Construction of virtual sites for reliability-based design

Construction de sites virtuels à des fins de conception fiabiliste

Phoon K.K.
National University of Singapore, Singapore

Ching J.
National Taiwan University, Chinese Taipei

ABSTRACT: This paper presents the construction of "virtual sites" using multivariate normal distributions calibrated from actual soil property databases. By doing so, the actual magnitude of uncertainty reduction from conducting better/more soil tests can be estimated realistically, rather than theoretically.

RÉSUMÉ: Cet article présente la construction de "sites virtuels" en utilisant des distributions normales à plusieurs variables calibrées à partir de bases de données de propriétés de sols réels. Par cette méthode, la réduction réelle de l'incertitude que l'on peut obtenir en augmentant le nombre et/ou la qualité des essais de sol peut être estimée de manière réaliste, et non plus seulement théorique.

KEYWORDS: virtual site; uncertainties; soil properties; correlation; site investigation; reliability-based design.

1 INTRODUCTION

This paper presents the concept of a "virtual site"; the purpose is to emulate site investigation efforts as realistically as possible. It is not possible to emulate every aspect of a real site at present. In this paper, the scope is to reproduce the information content arising from a typical mix of laboratory and field tests conducted in a site for the purpose of estimating a design undrained shear strength (s_u) for clays and friction angle (ϕ) for sands. The critical feature here is the consistent and realistic coupling of different test data, which is achieved using multivariate normal distributions. Data from different tests will be correlated, because they are measuring the same mass of soil, although they could be measuring different aspects of soil behavior under different boundary conditions and over different volumes. The purpose of developing a virtual site is not to replace actual site investigation. The purpose is to quantify the uncertainty reduction in s_u and ϕ by incorporating the test results from better and/or more tests.

The idea of simulating a "virtual site" is not new. For example, Jaksa et al. (2005) and Goldsworthy et al. (2007) used three dimensional random fields and Monte Carlo simulation to simulate the spatially variable elastic modulus of a "virtual" site. Each spatially variable realization constitutes a plausible full information scenario. Site investigation is then carried out numerically by sampling the continuous random field at discrete locations. The site investigation data so obtained constitute the typical partial information scenario commonly encountered in practice. The goal of these studies was to quantify the difference in the designs based on these full and partial information scenarios. In this paper, the virtual site simulation is based on multivariate normal distributions that couple soil parameters such as s_u, overconsolidation ratio, standard penetration test N-value, cone tip resistance, and Atterberg limits. The distinct features of this paper are: (a) a more realistic bag of multivariate information containing both laboratory and field data and (b) the probability model is constructed from an actual database of clays and sands. These features are critical to the objective of this paper, which is to quantify the uncertainty reduction in s_u and ϕ by incorporating the test results from better and/or more tests. This objective is only achievable if the information contained in the virtual site is comparable to that contained in a real site, not merely pertaining to a single laboratory/field parameter, but to a group of parameters that are correlated in a realistic way. By doing so, it is possible to evaluate the *actual* merits of reliability-based design approximately, rather than elaborate on the theoretical merits widely discussed in previous studies. This paper summarizes the current development of such virtual sites.

2 MULTIVARIATE GEOTECHNICAL DATA

Multivariate information is usually available in a typical site investigation. For instance, when undisturbed samples are extracted for oedometer and triaxial tests, SPT and/or piezocone test (CPTU) may be conducted in close proximity. Moreover, data sources such as the unit weight, plastic limit (PL), liquid limit (LL), and liquidity index (LI) are commonly determined from relatively simple laboratory tests on disturbed samples. These data could be correlated, and these correlations can be exploited to reduce the coefficient of variation of a design parameter. The impact on RBD is obvious. This section presents statistical characterization of multivariate geotechnical data.

Most soil parameters are not normally distributed, because they are positive valued. Let Y denote a non-normally distributed soil parameter. One well known cumulative distribution function (CDF) transform approach can be applied to convert Y into a standard normal variable X: $X = \Phi^{-1}[F(Y)]$, where $\Phi(.)$ is the CDF of a standard normal random variable, and F(.) is the CDF of Y. A set of multivariate soil parameters $\underline{Y} = (Y_1, Y_2, \ldots Y_n)'$ can be transformed into $\underline{X} = (X_1, X_2, \ldots X_n)'$. By definition, $X_1, X_2, \ldots X_n$ are *individually* standard normal random variables. It is crucial to note here that *collectively* $(X_1, X_2, \ldots X_n)'$ does not necessarily follow a multivariate normal distribution even if each component is normally distributed. Even so, recent studies by Ching et al. (2010) and Ching and Phoon (2012a) showed that the multivariate normal distribution is an acceptable approximation for selected parameters of clays, and Ching et al. (2012b) arrived at the same observation for selected parameters of sands.

The multivariate normal probability density function for $\underline{X} = (X_1, X_2, \ldots X_n)'$ can be defined uniquely by a correlation matrix:

$$f(\underline{X}) = |C|^{-\frac{1}{2}} (2\pi)^{-\frac{n}{2}} e^{-\frac{1}{2}\underline{X}'C^{-1}\underline{X}} \quad (1)$$

where C is the correlation matrix. For n = 3, the correlation matrix is given by:

$$C = \begin{bmatrix} 1 & \delta_{12} & \delta_{13} \\ \delta_{12} & 1 & \delta_{23} \\ \delta_{13} & \delta_{23} & 1 \end{bmatrix} \quad (2)$$

between X_i and X_j (not equal to the correlation between the original physical variable Y_i and Y_j). It is clear that the full multivariate dependency structure of a normal random vector only depends on a correlation matrix (C) containing bivariate correlations between all possible pairs of components, namely X_1 and X_2, X_1 and X_3, and X_2 and X_3. It is not necessary to measure X_1, X_2, and X_3 *simultaneously*. The practical advantage of capturing multivariate dependencies in any dimension (i.e., any number of random variables) using only bivariate dependency information is obvious.

It is simple to obtain realizations of *independent* standard normal random variables $\underline{U} = (U_1, U_2, U_3)'$ using library functions in many softwares. Realizations of *correlated* standard normal random variables $\underline{X} = (X_1, X_2, X_3)'$ can be obtained using $\underline{X} = L\underline{U}$, in which L is the lower triangular Cholesky factor satisfying $C = LL'$. Finally, each soil parameter is obtained using $Y_i = F^{-1}[\Phi(X_i)]$.

2.1 *Complete multivariate information (structured clays)*

A multivariate database of Y_1 = LI (liquidity index), $Y_2 = s_u$, $Y_3 = s_u^{re}$ (remolded undrained shear strength), $Y_4 = \sigma'_p$ (preconsolidation stress), and $Y_5 = \sigma'_v$ (effective vertical stress) is complied in Ching & Phoon (2012a). There are 345 data points of structured clays from 37 sites worldwide, covering a wide range of sensitivity, LI, and clay types, with simultaneous knowledge of $(Y_1, Y_2, \ldots Y_5)'$. The OCR values of the data points are generally small, mostly less than 4. Fissured and organic clays are mostly left out of the database. Because s_u values depend on stress state, strain rate, sampling disturbance, etc., all s_u values are converted into mobilized s_u values following the recommendations made by Mesri and Huvaj (1997). The marginal probability density functions (PDF) for $(Y_1, Y_2, \ldots Y_5)'$ and their statistics (mean of $Y_i = \mu_i$, COV of $Y_i = V_i$, mean of $\ln(Y_i) = \lambda_i$, standard deviation of $\ln(Y_i) = \xi_i$) are summarized in Table 1.

For lognormal Y, the CDF transform is:

$$X_i = \left[\ln(Y_i) - \lambda_i\right] / \xi_i \quad (3)$$

The transformed $(X_1, X_2, \ldots, X_5)'$ are individually standard normal random variables. The correlation matrix C for $(X_1, X_2, \ldots X_5)'$ is shown in Table 2, and $(X_1, X_2, \ldots X_5)'$ is assumed to be multivariate normal with the correlation matrix listed in the table.

The multivariate normal distribution is employed to simulate samples of (LI, s_u, s_u^{re}, σ'_p, σ'_v), shown in Figure 1 together with the calibration database. Not only the correlations among the original random variables (LI, s_u, s_u^{re}, σ'_p, σ'_v) are shown but the correlations among their derived (normalized) quantities, including $S_t = s_u/s_u^{re}$, OCR = σ'_p/σ'_v, s_u/σ'_v, are also shown. The multivariate normal distribution performs adequately, as the simulated samples closely mimic the correlation behaviors of the calibration database, even for those with nonlinear trends, e.g. LI-s_u^{re} and LI-S_t correlations.

Table 1. Distributions and statistics of $(Y_1, Y_2, \ldots Y_5)'$ for structured clays (Source: Ching & Phoon 2012a).

	Distribution	Mean	COV	Mean of $\ln(Y_i)$, λ_i	stdev of $\ln(Y_i)$, ξ_i
Y_1 = LI	Lognormal	1.25	0.49	0.122	0.459
$Y_2 = s_u$	Lognormal	31.01kN/m^2	0.95	3.051	0.898
$Y_3 = s_u^{re}$	Lognormal	2.51kN/m^2	1.52	0.226	1.191
$Y_4 = \sigma'_p$	Lognormal	105.82kN/m^2	0.98	4.311	0.835
$Y_5 = \sigma'_v$	Lognormal	66.63kN/m^2	0.80	3.891	0.823

Table 2. Correlation matrix C for $(X_1, X_2, \ldots X_5)'$ for the five selected parameters of structured clays (Source: Ching & Phoon 2012a).

	X_1 (LI)	X_2 (s_u)	X_3 (s_u^{re})	X_4 (σ'_p)	X_5 (σ'_v)
X_1 (LI)	1.000	-0.083	-0.824	-0.176	0.280
X_2 (s_u)	-0.083	1.000	0.276	0.915	0.801
X_3 (s_u^{re})	-0.824	0.276	1.000	0.365	0.453
X_4 (σ'_p)	-0.176	0.915	0.365	1.000	0.850
X_5 (σ'_v)	0.280	0.801	0.453	0.850	1.000

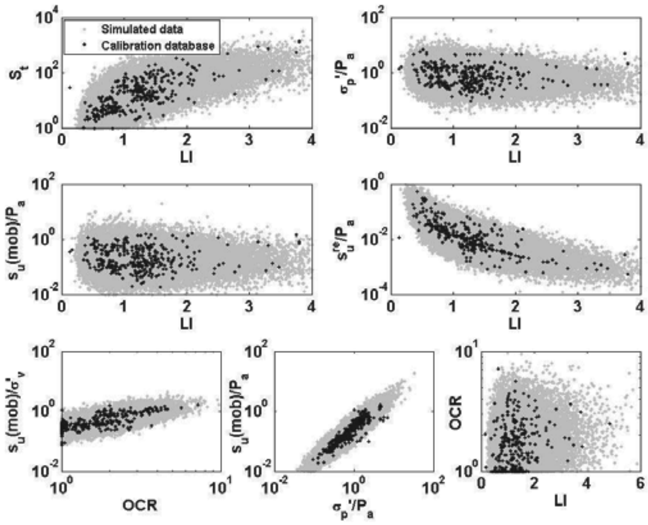

Figure 1. Comparisons between the calibration database and the simulated data points (Source: Ching & Phoon 2012a).

2.2 *Incomplete multivariate information (unstructured clays)*

Ching et al. (2010) presented another clay database containing four soil parameters: Y_1 = OCR, $Y_2 = s_u$ from CIUC test, $Y_3 = q_T - \sigma_v$ (net cone resistance), and $Y_4 = N_{60}$ (SPT N corrected for energy efficiency). The range of OCR of this database is wider – from 1 to 50. However, only bivariate data on (Y_1, Y_2) = (OCR, s_u), $(Y_3, Y_2) = (q_T - \sigma_v, s_u)$, and $(Y_4, Y_2) = (N_{60}, s_u)$ are available. Bivariate data on (Y_1, Y_3) = (OCR, $q_T - \sigma_v$), (Y_1, Y_4) = (OCR, N_{60}), and $(Y_3, Y_4) = (q_T - \sigma_v, N_{60})$ are missing, i.e., the bivariate correlations δ_{ij} are only partially known. Given that complete bivariate information is not available, it is not possible to apply the aforementioned CDF transform approach directly. It is accurate to say that although it is common to measure more than two soil parameters in a site investigation, it is uncommon to establish correlations between *all possible pairs* of soil parameters.

To deal with this difficulty of incomplete bivariate correlations, Ching et al. (2010) constructed a multivariate normal distribution using a Bayes net model which prescribed a dependency structure based on some postulated but reasonable conditional relationships between the soil parameters. They considered Y_1 = OCR as a given number and the remaining soil parameters $(Y_2, Y_3, Y_4)'$ are lognormally distributed random variables. Hence, $\ln(Y_2) = \ln(s_u) = \lambda_2 + \xi_2 X_2$, $\ln(Y_3) = \ln(q_T - \sigma_v) = \lambda_3 + \xi_3 X_3$, and $\ln(Y_4) = \ln(N_{60}) = \lambda_4 + \xi_4 X_4$, in which X_i are standard normal random variables. The simulation of $(Y_1, Y_2, Y_3, Y_4)'$ starts from OCR. The undrained shear strength, Y_2, is next simulated using this OCR sample and the SHANSEP model (Ladd and Foott 1974):

$$\ln(s_u) = 0.64\ln(\mathrm{OCR}) + \ln(\sigma'_{v0}) - 0.874 + 0.237U_1 \quad (4)$$

where 0.237 is the standard deviation of the transformation uncertainty, and U_1 is standard normal. The third step is to simulate N_{60} and q_T - σ_v using the s_u sample:

$$\ln(N_{60}) = 1.633\ln(s_u) - 0.403\ln(\sigma'_{v0}) - 3.845 + 0.456U_2 \quad (5a)$$

$$\ln(q_T - \sigma_v) = \ln(s_u) + 2.54 + 0.34U_3 \quad (5b)$$

where 0.456 and 0.34 are the standard deviations of the transformation uncertainties, and U_2 and U_3 are standard normal. Figure 2 shows the correlation plots for the simulated {OCR, s_u, N_{60}, q_T - σ_v} for a case where OCR is uniformly distributed over [5, 24].

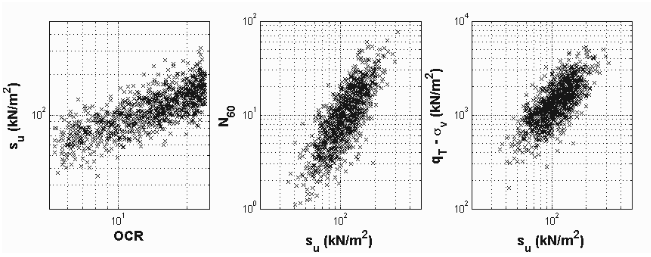

Figure 2. Correlation plots for {OCR, s_u, N_{60}, q_T - σ_v} samples.

Based on the results of Ching et al. (2010), Phoon et al. (2012) further assumed OCR to be lognormal with a reasonable COV = 0.25. Under this assumption, they showed that the underlying standard normal variables $(X_1, X_2, X_3, X_4)'$ have the correlation matrix shown in Table 3. The correlation matrix in Table 3 should be suitable for unstructured clays covering a fairly wide range of OCR.

Table 3. Correlation matrix C for $(X_1, X_2, X_3, X_4)'$ for the four selected parameters of unstructured clays (Source: Phoon et al. 2012).

	X_1 (OCR)	X_2 (s_u)	X_3 (q_T - σ_v)	X_4 (N_{60})
X_1 (OCR)	1.000	0.554	0.355	0.395
X_2 (s_u)	0.554	1.000	0.642	0.714
X_3 (q_T - σ_v)	0.355	0.642	1.000	0.458
X_4 (N_{60})	0.395	0.714	0.458	1.000

2.3 *Incomplete multivariate information (clean sands)*

Ching et al. (2012b) presented a study that is very similar to Ching et al. (2010) but for clean sands. The study was based on a database containing five selected parameters of normally consolidated clean sands: $Y_1 = \phi_{cv}$ (critical state friction angle), $Y_2 = I_R$ (dilatancy index, see Bolton 1986), $Y_3 = \phi_p$ (peak secant friction angle), $Y_4 = (q_c/P_a)/(\sigma'_v/P_a)^{0.5} = q_{c1}$ (corrected cone resistance), and $Y_5 = (N_1)_{60}$ (SPT N corrected for energy efficiency and overburden stress). They considered $Y_1 = \phi_{cv}$ and $Y_2 = I_R$ as given numbers and the remaining soil parameters $(Y_3, Y_4, Y_5)'$ are random variables: Y_3 is normal, while Y_4 and Y_5 are lognormal. Hence, $Y_3 = \phi_p = \mu_3 + \mu_3 V_3 X_3$, $\ln(Y_4) = \ln(q_{c1}) = \lambda_4 + \xi_4 X_4$, and $\ln(Y_5) = \ln[(N_1)_{60}] = \lambda_5 + \xi_5 X_5$, in which X_i are standard normal random variables. If we further assume ϕ_{cv} and I_R are normal with reasonable standard deviations of 3^o and 1^o, respectively, i.e., $Y_1 = \phi_{cv} = \mu_1 + 3X_1$ and $Y_2 = I_R = \mu_2 + X_2$, and also assume independence between ϕ_{cv} and I_R, it can be shown that the underlying standard normal variables $(X_1, X_2, X_3, X_4, X_5)'$ has the correlation matrix shown in Table 4. The correlation matrix in Table 4 should be suitable for normally consolidated clean sands.

Table 4. Correlation matrix C for $(X_1, X_2, X_3, X_4, X_5)'$ for the five selected parameters of clean sands (Source: Ching et al. 2012b).

	X_1 (ϕ_{cv})	X_2 (I_R)	X_3 (ϕ_p)	X_4 (q_{c1})	X_5 [$(N_1)_{60}$]
X_1 (ϕ_{cv})	1.000	0.000	0.642	0.491	0.536
X_2 (I_R)	0.000	1.000	0.642	0.491	0.536
X_3 (ϕ_p)	0.642	0.642	1.000	0.764	0.835
X_4 (q_{c1})	0.491	0.491	0.764	1.000	0.638
X_5 [$(N_1)_{60}$]	0.536	0.536	0.835	0.638	1.000

2.4 *Undrained shear strengths under various test procedures*

The undrained shear strength (s_u) of a clay is not a constant. In particular, s_u of a clay evaluated by different test procedures are different because these tests may have different stress states, stress histories, degrees of sampling disturbance, and strain rates. Ching & Phoon (2013) constructs the multivariate normal distribution of the s_u values from seven s_u tests (CIUC, CK_0UC, CK_0UE, DSS, VST, UU, UC) based on a large clay database consisting data points from 146 studies. Many s_u data points are associated with a known test mode (6310 points), a known OCR (4584 points), and a known plasticity index (PI) (4541 points). The geographical regions cover Australia, Austria, Brazil, Canada, China, England, Finland, France, Germany, Hong Kong, Iraq, Italy, Japan, Korea, Malaysia, Mexico, New Zealand, Norway, Northern Ireland, Poland, Singapore, South African, Spain, Sweden, Thailand, Taiwan, United Kingdom, United States, and Venezuela. The clay properties cover a wide range of OCR (mostly 1~10, few studies OCR > 10, but nearly all studies are with OCR < 50) and a wide range of sensitivity S_t (sites with S_t = 1~ tens or hundreds are fairly typical).

An important step for the construction of the multivariate distribution is to convert all s_u data points in the database into the following standardized form:

$$s_{u,NC,1\%,PI20}/\sigma'_v = (s_u/\sigma'_v)/(b_{OCR} \cdot c_{rate} \cdot d_{PI}) \quad (6)$$

where $s_{u,NC,1\%,P20}$ is the undrained shear strength of a NC clay with PI = 20 subjected to a 1% per hour strain rate; b_{OCR}, c_{rate}, and d_{PI} are modifier factors that adjust the reference normalized undrained shear strength for overconsolidation ratio, strain rate, and plasticity. Table 5 shows these factors (Ching et al. 2013; Ching & Phoon 2013). The standardized $s_{u,NC,1\%,P20}/\sigma'_v$ is be denoted by $Y_{\text{test mode index}}$. The test mode indices are respectively 1, 2, 3, 4, 5, 6, and 7 for CIUC, CAUC, CAUE, DSS, FV (field vane), UU, and UC. Hence, there are seven random variables $(Y_1, Y_2 \ldots Y_7)'$. Table 6 shows the statistics of Y_i. The Y data points for each test mode are roughly lognormally distributed, i.e., $X_i = [\ln(Y_i)-\lambda_i]/\xi_i$ is roughly standard normal. Given a test mode i, the scatter in the Y_i data points, quantified by the COV in Table 6, may be due to measurement errors in s_u and global inherent variability in s_u (s_u from different geographic locales) as well as the transformation uncertainties associated with the standardization steps for PI, strain rate, and OCR.

The Y_i data points are converted to standard normal variables $X_i = [\ln(Y_i)-\lambda_i]/\xi_i$. Table 7 shows the correlation matrix C for $(X_1, X_2, \ldots, X_7)'$. The estimated correlation coefficients δ_{ij} are quite sensible. The four triaxial compression (TC) test modes $(X_1, X_2, X_6, X_7)'$ seem mutually highly correlated (δ_{ij} > 0.8), with the exception of (X_6, X_7) having δ_{ij} = 0.59. The CAUE test mode (X_3) has weak correlation with TC test modes (δ_{ij} < 0.5), probably because it imposes a different stress state from TC tests. The correlation coefficients between FV and TC are relatively weak as well ($\delta_{ij} \leq 0.63$). Such relatively low correlation between FV and TC may be due to the fact that the FV test has several distinct aspects (stress state, drainage boundaries, strain rate, and failure mode). It is interesting that the correlation between FV and DSS is high (δ_{ij} = 0.73).

Table 5 b_{OCR}, c_{rate}, and d_{PI} factors (Source: Ching et al. 2013).

Factor	Test type	Formula
b_{OCR}	CIUC	$OCR^{0.602}$
	CAUC	$OCR^{0.681}$

	CAUE	$OCR^{0.898}$
	DSS	$OCR^{0.749}$
	VST	$OCR^{0.902}$
	UU	$OCR^{0.800}$
	UC	$OCR^{0.932}$
c_{rate}		$1.0+0.1\times\log_{10}$(strain rate/1%)
	CIUC	$(PI/20)^0 = 1$
	CAUC	$(PI/20)^0 = 1$
	CAUE	$(PI/20)^{0.178}$
d_{PI}	DSS	$(PI/20)^{0.0655}$
	VST	$(PI/20)^{0.124}$
	UU	$(PI/20)^0 = 1$
	UC	$(PI/20)^0 = 1$

Table 6 Statistics of Y data points (Source: Ching et al. 2013).

	# pts.	Mean	COV	Mean of $\ln(Y_i)$, λ_i	Stdev of $\ln(Y_i)$, ξ_i
Y_1 (CIUC)	637	0.404	0.316	-0.955	0.315
Y_2 (CAUC)	555	0.350	0.318	-1.090	0.280
Y_3 (CAUE)	224	0.184	0.324	-1.748	0.355
Y_4 (DSS)	573	0.241	0.399	-1.468	0.277
Y_5 (FV)	1057	0.275	0.416	-1.363	0.372
Y_6 (UU)	435	0.243	0.504	-1.523	0.463
Y_7 (UC)	387	0.223	0.611	-1.640	0.523

Table 7 Correlation matrix C for $(X_1, X_2, \ldots, X_7)'$ (Source: Ching & Phoon 2013).

	X_1 (CIUC)	X_2 (CAUC)	X_3 (CAUE)	X_4 (DSS)	X_5 (FV)	X_6 (UU)	X_7 (UC)
X_1 (CIUC)	1.00	0.84	0.47	0.72	0.63	0.88	0.85
X_2 (CAUC)	0.84	1.00	0.39	0.78	0.35	0.7*	0.6*
X_3 (CAUE)	0.47	0.39	1.00	0.45	0.41	0.4*	0.3*
X_4 (DSS)	0.72	0.78	0.45	1.00	0.73	0.6*	0.5*
X_5 (VST)	0.63	0.35	0.41	0.73	1.00	0.64	0.46
X_6 (UU)	0.88	0.7*	0.4*	0.6*	0.64	1.00	0.68
X_7 (UC)	0.85	0.6*	0.3*	0.5*	0.46	0.68	1.00

* insufficient data pairs, estimated based on judgments

3 REDUCING UNCERTAINTY IN DESIGN PARAMETER WITH BETTER AND/OR MORE TESTS

As mentioned earlier, it is simple to simulate virtual site investigation data $(Y_1, Y_2, \ldots, Y_n)'$. First, obtain realizations of *independent* standard normal random variables $\underline{U} = (U_1, U_2, \ldots, U_n)'$ using library functions in many softwares. Realizations of *correlated* standard normal random variables $\underline{X} = (X_1, X_2, \ldots, X_n)'$ can be obtained using $\underline{X} = L\underline{U}$, in which L is the lower triangular Cholesky factor satisfying $C = LL'$. Finally, each soil parameter is obtained using $Y_i = F^{-1}[\Phi(X_i)]$. For lognormal distribution, $Y_i = \exp(\lambda_i+\xi_i\times X_i)$. Figures 1 & 2 already showed the simulated data $(Y_1, Y_2, \ldots, Y_n)'$. This section will further discuss how to use the simulated data to quantify the uncertainty reduction in s_u and ϕ by incorporating the test results from better and/or more tests.

This is illustrated below using results presented in Figure 2. The histogram of the simulated s_u data for the same virtual site is given in the left plot of Figure 3, showing the simulated s_u data when no site-specific tests are conducted. Let us consider a site investigation program consisting oedometer, CPTU, and SPT N tests. Suppose the test results show that OCR is within [9.5,13.1], N_{60} within [7,9], and q_T-σ_v within [1100kN/m^2,1350kN/m^2]. Based on the above information, the conditional samples of s_u can be easily obtained by filtering out samples satisfying OCR$\in$[9.5,13.1], $N_{60}\in$[7,9], and q_T-$\sigma_v'\in$[1100kN/m^2,1350kN/m^2] simultaneously from the population at large. The s_u values associated with this filtered set of (OCR, s_u, N_{60}, q_T - σ_v) values are therefore the conditional s_u samples. The histogram of these conditional samples is shown in the right plot of Figure 3. It is clear that the uncertainty in s_u is significantly reduced, given the information from better and/or more tests.

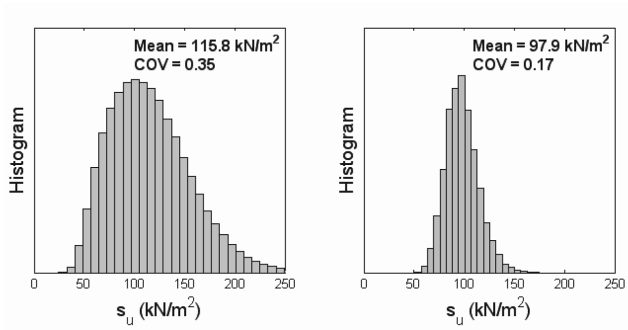

Figure 3. Histograms of the (conditional) s_u samples.

4 CONCLUSION

The construction of "virtual sites" are demonstrated in this paper using multivariate normal distributions calibrated from actual soil property databases. By doing so, it is possible to evaluate the reduction in the uncertainties associated with design parameters as a function of better and/or more tests. The practical goal is to establish an *actual* (not theoretical) link between the cost of a site investigation program and the potential design savings accrued from reliability-based design.

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