Consolidation theory for combined vacuum pressure and surcharge loading

Théorie de la consolidation sous l'action combinée du vide et d'un pré-chargement

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ABSTRACT: Atheory describing the consolidation of a uniform clayey deposit with and without prefabricated vertical drain (PVD) improvement under the combination of a vacuum pressure and a surcharge load has been developed and expressed as closed-form equations. For the case of a soil layer without PVD improvement, both one-way and two-way drainage boundary conditions are considered. Laboratory consolidation tests using combinations of vacuum pressure and surcharge load were conductedunder oedometer conditions with vertical or radial drainage. The measured excess pore water pressures are compared with values predicted by the theory presented in the paper. It has been demonstrated that the theory is valid and can be used for designing preloading projects that involve the combination of a vacuum pressure and a surcharge load.

RÉSUMÉ : Une théorie décrivant la consolidation d'un dépôt argileux uniforme avec et sans amélioration par drains verticaux préfabriqués (DVP) sous l'action combinée du vide et d'un pré-chargement a été développée avec un système fermé d'équations. Pour le cas d'une couche de sol sans amélioration par DVP, des conditions aux limites drainantes par un côté et par deux côtés sont considérées. Des essais de consolidation au laboratoire sous des conditions oedométriques ont été réalisés sous vide et pré-chargement avec des drains verticaux ou radiaux. La surpression interstitielle mesurée est comparée avec les valeurs prévues par la théorie présentée dans le présent article. Il a été démontré que la théorie est valable et peut être utilisée pour définir des projets de pré-chargement qui impliquent l'utilisation combinée du vide et d'un pré-chargement.

KEYWORDS:consolidation, vacuum pressure, embankment, laboratory test, soft clay

1 INTRODUCTION

Preloading a soft clayey deposit with the combination of a vacuum pressure and a surcharge load (embankment fill) has several advantages, such as increasing the preloading pressure and reducing lateral displacements of the deposit, etc. (e.g., Chai et al. 2006). Its use in engineering applications has increased in recent years (e.g., Kelly and Wong 2009; Hirata et al. 2010; Indraratna et al. 2011).

Vacuum consolidation has different characteristics compared with consolidation induced by direct application of a surcharge load (Chai et al. 2009). For a soil deposit without any improvement in consolidation performance that might result from the installation of prefabricated vertical drains (PVDs), when a vacuum pressure is applied water is drained out of the soil layer only at the boundary where that vacuum pressure is applied. This applies for both cases of one-way and two-way drainage conditions. However, for a deposit with one-way drainage constrained to deform under one-dimensional (1D) conditions, the final state is a uniform vacuum pressure distribution throughout the deposit and consequently zero flow rate. But for a deposit with two-way drainage, at the bottom boundary the excess pore water pressure is fixed at zero and effectively no vacuum pressure can be applied at this location, and so the final state involves the steady flow of pore water toward the boundary at which the vacuum pressure is applied. Considering these complicating factors, Chai and Carter (2011) recently derived a consolidation theory for soils subjected to vacuum pressure. However, their theory cannot be applied directly for cases that involve a combination of vacuum pressure and surcharge loading, and therefore there is a need to develop a reliable theory for such cases.

This paper presents a newly developed consolidation theory applicable to soils subjected to a combination of vacuum pressure and surcharge loading. This theory is applicable to the case of a uniform soil deposit with or without PVD improvement. Predictions obtained using this theory are compared with the results of laboratory tests conducted under oedometer conditions, for cases that involve both vertical and radial drainage conditions, with the latter designed to simulate the consolidation of a deposit improved by PVDs. It has been shown that the theory is valid and can be used for designingpreloading projects that involve a combination of vacuum pressure and surcharge loading.

2 CONSOLIDATION THEORY

2.1 *Uniform layer without PVDs*

Under the same assumptions as those made in Terzaghi's 1D consolidation theory (Terzaghi 1943), the governing equation and the boundary conditions for the generation and dissipation of excess pore water pressure in a saturated soil layer under a combination of vacuum pressure and surcharge load are as follows:

$$c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \quad (1)$$

$$u(0,t) = -p_{vac} \quad (2)$$

$$\frac{\partial u(H,t)}{\partial z} = 0 \text{ for } t > 0 \text{ (one-way drainage)} \quad (3)$$

$$u(H,t) = 0 \text{ for } t > 0 \text{ (two-way drainage)} \quad (3a)$$

where$z$ = the spatial coordinate; t = time; u = the excess pore water pressure; c_v = the coefficient of consolidation of the soil; p_{vac} = the magnitude of the applied vacuum pressure at $z = 0$; and H is the thickness of the deposit.

With the presence of a vacuum pressure, the final state is not a condition with zero excess pore pressure in the deposit. Therefore, the solution to the governing equation must consist of two parts, namely the steady state solution ($Y(z)$) and the transient solution ($v(z,t)$) (Chai and Carter 2011). With the boundary condition defined by Eq. (2), $u(z, t)$ can be expressed in the following form:

$$u(z,t) = -p_{vac}Y(z) + p_{vac}v_1(z,t) + p_s v_2(z,t) \quad (4)$$

wherep_s = the magnitude of the applied surcharge load. The term $-p_{vac}Y(z)$ is the final steady state excess pore water pressure distribution and ($p_{vac}\ v_1(z,\ t)+\ p_s v_2(z,\ t)$) is the time-dependent component of the excess pore water pressure.

2.1.1*One-way drainage*

For this case the excess pore water pressure distribution is given by:

$$u = -p_{vac} + (p_{vac} + p_s)\cdot\left(\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin(a_n z)e^{-a_n^2 \cdot c_v \cdot t}\right) \quad (5)$$

where$a_n = (2n\text{-}1)\pi/(2H)$. In this case, $Y(z) = 1$, and the $v_1(z,\ t) = v_2(z,\ t)$ and its expression is given in the last set of parentheses of Eq. (5). The average degree of consolidation is given by:

$$U = 1 - \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}e^{-\frac{c_v \cdot t}{4H^2}(2n-1)^2\pi^2} \quad (6)$$

2.1.2*Two-way drainage*

In this case the excess pore water pressure distribution in the soil is given by:

$$u(z,t) = -p_{vac}\left(1-\frac{z}{H}\right) + \frac{2}{\pi}\sum_{n=1}^{\infty}\left[\begin{array}{l}\frac{p_{vac}+p_s}{n}\sin(\lambda_n z)\\ +\frac{p_s}{n}\sin(\lambda_n(H-z))\end{array}\right]e^{-\lambda_n^2 \cdot c_v \cdot t} \quad (7)$$

where $\lambda_n = n\pi/H$. The average degree of consolidation is given by:

$$U(t) = 1 - \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}e^{-\frac{c_{vs} \cdot t}{H^2}(2n-1)^2\pi^2} \quad (8)$$

2.2 *Uniform layer with PVD improvement*

The theory for a PVD-improved soil deposit is derived here for the case of one-way drainage conditions using a unit cell model, as shown in Fig. 1. The governing equation for consolidation is as follows:

$$\frac{\partial u}{\partial t} = c_h\left[\left(\frac{\partial^2 u}{\partial r^2}\right) + \frac{1}{r}\left(\frac{\partial u}{\partial r}\right)\right] \quad (9)$$

where$r$ = the radial distance and c_h = the coefficient of consolidation in the horizontal direction. The boundary conditions are:

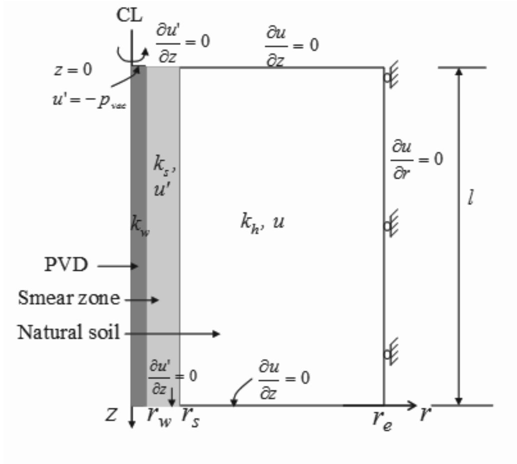


Figure 1. Unit cell model and boundary conditions

$$\frac{\partial u(r_e, z, t)}{\partial r} = 0 \quad (10)$$

$$\frac{\partial u(r,0,t)}{\partial z} = 0\,,\ \frac{\partial u'(r,0,t)}{\partial z} = 0 \quad (11)$$

$$\frac{\partial u(r,l,t)}{\partial z} = 0\,,\ \frac{\partial u'(r,l,t)}{\partial z} = 0 \quad (12)$$

$$u' = (r_w, 0, t) = -p_{vac} \quad (13)$$

where$u$ and u' = the excess pore water pressures in the undisturbed zone and the smear zone, respectively (Fig. 1), z = depth from the ground surface, r_w = equivalent radius of a PVD, and r_e = radius of the unit cell. The solutions for u and u' can be expressed as:

$$u(r,z,t) = -p_{vac} + (p_{vac} + p_s)v(r,z,t) \text{ for } (r_s<r\le r_e) \quad (14)$$

$$u'(r,z,t) = -p_{vac} + (p_{vac} + p_s)v'(r,z,t) \text{ for } (r_w<r\le r_s) \quad (15)$$

wherer_s = radius of the smear zone. The additional conditions for getting explicit expressions for v and v' are the following water flow continuity conditions.

(1) The total inflow of pore water through the boundary of a cylinder with a radius of r has to be equal to the change in volume of the hollow cylinder with outer radius of r_e and inner radius of r.

(2) The pore water flow into the PVD from a horizontally cut soil slice is equal to the change of vertical flow rate in the PVD.

At the interface between the smear zone and the undisturbed zone, the radial flow rate from the undisturbed zone is equal to the flow rate into the smear zone.

With these conditions and using the same assumptions as those adopted in obtaining Hansbo's (1981) solution, it can be shown that the expressions for $v(\mathrm{r}, z, t)$ and $v'(\mathrm{r}, z, t)$ are as follows:

$$v'(r,z,t) = \frac{k_h}{k_s \cdot r_e^2 \cdot \mu}\left[\begin{array}{l} r_e^2 \ln\frac{r}{r_w} - \frac{r^2 - r_w^2}{2} \\ + \frac{k_s}{k_w}(n^2-1)(2lz - z^2)\end{array}\right]\exp\left(\frac{-8T_h}{\mu}\right) \text{ for } (r_w \le r \le r_s) \quad (16)$$

$$v(r,z,t)=\frac{1}{r_e^2\cdot\mu}\left[\begin{array}{l} r_e^2\ln\frac{r}{r_s}-\frac{r^2-r_s^2}{2}+\frac{k_h}{k_s}\left(\begin{array}{l} r_e^2\ln s\\ -\frac{r_s^2-r_w^2}{2}\end{array}\right)\\ +\frac{k_h}{k_w}\left(n^2-1\right)\left(2lz-z^2\right)\end{array}\right]\exp\left(\frac{-8T_h}{\mu}\right)\text{ for}$$

$$(r_s < r \le r_e)\quad(17)$$

where: $n = r_e/r_w$, $s = r_s/r_w$, k_h and k_s = the hydraulic conductivities in the horizontal direction of the undisturbed zone and the smear zone respectively, k_w = the hydraulic conductivity of the drain (PVD), l = the drainage length of a PVD, and $T_h = c_v{\cdot}t/(4r_e^2)$. Parameter μ represents the effects of PVD spacing, smear zone and well resistance. Adopting an average well resistance and with some approximation, the expression forμ is as follows (Hansbo 1981):

$$\mu=\ln(n/s)+(k_h/k_s)\ln(s)-\frac{3}{4}+\frac{2l^2\cdot k_h}{3r_w^2k_w}\qquad(18)$$

The average degree of consolidation (U_h) of the unit cell is (Hansbo 1981):

$$U_h=1-\exp\left(-8T_h/\mu\right)\qquad(19)$$

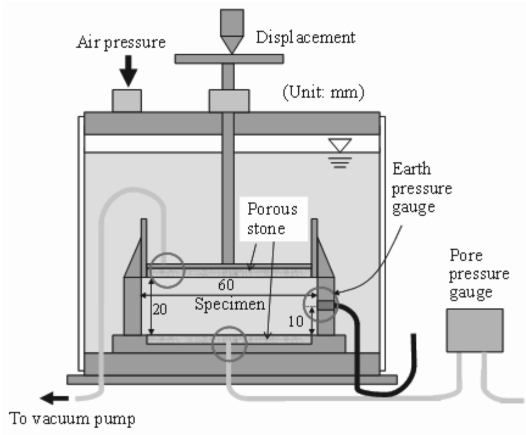
3 COMPARISON OF TEST RESULTS AND PREDICTIONS

Laboratory consolidation tests involving the combination of a vacuum pressure and a surcharge load have been conducted under oedometer conditions with both verticaland radial drainage (the latter to simulate the effects of PVD drainage), and the measured excess pore pressures have been compared with the predicted values.

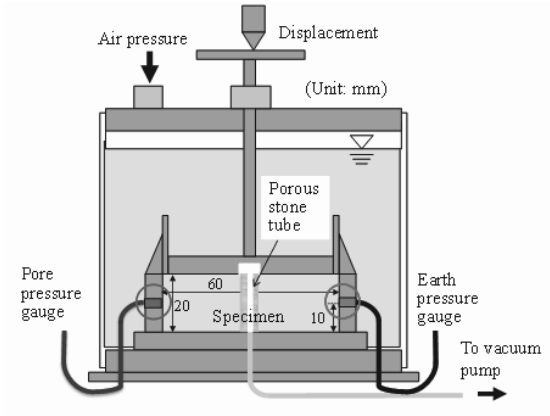
3.1*Test details*

Figures 2(a) and (b) show the set-up of the tests, with vertical (V-test) and radial (R-test) drainage conditions,respectively. During testing, the settlement, the excess pore water pressure at the bottom of the sample (V-test) or the middle height of the consolidation ring (R-test), and the horizontal earth pressure at the middle height of the consolidation ring can be measured. For the R-test, the centre drainage porous stone tube has an outer diameter of 8 mm, which is inserted into a predrilled hole at the center of a sample with a filter paper placed between the soil sample and the tube. The soil samples were re-consolidated from Ariakeclay slurries under a surcharge pressure of 20 kPa.

Two series of tests, V-tests and R-tests, were conducted. Here only one test from each serieshas been chosen to compare with the values predicted by the theory presented above. In the case of the V-test, the test with one-way drainage conditions has been selected, because for two-way drainage conditions no pore water pressures were measured with the device used. The two series of tests were conducted at different times and different soil samples were used. Some of available soil properties as well as the test conditions are listed in Table 1. In this table, the vertical effective stress, σ'_{v0}, indicates that the soil sample was first consolidated under σ'_{v0} (simulating the initial effective stress of the soil sample at a specified depth in the deposit) and then the consolidation test was conducted by applying additional incremental consolidation pressures (vacuum pressure and surcharge load).



(a) Vertical drainage test



(b) Radial drainage test

Figure 2. Sketch of the set-up of the tests

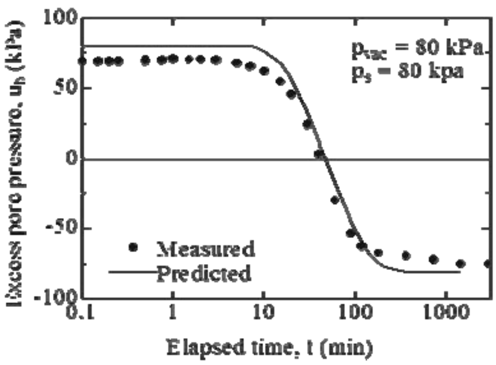


Figure 3. Comparison of predicted and measured u_b values

3.2*Comparison of measured and predicted pore pressures*

3.2.1*V-test*

After initial consolidation under σ'_{v0} = 40 kPa, the thickness of the sample was 18.7 mm (or compression of about 1.2 mm). Further, under 80 kPa vacuum pressure and 80 kPa surcharge load, the additional compression was about 3.2 mm. Since the thickness of the sample is also the vertical drainage path length, in the predictions an average sample thickness of 17.2 mm was adopted. Comparison of the measured and the predicted excess pore water pressures at the bottom of the sample (u_b) is shown

Table 1. Some soil properties and test conditions

Test	Soil	Plasticity limit, W_p (%)	Liquid limit, W_L(%)	Coefficient of consolidation c_v or c_h (m^2/min)	C_c	e_0♣	σ'_{v0} (kPa)	p_{vac} (kPa)	p_s (kPa)
V-test	Ariake clay-1	60.3	120.5	2.3×10^{-5}#	0.75	2.5	40*	80	80
R-test	Ariake clay-2	56.8	120.3	5.0×10^{-6}	-	-	0	80	80

*: Initial vertical effective stress in the sample; #: The value was obtained by fitting the measured consolidation rate; ♣: After pre-consolidation under 20 kPa pressure.

in Fig. 3. Except for the fact that the measured initial value of $u_b$of about 72 kPa is slightly lower than the 80 kPa applied surcharge load, the prediction almost matches the measured data. The slightly lower initial u_b value may indicate that the specimen was not 100% saturated.

Although two-way drainage test was not conducted, using the same soil parameters as for one-way drainage test, and assuming the thickness of the soil sample is 20 mm, the predicted excess pore water pressure (u) distribution within the sample at different elapsed times are given in Fig. 4 to demonstrate the capacity of the proposed theory.

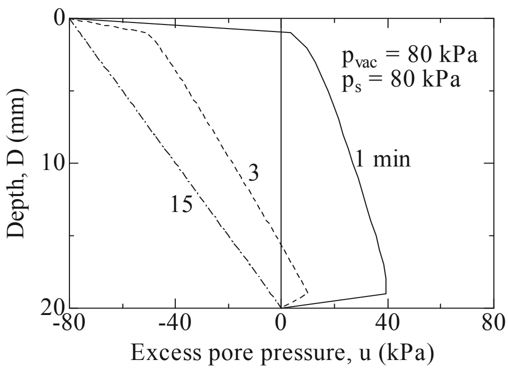


Figure 4. Predicted u variation in soil sample under two-way drainage boundary condition

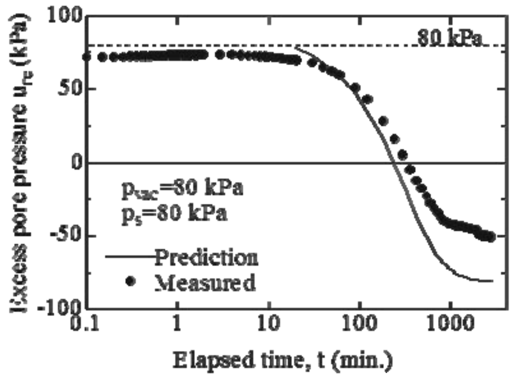


Figure5.Comparison of predicted and measured u_{re}

3.2.2 *R-test*

The geometric parameters required to calculate the predictions for this case are: r_e = 30 mm; r_w = 4 mm; and l = 20 mm. The assumed radius of the smear zone, r_s = 7 mm; the hydraulic conductivity ratio, k_h/k_s = 5; $k_h = 10^{-9}$ m/s; and $k_w = 10^{-4}$ m/s. In the case of radial drainage, with Eqs. (16) and (17) the initial condition of a uniform excess pore water pressure (u_0) distribution in a unit cell is not satisfied (which is a particular limitation of this theory). These equations only ensure that the average initial value of u_0 is the same as the applied value. The predicted initial value at the periphery of the sample (unit cell) is higher than the applied value. The predicted values are compared with the measured data from the time at which the predicted value at the periphery was equal to the applied initial value. In the physical test at the corresponding time, the pore water pressure at the periphery of the sample starts to reduce. Comparison of the excess pore water pressures at the periphery of the sample (u_{re}) is given in Fig. 5. For this case, during the consolidation period the measured excess pore water pressure initially decreased but then increased for a brief period before finally exhibiting further dissipation. Furthermore, the measured final excess pore water pressure did not reach the applied vacuum pressure of 80 kPa. Nevertheless, the trends of both the measured and the predicted dissipation curves are similar.

From the above comparisons, it can be seen that the theory provides reasonable predictions of the measured soil behaviour and so it should be able to be used reliably for designing preloading projects that adopt a combination of vacuum pressure and surcharge load to consolidate the soil deposit.

4 CONCLUSIONS

A consolidation theory, expressed in closed-form equations, for soil consolidation under the combination of a vacuum pressure and a surcharge load has been developed for a uniform clayey deposit with and without prefabricated vertical drain (PVD) improvement. For cases without PVD improvement, both one-way and two-way drainage boundary conditions have been considered.

Laboratory consolidation tests were conducted, adopting a combination of vacuum pressure and surcharge loading under oedometer conditions with both vertical and radial drainage. The excess pore water pressures measured in these test were compared with values predicted by the suggested theory. It has been demonstrated that the theory is valid and can be used for designing preloading projects that adopt a combination of vacuum pressure and surcharge load to pre-consolidate soft soil deposits.

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