

A smart adaptive multivariable search algorithm applied to slope stability in locating the global optima

Un algorithme adaptatif multivariable de recherche d'optimum global appliqué à la stabilité des pentes

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ABSTRACT: The paper addresses the topic of single objective optimisation of a three dimensional real-world problem and introduces a hybrid technique of an iterative random population search within a geometrically shrinking hypercube and a sort of simplified Design of Experiments (DOE). A 'population' of design variables are generated and augmented with the multivariable objective function, and the design variables pertaining to the local optima are perturbed by a factor (Δk) sequentially in both positive and negative directions to create $2(2^N-1)$ offspring in the neighbourhood of local optima to hopefully produce some better progeny. The 'fittest' perturbed offspring decides a new contracted search interval for the consecutive generation according to a geometric decay rule commensurate with the generation number. A 'simple hill-climbing' strategy in Artificial Intelligence context is followed subsequently and the loop is continued to produce fresh generations of refined offspring till the outcome converges to the global optimum. The method is applied in searching the critical slip-surface of a vulnerable soil-slope and it was revealed that the optimum found by this method is superior to that found by traditional and non-traditional (genetic algorithms) optimization techniques while using much less computational resources.

RÉSUMÉ: L'article traite de l'optimisation à une seule fonction objectif pour un problème réel tridimensionnel et introduit une technique hybride de recherche itérative à partir d'une population aléatoire au sein d'un hyper cube de taille décroissante, selon une méthode simplifiée de plan d'expérience. Une "population" de variables de design est générée, et étendue grâce à la fonction objectif multi variable. Les variables de design correspondant à l'optimum local sont alors perturbées par un facteur (Δk) de manière séquentielle dans les directions à la fois positives et négatives pour créer $2(2^N-1)$ individus de génération suivante dans le voisinage de cet optimum, dans l'espoir de produire une meilleure génération. La génération perturbée la plus satisfaisante selon l'objectif définit un nouvel intervalle de recherche, de taille réduite pour la génération suivante, selon une règle de décroissance géométrique en rapport avec le rang de génération. Une stratégie simple de plus grande pente dans un contexte d'intelligence artificielle est suivie pas à pas et une boucle produit de nouvelles générations améliorées jusqu'à ce que le résultat converge vers l'optimum global, indépendamment de la population initiale. La méthode est appliquée à la recherche de la surface de glissement critique d'un sol en pente vulnérable. Il a été constaté que l'optimum trouvé par cette méthode est plus critique que celui donné par les méthodes traditionnelles et non traditionnelles (algorithmes génétiques) et de plus, cette méthode est moins exigeante en terme de capacité de calcul.

KEYWORDS: hybrid technique, random population search, optimization algorithm, slope stability.

1 INTRODUCTION

The stability of slopes has received wide attention due to its practical importance in the design of excavations, embankments, and dams. There are numerous methods available for stability analysis and the majority of analyses performed in practice still use traditional limit equilibrium approaches. By the advent of computers, the use of optimization techniques in locating the critical slip surface has been a major topic for the researchers. Duncan (1996) presented a comprehensive review of both limit equilibrium and finite-element analysis of slopes. Malkawi et al (2001) developed an effective approach for locating the critical circular slip surface based on Monte-Carlo techniques. Non-traditional optimization algorithms simulating processes drawn from nature like genetic algorithm (GA) and simulated annealing (SA) have proved to be efficient in locating the global optima. GA mimics the principles of Darwin's natural selection and survival of the fittest rule, in which an optimum solution evolves through a series of generations of population and has the super ability of global convergence and parallel searching. SA is the stochastic evolution of thermodynamic state of slow cooling of molten metals to achieve a crystalline absolute minimum energy state, where a perpetual decreasing sequence of temperature controls the reproduction rate, which is very efficient in neighborhood search. Li et al (2009), Author (2011) proposed hybrid global search procedures combining GA with SA. While summarizing

the state of the art techniques for evolutionary algorithm (EA) parameter tuning, T. Beielstein (2003) exclaims that "*real world optimization problems allow only a few preliminary experiments to find good EA settings. As the commonly used 'one factor at a time approach' is considered as inefficient and ineffective, we recommend DOE methods*". The present paper introduces a new Soft Computing algorithm- a hybrid technique of an iterative *random population based search embedded with simulated annealing (SA) features within a geometrically shrinking hypercube coupled with simplified Design of Experiments (DOE)*.

2 APMA-A NOVEL OPTIMIZING TOOL

A smart adaptive population based multivariable function optimization algorithm proposed herein, and referred to as APMA hereinafter, is a simple yet robust optimization procedure basically of heuristic nature. Before plunging into details, a fitness function is defined to maintain uniformity over various problem domains and to map the 'goodness' of the objective function (here FOS) value to a fitness value. The fitness of an individual is calculated as the worst objective function objective function (FOS) value of the whole population subtracted from the individual's objective function value. Hence, this fitness function is computed for the individual as $F_i = \max. \{f(x)_j | j = 1, 2, \dots, n\} - f(x)_i$. Where; 'n' is the population

size. $f(x_i)$ is the objective function (here FOS of i^{th} individual). The technique is known as windowing as it eliminates the worst individual (FOS)-the probability comes to zero, and stimulates the better ones.

Within the random generated set of population (n) of design multivariable (N-dimensions or feature vectors), $f(x_i, y_i, z_i, \dots, w_i)$ within wide deterministic search boundaries for each variable, and subsequent objective function evaluations, the minimum is located. With a view to exploit the search space neighbourhood

the N-multivariable set $f(x_{i_{\min}}, y_{i_{\min}}, z_{i_{\min}}, \dots, w_{i_{\min}})$ creating this minimum is perturbed by a factor (Δk) sequentially in both directions (positive as well as negative directions) $f(x_{i_{\min}} \pm \Delta k), (y_{i_{\min}} \pm \Delta k), (z_{i_{\min}} \pm \Delta k), \dots, (w_{i_{\min}} \pm \Delta k))$ to

$$2 \sum_{i=1}^{i=N} {}^N C_i = 2 \sum_{i=1}^{i=N} \frac{N!}{(i!)(N-i)!} = 2(2^N - 1)$$

create $\{ \text{Where, } N! = N.(N-1).(N-2) \dots \dots 3.2.1 \}$ offspring in the neighbourhood of the local minimum in an attempt to generate some superior offspring. Hence, for a three variable function (like slope-stability problem) $f(x_i, y_i, z_i)$, a population (of n individuals) are randomly generated and their local minimum $f(x_{i_{\min}}, y_{i_{\min}}, z_{i_{\min}})$ located for the first generation.

Thereafter, this local minimum is perturbed initially in the positive direction and function evaluations are made at $f(x_{i_{\min}} + \Delta k), (y_{i_{\min}}, z_{i_{\min}}); f(x_{i_{\min}}, (y_{i_{\min}} + \Delta k), z_{i_{\min}}); f(x_{i_{\min}}, y_{i_{\min}}, (z_{i_{\min}} + \Delta k)); f(x_{i_{\min}} + \Delta k), (y_{i_{\min}} + \Delta k), z_{i_{\min}}); f(x_{i_{\min}}, (y_{i_{\min}} + \Delta k), (z_{i_{\min}} + \Delta k)); f(x_{i_{\min}} + \Delta k), (y_{i_{\min}} + \Delta k), (z_{i_{\min}} + \Delta k)); f(x_{i_{\min}} + \Delta k), (y_{i_{\min}} + \Delta k), (z_{i_{\min}} + \Delta k));$

and finally in the negative direction, by changing the sign of Δk in above expressions. Hence, apart from objective function evaluations of all n number of population individuals in each generation, the algorithm requires further function evaluations at $2(2^N-1)$ points around the local minimum of each generation. Now as fresh generations are produced, often different selection pressures of reproduction are needed at successive generations. Hence, the choice of a value of the sequential perturbing factor (Δk) becomes of paramount importance, in a sense that if it is too large it will be adequate in the first phase of the search but not in the final phase and vice versa if it is too small. Further, this Δk is likely to be different for different design variables commensurate with their individual feasible search intervals. As such, the situation calls for some sort of normalization of Δk in initial phase for application in various problem domains, and further it should possess the flexibility of shrinking itself automatically in successive generations according some decay rule in compliance with the selection pressure criteria. In this study, this perturbing parameter is defined as $\Delta k = \kappa.(x_i^U - x_i^L) \Delta_i$, where, κ is a problem specific constant; to be fixed after some initial trials. x_i^U and x_i^L are the upper and lower bounds of a design variable and $\Delta_i [=G_i(1-r)^{(G_i-1)}]$ is the size reduction parameter, where, G_i is the generation number and 'r' a constant less than unity. As the number of design variables (N-dimensions) increases, more trials are needed for fixing the value of κ . The 'fittest' perturbed offspring

$f(x_{i_{\min}}^p, y_{i_{\min}}^p, z_{i_{\min}}^p, \dots, w_{i_{\min}}^p)$ of the i^{th} generation decides a new contracted search interval for the consecutive generation, wherein the value each design variable corresponding to the fittest perturbed offspring is assigned the central value of the interval. The two new extreme bounds of the fresh search interval is the product of the positive and negative value of this central value and the size reduction parameter, Δ_i . Thus, if the local minimum objective function of the previous generation be

$$f(x_{(i-1)_{\min}}^p, y_{(i-1)_{\min}}^p, z_{(i-1)_{\min}}^p, \dots, w_{(i-1)_{\min}}^p), \text{ the current (that is, } i^{\text{th}} \text{ search interval becomes, } \left[\left\{ x_{(i-1)_{\min}}^p \pm G_i(1-r)^{G_i-1} x_{(i-1)_{\min}}^p \right\}, \left\{ y_{(i-1)_{\min}}^p \pm G_i(1-r)^{G_i-1} y_{(i-1)_{\min}}^p \right\}, \left\{ z_{(i-1)_{\min}}^p \pm G_i(1-r)^{G_i-1} z_{(i-1)_{\min}}^p \right\}, \dots, \left\{ w_{(i-1)_{\min}}^p \pm G_i(1-r)^{G_i-1} w_{(i-1)_{\min}}^p \right\} \right]$$

A new population of random variables (of size n) is again generated from scratch within the new reduced stochastic search interval and again objective function evaluations are made for each set of new random variables so generated. The generated set replaces the initial one and the loop is continued to produce fresh generations of refined offsprings till the outcome converges to the global optima.

2.1 APMA efficacy checked with benchmark test functions

With a view to examine the performance of the algorithm, APMA is initially applied to some benchmark unconstrained global optimization test functions like Goldstein-Price's function, Hartman's function, Beale's function, Perm's function, Booth's function, Bohachevsky's function (A. Hedar), Six hump camel's back function and Xin-She-Yang's function (X.-S. Yang, 2010) and promising results were obtained. APMA successfully captured all the four optima of the multimodal Himmelblau function (Deb K., 2000): $f(x_1, x_2) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$, [the optima being (-2.805, 3.131); (-3.779, -3.283); (3.584, -1.848); (3,2)] in each and every simulation run while finally converging to the global minimum at (3,2) [The simulation runs has been done with $\kappa=80-90\%$ & $r=0.10$]. The method may be regarded as a basic thrust of 'Artificial Intelligence', that is to get the computer to perform tasks fast and automatically. The method is independent of the initial vector and as no specific search direction is used in this method, this random search method is expected to work efficiently in many classes of problems.

3 THE PROBLEM DEFINITION

A problem cited by Spencer (1967) is chosen for analysis. The problem parameters, soil-data and search boundaries are depicted in Fig.1. In the search process, the three independent design variables are the abscissa (CX) and ordinate (CY) of the circle centre and the depth factor (N_d) of the circular failure surface. The base width (B) and height (H) of slope are assumed as 60 meters and 30 meters respectively.

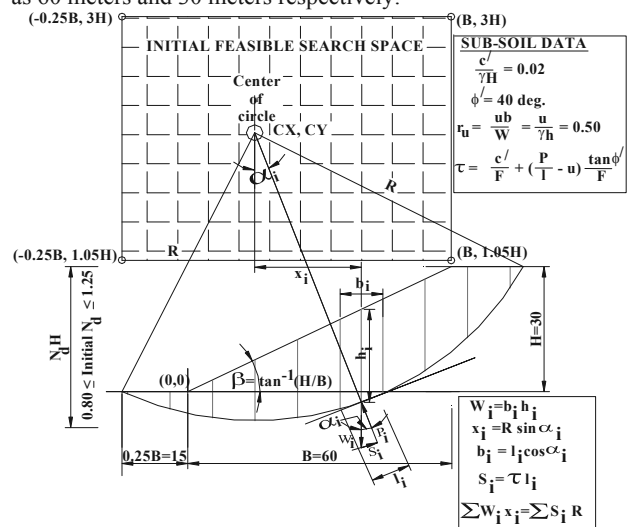


Fig.1. Initial variable bounds of the tri-variable & soil data used in search for critical circle-The Slope-Stability problem definition.

The radius $R=f\{CX, CY, N_dH\}$. Based on a few trials, the feasible bounds of the design variables, has been identified as: -

$0.25B \leq CX \leq B$, $1.05H \leq CY \leq 3H$, $0.80 \leq N_d \leq 1.25$. In the widely used limit equilibrium methods of slope analysis, the potential slip surface and the sliding mass are divided into segments or slices. The FOS, (F) is related to the total height of the slope H, the effective subsoil parameters c' , ϕ' and γ , the pore pressure ratio r_u ($= u/\gamma h$), the individual slices of width b_i , height h_i and α_i -the inclination of slice on the failure arc with the horizontal, by the following equation (Bishop, 1955):

$$F = \frac{\sum_{i=1}^n \left[\left\{ \left(\frac{c'}{\gamma H} \right) \left(\frac{b_i}{H} \right) + \left(\frac{b_i}{H} \right) \left(\frac{h_i}{H} \right) (1 - r_u) \tan \phi' \right\} \frac{\sec \alpha_i}{1 + \tan \alpha_i \tan \phi'} \right]}{\sum_{i=1}^n \left(\frac{b_i}{H} \right) \left(\frac{h_i}{H} \right) \sin \alpha_i}$$

A modest population size (n) of 20 is adopted. The design variables of the fittest population genre (local FOS_{min}) is perturbed sequentially by a factor Δk in both directions, resulting in $2(2^3-1)=14$ ‘offsprings’. **Initial value of Δk is fixed at 5% of the search interval** for each variable after some initial trials. Hence, Δk works out to be $3.75 [= .05x\{(60.00)-(-15.00)\}]$ for x_i -the abscissa of the slip-circle centre, $3.225 [= .05x(94.50-30.00)]$ for y_i -the ordinate of the slip-circle centre and $0.0225 [= .05x(1.25-0.80)]$ for z_i -the depth factor of the slide (refer Fig.2). **This Δk is further shrunk in successive generations by multiplying it with the size reduction parameter, $\Delta_i = G_i(1-r)^{(G_i-1)}$.** Again, the objective function evaluations are made for the perturbed individuals and the minimum of these 14 points is located. This local minimum point (offspring) acts as the ‘mother’ of the next generation, and the corresponding design variables acts as the mean of the search space of next generation.

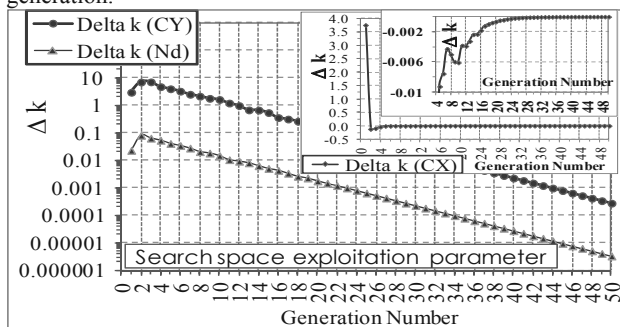


Fig.2. Search space exploitation in line with Design of Experiments.

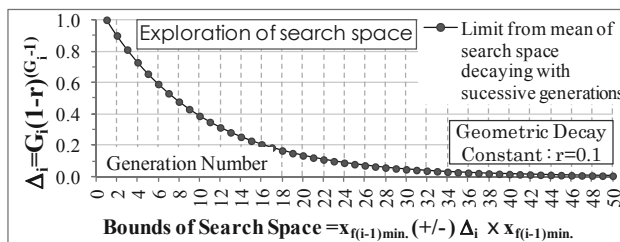


Fig.3. Search space exploration analogous to simulated annealing schedule by contraction of search boundary in successive generations.

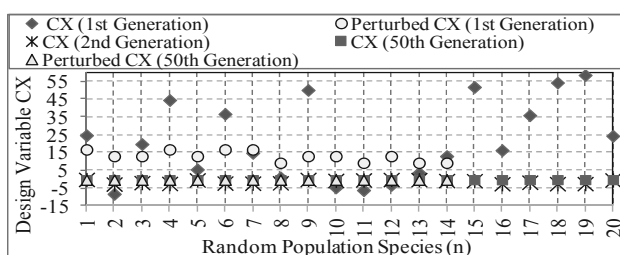


Fig.4. Initial deterministic search space turns heuristic at 2nd Generation—a quick shift towards the best part of search space.

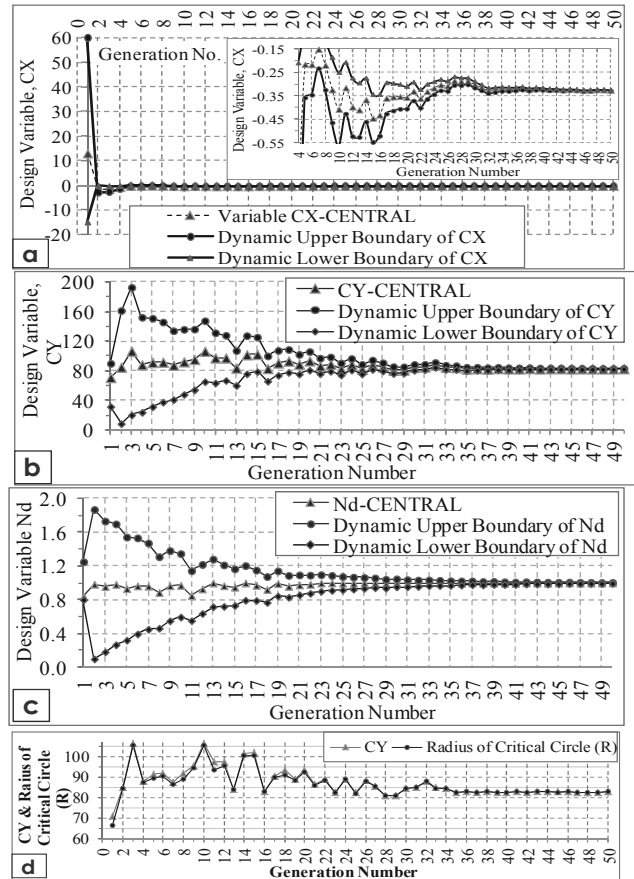


Fig.5. Robust movement of variable bounds along with the central value of tri-variables (CX, CY, N_d) shown in (a), (b), (c) respectively. In (a), a further close-up view from 2nd generation is shown. From (d) it emerges that $R \approx CY$ for critical circle.

4 COMPUTER SIMULATION & GRAPHICAL DEPICTION OF THE SMART ADAPTIVE PROCESS

Fig.3 shows how the limit of search boundary shrinks towards the mean (that is, the central value or the best point of the preceding generation) of the search space. Fig.4 shows a typical result of 1st, 2nd and 50th randomly generated population along-with the perturbed population set of 1st and 50th generations. **It may be noted that initial wide deterministic search space turns heuristic at 2nd generation with a quick shift towards the best part of search space.** Fig.5(a) to (c) depict the robust movement of variable bounds along with the central value of the three design variables (CX, CY & N_d), illustrating the generate-and-test heuristic search technique that exploit domain-specific knowledge. It emerged that whatever be the initial search bounds specified deterministically, the algorithm adjusts itself to move to the best part of search space in the immediate 2nd generation. It is revealed that **the bounds of CX are drastically reduced (Fig.5a), and that of CY & N_d are radically expanded (Fig.5b&c) in immediate 2nd generation**, and thereafter the bounds move steadily with successive generations that are guided by the mean of the search space, while maintaining a heuristic character. It emerged that the value of radius of critical circle (R) almost merges with the ordinate (CY) of the critical slip circle (Fig.5d). Fig.6 depicts the change in average fitness, standard deviation and variance of fitness function of population with successive generations. The variance of fitness decreases steadily with increasing generations maintaining its randomness. Fig.7 shows the maximum fitness of each generation vs. search space size reduction parameter in log-log scale. Fig.8 portrays the stochastic movement of Min. FOS in successive generations to converge to global optimum, wherein results of five simulation runs are superimposed. The inherent

stochastic nature of the algorithm showing value convergence with increasing generations is reflected in all the results.

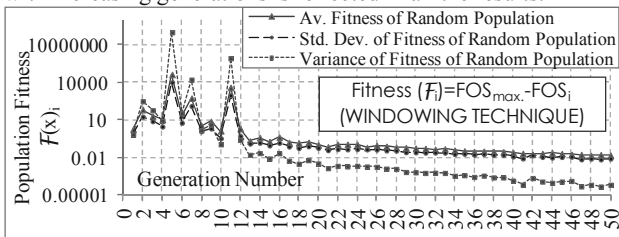


Fig.6. Change in average fitness, standard deviation & variance of population fitness function with successive generations.

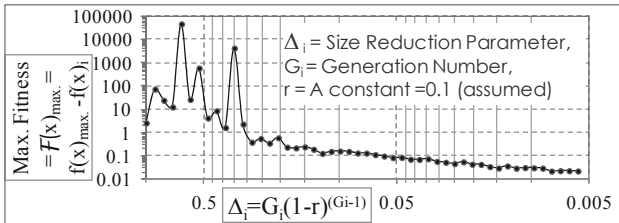


Fig.7. Max^m fitness of each gen. vs. search space size red. parameter.

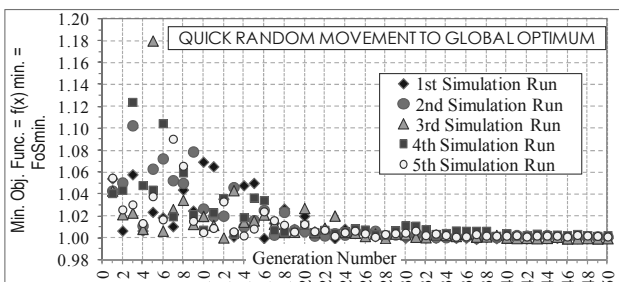


Fig.8. Stochastic movement of Min. FOS (obj. func.) in successive gens. to converge to global optimum (Results of five simulation runs).

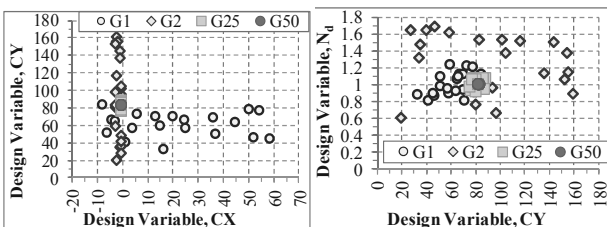


Fig.9. CX vs. CY & CY vs. N_d: The artificial intelligent character depiction. Fast random movement of candidate solutions to best part of the search space with increasing generations.

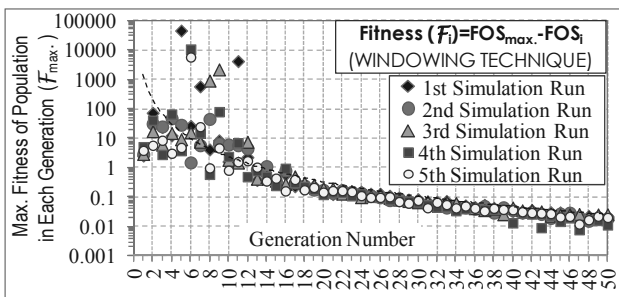


Fig.10. Stochastic decrease of maximum fitness in successive generations (Results of five simulation runs superimposed).

Fig.9 depicts the artificial intelligent character of the algorithm wherein a fast random movement of candidate solutions with increasing generations to best part of the search space is noticed. Fig.10 shows the stochastic decrease of maximum fitness in successive generations in 5 simulation runs. Fig.-11 gives the validation of results against other studies reported in literature.

5 CONCLUSIONS

A global population based search procedure (APMA) is developed and successfully applied to slope-stability problem. It does not require problem specific knowledge in searching the critical slip-surface of a soil-slope and is a heuristic technique based on the ‘generate-and-test’ strategy. Diverse aspects are presented to demonstrate its efficiency and robustness. The spectrum of application area of APMA is widespread as it is a direct search method where no specific search direction is used and multivariable functions, both continuous and discontinuous can be handled. Function value evaluations at discrete points only enable it to handle non-differentiable functions at ease. The beauty of the process is that, it handles a number of designs in each simulation run. The result of some simulation runs revealed minimum factor of safety obtained by APMA is less than that found by directed grid search, variational method, GA and GA-SA hybrid.

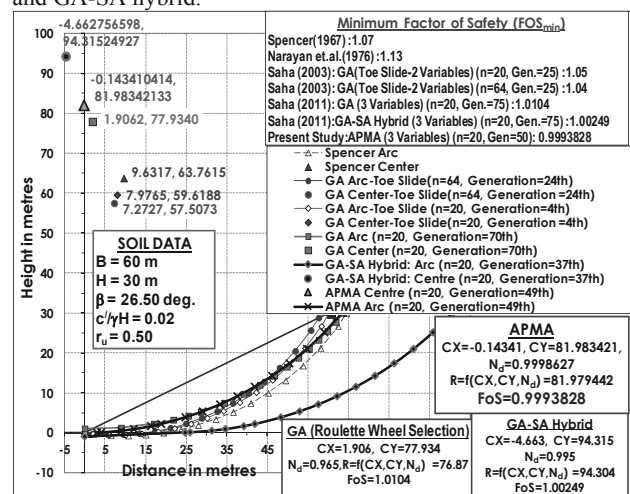


Fig.11. Comparison of results obtained by various methods.

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