuum over the cylindrical periphery of the element, the displacement, $\rho_{s,ij}$, of the soil adjacent to the centre of the i^{th} element due to stress, τ_j, acting on the element, j, considering deformation modulus, E_s, of the soil, increases linearly with the depth as $E_s(z)=E_{so}\left(1+\alpha_s \frac{z}{L}\right)$ is obtained as

$$\rho_{S,ij}=\frac{d}{E_{so}}.\frac{I_{S,ij}}{\left(1+\alpha_s.\left(\frac{z_i}{L}\right)\right)}.\tau_j \quad (1)$$

where $I_{s,ij}$ – is the soil displacement influence coefficient. The total soil displacement, $\rho_{s,i}$, adjacent to node 'i' due to stresses on all the elements of the GPA, is obtained by summing up all the displacements at node 'i', as

$$\rho_{S,i}=\frac{d}{E_{s0}}\sum_{j=1}^{n}\frac{I_{S,ij}}{\left(1+\alpha_s.\left(\frac{z_i}{L}\right)\right)}\tau_j \quad (2)$$

The vertical soil displacements adjacent to all the nodes are collected to arrive at

$$\{\rho_S\}=\frac{d}{E_{so}}\left[I_S'\right]\{\tau\} \quad (3)$$

where $\{\rho_s\}$ and $\{\tau\}$ are respectively the soil displacement and stress vectors of size, n, and elements of the matrix $\left[I_S'\right]$ are $I'_{S,ij}=I_{S,ij}/\left(1+\alpha_s.z'_i\right)$ - non-dimensional soil displacement influence coefficient of GPA, where $Z'_i=Z_i/L$- normalized depth at i^{th} element.

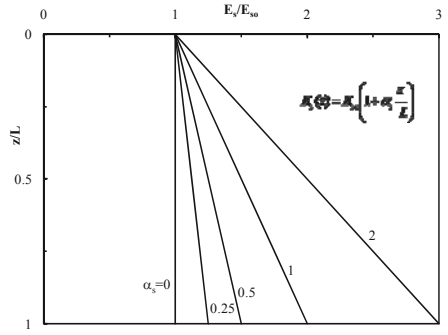

Fig. 3 Variation of E_s with Depth – Effect of α_s

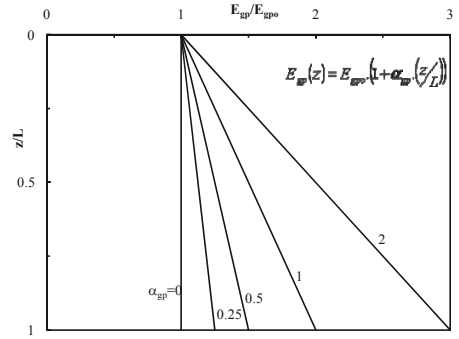

Fig. 4 Variation of E_{gp} with Depth – Effect of α_{gp}

2.1 DISPLACEMNTS OF GPA

The vertical displacements of GPA are obtained considering it to be compressible. Figure 5 depicts the stresses on an infinitesimal element of GPA of thickness, Δz. Poulos and Davis (1980) have established that lateral/radial stresses have negligible effect on the vertical displacements. Considering the deformation modulus of the granular material, E_{gp}, to increase linearly with depth, z, as

$$E_{gp}(z)=E_{gpo}.\left(1+\alpha_{gp}.\left(\frac{z}{L}\right)\right) \quad (4)$$

The equilibrium of forces in the vertical direction reduces to

$$\frac{d\sigma_z}{dz}+\frac{4}{d}\tau=0 \quad (5)$$

where σ_z is the normal stress in to the GPA. The stress-strain relationship for GPA material, is

$$\sigma_z=E_{gp}.\varepsilon_z=-E_{gp}.\frac{d\rho_{gp}}{dz} \quad (6)$$

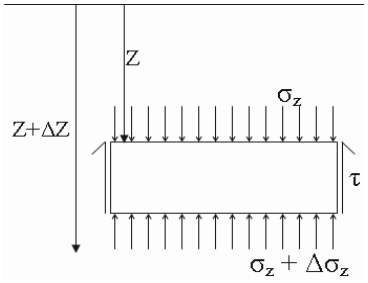

Fig. 5 Stresses acting on an Infinitesimal Element

where ε_z and ρ_{gp} are respectively the axial strain and GPA displacement. Substituting for modified E_{gp} the stress-strain Equation 6 is modified as

$$\sigma_z=E_{gp}.\varepsilon_z=-E_{gpo}\left(1+\alpha_{gp}.\left(\frac{z}{L}\right)\right).\frac{d\rho_{gp}}{dz} \quad (7)$$

Differentiating Equation 7 with respect to depth, z,

$$\frac{d\sigma_z}{dz}=-E_{gpo}\left(\frac{\alpha_{gp}}{L}\right)\frac{d\rho_{gp}}{dz}-E_{gpo}\left(1+\alpha_{gp}.\left(\frac{z}{L}\right)\right).\frac{d^2\rho_{gp}}{dz} \quad (8)$$

On simplification the differential Equation 8 becomes

$$\frac{d\sigma_z}{dz}=-E_{gpo}\left[\left(\frac{\alpha_{gp}}{L}\right)\frac{d\rho_{gp}}{dz}+\left(1+\alpha_{gp}.\left(\frac{z}{L}\right)\right).\frac{d^2\rho_{gp}}{dz}\right]. \quad (9)$$

Combining Equations 5 and 9 simplify

$$-E_{gpo}\left[\left(\frac{\alpha_{gp}}{L}\right)\frac{d\rho_{gp}}{dz}+\left(1+\alpha_{gp}.\left(\frac{z}{L}\right)\right).\frac{d^2\rho_{gp}}{dz}\right]+\frac{4}{d}\tau=0 \quad (10)$$

Equation 10 is solved along with the boundary conditions: at z =0 (i.e. at the top of GPA) P=0 (Free boundary) and at z=L (tip of the GPA), $P=P_0$ (the applied load). Equation 10 written in finite difference form reduces to

$$\left\{\frac{\alpha_{gp}}{L}\frac{(\rho_{gpi-1}-\rho_{gpi+1})}{2(\Delta z)}+(1+\alpha_{gp}.z_i')\frac{(\rho_{gpi-1}-2\rho_{gpi}+\rho_{gpi+1})}{(\Delta z)^2}\right\}\frac{4}{E_{gpo}d}\tau_i=0 \quad (11)$$

where Δz= (L/n) – length of differential element of GP. Rearranging the terms in Eq. 11

$$\frac{n^2}{L^2}\left\{\frac{\alpha_{gp}}{2.n}(\rho_{gp,i-1}-\rho_{gp,i+1})+(1+\alpha_{gp}.z_i').(\rho_{gp,i-1}-2\rho_{gp,i}+\rho_{gp,i+1})\right\}-\frac{4}{E_{gpo}.d}\tau_i=0 \quad (12)$$

Eq. 12 is written as

$$\left\{a_i.\rho_{gp,i-1}-2.b_i.\rho_{gp,i}+c_i.\rho_{gp,i+1}\right\}-\frac{4L^2}{n^2.E_{gpo}.d}\tau_i=0 \quad (13)$$

where $a_i=\left(1+\alpha_{gp}.z_i'-\frac{\alpha_{gp}}{2.n}\right)$ $b_i=\left(1+\alpha_{gp}.z_i'\right)$ $c_i=\left(1+\alpha_{gp}.z_i'+\frac{\alpha_{gp}}{2.n}\right)$

a_i, b_i and c_i are displacement influence coefficients.

$\rho_{gp,i}$ and τ_i are respectively the displacement at the centre of node ‘i’ and the shear stress on the interface of element, ‘i’, of the GPA. Eq. 13 is written for nodes i = 2 to (n-1). Invoking the first boundary condition, P=0 implies σ_z=0 and hence strain, ε_z= 0, leads to i.e.,

$$\rho_{gp,1} = \rho_{gp,1'} \quad (14)$$

where $\rho_{gp,1'}$–displacement at the imaginary node 1’ above the GPA (Fig. 2a). Eqs. 13 and 14 are combined to arrive at the finite difference equation for node ‘1’, as

$$\left\{a_1.\rho_{gp,1'}-2.b_1.\rho_{gp,1}+c_1.\rho_{gp,2}\right\}-\frac{4L^2}{n^2.E_{gpo}.d}\tau_1=0 \quad (15)$$

Eq. 15 reduces to

$$\left\{(a_1.-2.b_1).\rho_{gp,1}+c_1.\rho_{gp,2}\right\}-\frac{4L^2}{n^2.E_{gpo}.d}\tau_1=0 \quad (16)$$

All the equations for nodes 1 to (n-1) are collated as

$$\left[I_{gp}'\right]\left\{\rho_{gp}\right\}-\frac{4.L^2}{E_{gp}.n^2.d}.\{\tau\}=0 \quad (17)$$

where $\left[I_{gp}'\right]$ is the displacement coefficient matrix.

The pile displacements equations for nodes 1 to (n-1) are collated and summarized in Eq. 17. The pile displacement influence coefficients are

$$\begin{bmatrix}(a_1-2b_1) & c_1 & 0 & . & . & . & . & . & . & 0\\ a_2 & -2b_2 & c_2 & 0 & . & . & . & . & . & 0\\ 0 & a_3 & -2b_3 & c_3 & 0 & .. & . & . & . & 0\\ 0 & 0 & a_4 & -2b_4 & c_4 & 0 & . & . & . & 0\\ . & . & . & . & . & . & . & . & . & .\\ . & . & . & . & . & . & . & . & . & .\\ . & . & . & . & . & . & . & . & . & ..\\ . & . & . & . & . & . & . & . & . & .\\ . & . & . & . & . & . & a_{n-1} & -2b_{n-1} & c_{n-1}\end{bmatrix} \quad (18)$$

Considering the compatibility of displacements in soil and GPA

$$\left\{\rho_S\right\}=\left\{\rho_{gp}\right\} \quad (19)$$

Combining Eqs. 3 and 17 with Eq. 19

$$\left[I_{gp}'\right]\frac{d}{E_{so}}\left[I_S'\right]\{\tau\}-\frac{4.L^2}{E_{gpo}.n^2.d}.\{1\}.\{\tau\}=0 \quad (20)$$

where {1} is the unit vector.

3 RESULTS AND DISCUSSION

Equation 20 is solved for the displacements in GPA. The displacements generated along the GPA length are extrapolated to obtain the top, ρ_0, and the tip, ρ_L, displacements considering the 1st, 2nd and 3rd elements for the top and n-2, n-1and nth elements for the tip displacements in the GPA, respectively. The results are presented for the following ranges of parameters. L/d: 5, 10, 25 and 50; K: 10 to 10,000; Poisson’s ratio, ν_s: 0.5, α_s = 0, 0.25, 0.5, 1and 2; and α_{gp}=0, 0.25, 0.5, 1 and 2.

The influence of α_s on the variation of the shear stresses with depth is presented in Figure 6 for L/d=10, K=50, ν_s=0.5 and α_{gp}=0, 0.5 and 1. The variations of the shear stresses with depth are magnified at top as shown in Figure 6(b). The variations of shear stresses with depth are very similar for both values of α_s = 0 and 0.5 and decrease with increasing values of α_{gp}. The shear stresses at the tip decrease from 6.65 to 5.63 and from 5.25 to 4.39 for α_s = 0 & 0.5 with α_{gp} increasing from 0 to 1 respectively. On the contrary, the shear stresses at the top increase from 0.35 to 0.41 and 0.77 to 0.86 for α_s = 0 & 0.5 with α_{gp} increasing from 0 to 1 respectively.

The variations of shear stresses with depth as a function of α_{gp} are presented in Figure 7 for L/d=10, K=50, ν_s=0.5 and for α_s = 0, 0.5 and 1.0. The plots are magnified for the stresses at the top in Figure 7(b). The variation of shear stresses with depth for α_{gp} =0 & 0.5 are very similar for all α_s.

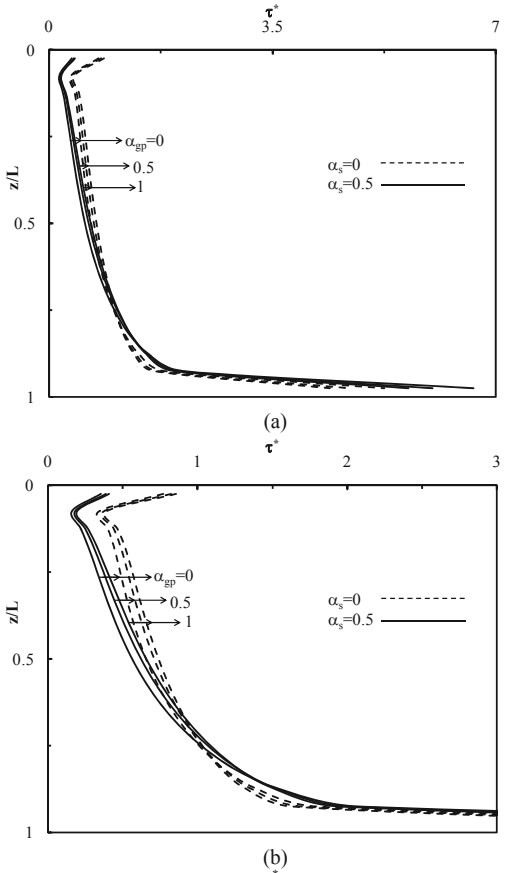

Fig. 6 Normalized shear stress, τ^* vs. Depth, z/L for L/d = 10, K=50 & ν_s=0.5 – (a) Effect of α_s & α_{gp}.(b) Enlarged at top.

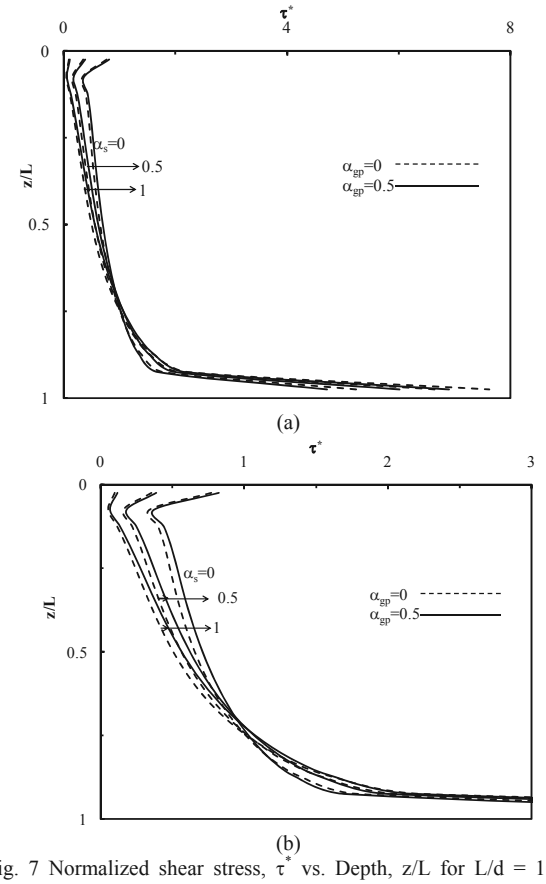

Fig. 7 Normalized shear stress, τ^* vs. Depth, z/L for L/d = 10, K=50 & ν_s=0.5 – (a) Effect of α_s & α_{gp}.(b) Enlarged at top.

The variations of normalized tip displacements with depth for α_s = 0 (E_s constant with depth) and 0.5 are presented in Figure 8 for L/d=10, K=50, ν_s=0.5 and α_{gp}=0, 0.5 and 1. The displacement coefficients decrease with α_{gp} at the tip, and increase at the top with α_{gp} for α_{gp} increasing from 0 to 1. The variation of I_U with depth is relatively large for smaller values of α_{gp} compared to that for α_{gp}=1. The displacement coefficient at the tip decreases from 1.91 to 1.47 for α_s =0 and from 1.732 to 1.34 for α_s =0.5. I_U decreases by about 30% for α_s =0.5 in comparison to that for α_s =0 (homogenous soil).

The influence of α_{gp}, the rate of increase of deformation modulus of granular material with depth on the variations of the displacement coefficients, I_U with depth for varying α_s from 0 to 1, L/d=10, K=50 and ν_s=0.5 is presented in Figure 9. The displacement coefficient, I_U, for α_s=0, at the tip decreases from 1.91 to 1.8 and increases from 1.12 to 1.17 for α_{gp} = 0 and 0.5 respectively. Similarly for α_s=0.5 and 1, the displacements at the tip decrease from 1.47 to 1.39 and 1.14 to 1.10 while at the top they increase from 0.87 to 0.90 and 0.71 to 0.74 for α_{gp} = 0 & 0.5 respectively.

4 CONCLUSION

Analysis of the GPA under the influence of the non-homogeneities of the deformation moduli of the soil and granular material is presented in this paper. The shear stresses near the top of GPA are significantly less for α_s increasing with depth. Displacements reduce for the deformation moduli parameters of soil and granular material increasing linearly with depth (α_s=0.5 & α_{gp}=0.5). The displacements reduce by 30% with depth for α_s=0.5 and 6% for α_{gp}=0.5 in comparison to those for α_s=0 and α_{gp}=0 respectively.

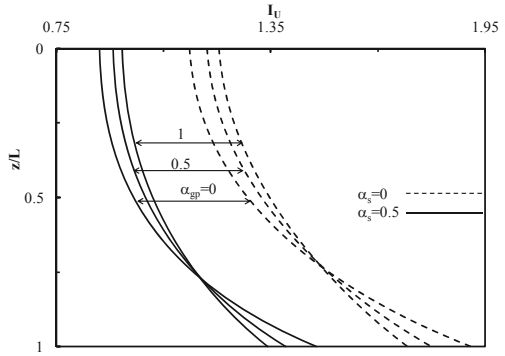

Fig. 8 Normalized displacement coefficient, I_U vs. Depth, z/L for L/d=10, K=50 & ν_s=0.5 – Effects of α_s & α_{gp}.

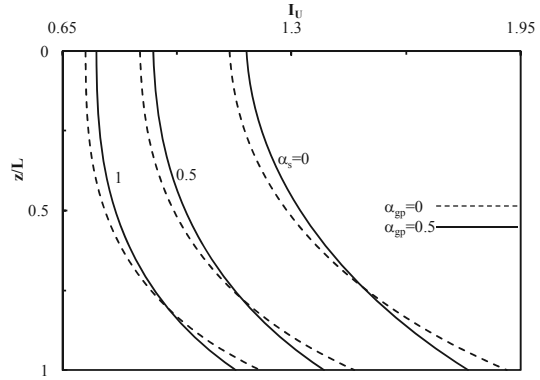

Fig. 9 Normalized displacement coefficient, I_U vs. Depth, z/L for L/d=10, K=50 & ν_s=0.5 – Effect of α_{gp} & α_s.

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