Calculation of slopes stability based on the energy approach

Calcul de la stabilité des pentes sur la base de l'approche énergétique

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ABSTRACT: For quantitative evaluation of slope's stability value safety factor coefficient K is used, defined by various known methods. However, the results ensued in this way for identical objects may substantially differ. The reason for this are numerous assumptions and simplifications adopted in the calculations. Approach based on the analysis of the stress fields and displacement of soil mass points and principle of virtual displacements allows to get adequate results while solving the corresponding task. The used method of sliding surface constructing ensures realization of minimal condition of K value at each point. Analysis of the stress-strain state of homogeneous and heterogeneous slopes is accomplished by using finite element method (FEM), and the boundary conditions imposed on the design scheme FEM are processed based on the analytical solutions of the first basic boundary value problem of elasticity theory for homogeneous weight simply connected domain, get by the authors.

RÉSUMÉ : Pour l'évaluation quantitative de la stabilité des pentes on utilise la valeur du facteur de sécurité K, définie par diverses méthodes connues. Cependant, les résultats obtenus de cette manière pour des objets identiques peuvent différer substantiellement. Les raisons de cette situation sont de nombreuses hypothèses et simplifications adoptées dans les calculs. Obtenir des résultats satisfaisants lors de la résolution du problème correspondant devient possible grace à l'aproche basée sur l'analyse des champs de tensions et de déplacements des points du massif de sol et aussi sur le principe des déplacements virtuels. La méthode utilisée de la construction de la surface de glissement assure l'accomplissement d'une condition de minimalité pour la valeur K dans chaque son point. L'analyse de l'état précontraint et déformé des pentes homogènes et hétérogènes est realisée en utilisant la méthode des éléments finis (FEM), tandis que les conditions aux limites, imposées au schema de calcul FEM, sont traitées sur la base des solutions analytiques du premier problème aux limites faisant partie de la théorie de l'élasticité pour une domaine homogène simplement connexe, obtenues par les auteurs.

KEYWORDS: slope, safety factor, stresses, displacements, work, restraining and moving forces

1. QUESTION STASUS

Principle of virtual displacements - one of the basic principles of mechanics, evaluating general condition of equilibrium of the mechanical system. It is widely used in static studies of material systems, with the effect imposed on the system connections, considered by adding appropriate reactions. This principle has been used previously in the stability of slopes calculation (Goldstein 1969, Dorfman 1975, Magdeev 1973, etc.), however, movements of the points lied on the most probable slip surface, are not appointed on the basis of the stress-strain state analysis of near the slope area and arbitrarily vary in magnitude until the condition K=Kmin is achieved.

In addition the following quite not correct assumptions can be provided:

– sliding prism splits into separate blocks, the interaction force and the friction between them is rarely taken into account; orientation of this forces and the position of the points of their application does not settle, and set arbitrarily to make a statically indeterminate task about equilibrium of blocks, statically determinate;

– the form and position of the fracture surface are taken in advance, in the process of problem solving their direct link with physical and mechanical properties of soils, slope geometry, surface loads is not established;

– almost always only one (and approximately in the form of bay weight) vertical σz component of the stress at near the slope area is taken into account;

– semi-infinite slope is considered (the fact of stress concentration in the transition region of the slope into the base is left out of the account); well-known calculation methods doesn't allow to take into account mutual influence of embankment's slopes and excavations on their stability, as in the design scheme base of the slope is not considered, and the daily surface of the slope is infinite, etc. and so on;

– in stability calculation such important value as the coefficient of lateral earth pressure ξo is not considered and etc.

All this makes it necessary to look for new solutions of the slope's stability problem, in particular, based on the analysis of the stress-strain state.

Known method (Potapova 2001), when the value of K is defined as algebraic sum of work of restraining and shearing forces acting at the points of most likely slip line

$$K = \frac{\Sigma A_{i\,y\partial}}{\Sigma A_{i\,c\partial}} = \frac{\Sigma(F_{i\,y\partial}\delta_i \cos\alpha_{i\,y\partial})}{\Sigma(F_{i\,c\partial}\delta_i \cos\alpha_{i\,c\partial})}, \qquad (1)$$

where $F_{i\,y\partial}; F_{i\,c\partial}$ - restraining and shearing forces agreeably; δ_i - complete movement; $\alpha_{i\,y\partial}; \alpha_{i\,c\partial}$ - angles between positive directions of the restraining and shearing forces and direction of the total displacement vector, respectively, in the i-th point of the sliding surface. Equation (1), in our opinion, is not quite correct, because in this equation full displacements of the points made of movements of the external load and dead weight of the soil are taken into account. Movements from dead weight of soil are formed during all period of the soil mass existence, and therefore their true values cannot be reliably determined using the theory of elasticity, which may affect the validity of the getting result. Ignoring of this fact can lead to getting while calculation overstated or, conversely, understated safety factors that may result in additional costs of material resources, or emergency initiation.

2. PROPOSED APPROACH

The following algorithm for calculating the value of the slope safety factor K of loaded soil slope, based on the joint application of the finite element method (FEM) and methods of complex function theory (Kolosov 1934, Muskhelishvili, 1966):

a) design scheme of FEM is made up provided with maximum possible degree of homogeneity of the finite element mesh observance, which provides the minimum width of the stiffness matrix of the system;

b) based on an analytical solution of the first fundamental boundary value problem of elasticity theory for a homogeneous isotropic half-plane with a curvilinear boundary (Bogomolov 1996) data handling of boundary conditions applied to uniform FEM scheme so that the stress values $\sigma_x; \sigma_z; \tau_{zx}$ at the corresponding points of the investigating region, found by FEM and analytical solution, coincided with the maximum degree of accuracy;

c) in the case of heterogeneous geological structure of the slope, therein geological elements endowing with corresponding physical and mechanical properties (solid weight γ, specific cohesion C; angle of internal friction φ, coefficient of elasticity E; coefficient of lateral pressure ξ_o (Poisson's ratio μ)) are extracted;

d) to the study area *n* external loads are applied, which according to all parameters coincide with loads specified in problem specification, but the values of their intensities $q_n \to 0$. If in the near the slope area plastic deformation regions are absent, the construction of most likely slip surface is performed by the method (Tsvetkov 1979), ensuring fulfillment of the condition of minimum value of K at each point. At the same time the safety factor value K_i in the i-th point of the soil mass is determined by the following expression

$$K_i = \frac{[0{,}5(\sigma_{iz} - \sigma_{ix})\cos 2\alpha_i + 0{,}5(\sigma_{ix} - \sigma_{iz})]tg\varphi}{0{,}5(\sigma_{ix} - \sigma_{iz})\sin\alpha_i + \tau_{ixz}\cos 2\alpha_i} + \to$$
$$\to + \frac{[\tau_{ixz}\sin 2\alpha_i + 2\sigma_{\text{св}}]tg\varphi}{0{,}5(\sigma_{ix} - \sigma_{iz})\sin\alpha_i + \tau_{ixz}\cos 2\alpha_i} \tag{2}$$

where: $\sigma_{\text{св}} = C/tg\varphi$ - cohesion pressure, numerator and denominator of the formula (2) determine, respectively, the numerical values of restraining $F_{y\partial i}$ and shearing $F_{c\partial i}$ forces acting in the i-th point of the slip surface along the most likely site of shift. Analysis of formula (2) shows that the value of K_i depends on the components of the stress and angle α_i most likely site shift at each point of sliding surface, as well as physical properties of soil.

The value of α_i is defined by formulas (4), which arise from the boundary condition (3)

$$\left.\begin{aligned} &\frac{\partial K}{\partial\alpha} = 0; \\ &\frac{\partial^2 K}{\partial\alpha^2} > 0. \end{aligned}\right\} \tag{3}$$

$$\sin 2\alpha_i = 2N\tau_{ixy} + (\sigma_{iy} - \sigma_{ix})\sqrt{\frac{1}{L} - N^2} \tag{4}$$

where:

$$L = (\sigma_{ix} - \sigma_{iy})^2 + 4\tau_{ixy}^2;\ N = (\sigma_{iy} + \sigma_{ix} + 2\sigma_{\text{св}})^{-1}$$

If the plastic deformation regions have been developed, but their sizes are small, then the construction of the most probable slip surface is carried out by a method built on the basis of approximate solutions of the mixed problem of elasticity theory and plasticity theory of soil (Bogomolov 1996). In this case, the stress at the plastic region (marked by a prime) are given by

$$\left.\begin{aligned} \sigma'_{iz} &= \sigma_{iz}; \\ \sigma'_{ix} &= \frac{\sigma_{iz}(l - \sin\varphi) - 2\sigma_{\text{св}}\sin\varphi}{l + \sin\varphi}; \\ \tau'_{ixz} &= \frac{(\sigma_{iz} + \sigma_{\text{св}})b\sin\varphi}{l + \sin\varphi}, \end{aligned}\right\} \tag{5}$$

where $b = tg2\alpha_i^*; l = (1 + b^2)^{0,5}; \alpha_i^* = \alpha_i - (\pi/4 + \varphi/2)$.

The value of the safety factor coefficient K_i' at the point of the slope, which is in the plastic area (obviously $K_i' = 1$) is given by

$$K_i' = \frac{(\sigma_{iz} + \sigma_{\text{св}})\sin\varphi(\cos 2\alpha_i^* + b\sin 2\alpha_i^*) + \sigma_{iz}l + \sigma_{\text{св}}b}{(\sigma_{iz} + \sigma_{\text{св}})(b\cos 2\alpha_i^* - 2)\cos\varphi} . \tag{6}$$

The numerator and denominator of formula (6) determine values of appropriate restraining and shearing forces. In m points of sliding surface with the help of FEM total displacements from the gravity forces and the external load are calculated. Note that construction of the most probable slip surface is carried out with such a step, that the difference between the horizontal coordinates of neighboring points on it is $x_{k+1} - x_k = ...x_{m+1} - x_m = 0{,}01\text{H}$ (where H - the height of the slope).

e) the value of the intensity of the external load is increased to the calculated value. Once again we construct the most probable slip surface and the calculation of the total displacements in its *m* points.

f) the principle of virtual displacements is used, at the same time the role of the possible movements is carried out by *differences* of movements in corresponding *m* points of the sliding surface, obtained under the use of two previous steps of the algorithm. The role of active forces is played by restraining and shearing forces acting at the same *m* points of slip surface. The formula for calculating the safety factor value of the slope, provided that the plastic deformation is absent, has the form

$$K = \frac{\Sigma A_{i\,y\partial}}{\Sigma A_{i\,c\partial}} = \frac{\Sigma(F_{i\,y\partial}(\Delta_i - \delta_i)\cos\alpha_{i\,y\partial})}{\Sigma(F_{i\,c\partial}(\Delta_i - \delta_i)\cos\alpha_{i\,c\partial})}, \tag{7}$$

where: δ_i and Δ_i - respectively the total displacement of i-th point of the most probable slip surface, calculated from the action of its own weight of soil, provided that $q_n \to 0$, and taking into account dead weight of soil, provided that the value of the intensity of the external loads are equal to calculated values, i.e. $q_n = q_p$.

The value of the safety factor coefficient of the slope in the case of plastic regions in the near the slope area is determined by the formula

$$K = \frac{S'_{\text{уд}} + S_{\text{уд}}}{S'_{\text{сд}} + S_{\text{сд}}} \qquad (8)$$

where: $S'_{\text{уд}}$ and $S'_{\text{сд}}$ – squares of restraining and shearing force's diagrams, built along a section of the most probable slip surface that is in the region of plastic deformation, $S_{\text{уд}}$ and $S_{\text{сд}}$ - squares of restraining and shearing force's diagrams, built for the area most likely slip surface, which is outside the boundaries of the plastic deformation regions. In that way, if in a soil mass there are no plastic deformation regions or their sizes are small, then the proposed procedure excludes from examination of displacement from dead weight of the soil, which, as noted above, it is impossible to determine exactly what increases the reliability of the getting results.

3. PROCEDURE FORMALIZATION

Described above procedure is formalized in a computer program (Bogomolov and others, 2010), which allows to calculate stability of homogeneous slopes and slopes of complex geological structure with any configuration of the outer boundary. At the same time all strength and deformation characteristics of the soil are taken into account, to the surface of the slope can be applied almost any number of point and distributed loads of any orientation, length and intensity. It is possible inside the design schemes automatically generate fields of various shapes, endowed with certain properties, including the voids. Figures 1 and 2 show the most probable slip surfaces in the absence of the plastic deformation areas and while their development. Here

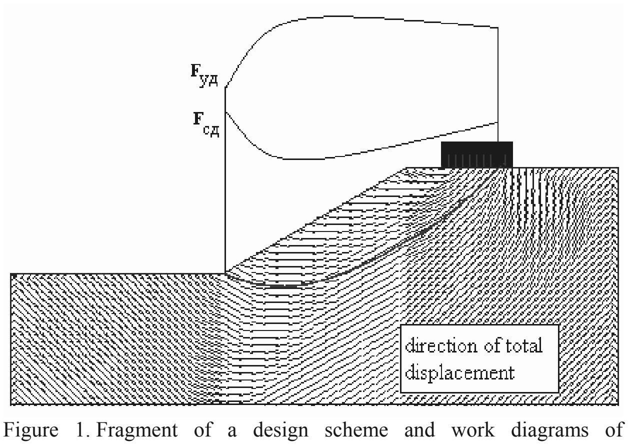

Figure 1. Fragment of a design scheme and work diagrams of restraining and shearing forces (plastic deformation areas are not presented)

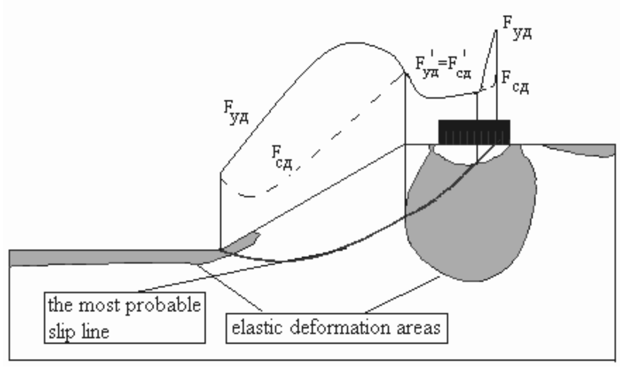

Figure 2. Work diagrams of restraining and shearing forces provided formation of plastic deformation areas

built in the computer program shell developed by the authors work diagrams of restraining and shearing forces acting at the points of the sliding surfaces are showed. Areas of diagrams are calculated automatically and are used in calculations of the safety factor value (see equation (8)).

Note one more circumstance. If the slope is uniform, then safety factor value calculated by the proposed method is independent from the value of deformation modulus of soil E_o, as in this case, displacements linearly depend on the numerical value. If the slope is not uniform, then the stresses at the points of soil mass will depend on size of the deformation modules and coefficient of lateral earth pressure of the nearby geological engineering elements. Consequently, the safety factor value will also be a function of these variables. This fact cannot be accounted by any of these methods, during the implementation of which analysis of the stress-strain state of the soil mass is not conducted.

4. COMPARISON OF NUMERICAL RESULTS WITH EXPERIMENTAL DATA AND FIELD OBSERVATIONS

We (Bogomolov & Vereshchagin, 1990) conducted experiments on the destruction of loaded slopes, made on the models from equivalent materials, as a basis for which were used: a) a mixture of river sand (97%) and motor oil (3%) having the following physical properties: γ=1,55 t/m^3; C=0,49 MPa; φ=24°; b) gelatinegel XC having the following characteristics: mass concentration of gelatine 30% - γ=1,15 t/m^3; C=72 MPa; φ=25,5°, and at 15% - γ=1,078 t/m^3; C=34,8 MPa; φ=13,5°. Value ξo for sand-oil mixture is set by pulling steel strips in a tray filled with the material which is under study (Terzaghi 1961), equal ξo=0,75.

Models from sand and oil mixture have height H=0,3 m, width L=0,6 m, β=75°. They were loaded with uniformly distributed load over the stamp which has the following dimensions in plan 0,6×0,5 m, consistently placed at a distance b=0;0,25H;0,5H from the edge of the escarpment. When b=0 the average value (based on 10 experiments) of the intensity of the breaking load q_p=1,84 *kPa*, when b=0,25H - q_p=0,78 *kPa*, and when b=0,5H - q_p=1,39 *kPa*. Calculated safety factor values for each of the three variants were found to be K_1=1,11; K_2=0,99; K_3=1,02. Evidently these values differs from limit value K=1 for no more than 11%.

Models from gelatingel *XC* formed in dismountable container made of organic glass, in the same forms loading of models up to failure was conducted, and at this moment loading size was fixed. Safety factor values, calculated on the basis of our proposals for the moment of destruction, were found to be $K_{30\%}$=1,08 and $K_{15\%}$=1,1, i.e. differ from the limiting value of K=1, on 8% and 10%.

Figure 3 show the most probable surface of failure obtained theoretically (1 and 2) and in experiments (1 'and 2'), which are almost identical to the corresponding models.

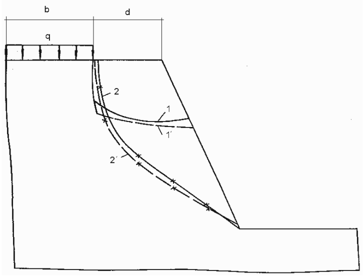

Figure 3. Theoretical and experimental slip lines in model made of equivalent materials

A case (Mochak 1964) of landslides formation in the quarry in Tsinzendorf (Germany) in the excavator ledge with the angle β=36° was described. Soils - ribbon clay with the following content 42,9-58,5 of silt and 30,6-43,5% of clay, consistency – flexibly soft, with φ=21°, C=17,2 kPa. The intensity of the load created by the excavator, which was at the edge of the slope (d=0), q=4,4 kPa, its width b=7 m.

Author (Mochak 1964) calculated the slope stability by the method of K. Tercagi, resulting in safety factor value determined to be KT=1,19. The calculation by the proposed method results in K=0,97, which is only 3% different from the limit.

5 CONCLUSIONS

Method for calculating of loaded slope’s stability, based on the combined use of the finite element method, complex function theory and the principle of virtual displacements, which is formalized in a computer program was suggested. The results of calculations with the help of the program that was announced with sufficient accuracy for engineering practice with the experimental data and behavior of landslide danger objects specifically.

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