Structural Optimization in Geotechnical Engineering

Optimisation de la structure dans la géotechnique

Pucker T., Grabe J.
Hamburg University of Technology, Institute of Geotechnical Engineering and Construction Management

ABSTRACT: Structural optimization methods are used for a wide range of engineering problems. In geotechnical engineering however, only limited experience exists with these methods. The possibilities and difficulties in applying such techniques to geotechnical problems are discussed in this paper and the adaption of the commonly known SIMP-method (Solid Isotropic Material with Penelization) to geotechnical problems is introduced. An application example is used to demonstrate the potential of structural optimization in geotechnical engineering.

RÉSUMÉ : Des méthodes de l'optimisation de la structure sont employés dans beaucoup de disciplines d'ingénieurs. Mais il y a quand-même peu d'expériences dans le domaine de la géotechnique. Les possibilités et les difficultés de l'application de ces procédures d'optimisation dans la géotechnique sont discutées dans cet article. L'application de la méthode SIMP est présenté. Un exemple est présenté pout montrer le potentiel de l'application de cette méthode.

KEYWORDS: topology, optimization, optimisation

1 INTRODUCTION

Structural optimization methods are used for a wide range of engineering problems, for instance in aviation and automotive engineering. In geotechnical engineering however, only limited experience exists with such methods. The possibilities and difficulties in applying these techniques to geotechnical problems are discussed in this paper and the adaption of the commonly known SIMP-method to geotechnical problems is introduced.

1.1 *Structural Optimization*

Structural Optimization can be classified in three categories. The first category is called topology optimization and describes the main geometry of a design. Topology is a mathematical field used to describe geometrical structures. A geotechnical example of different topologies to stabilize a slope is shown in Figure 1.

Figure 1a) shows the design problem. Different constructions can be used to solve this task, for example a gravity wall (Figure 1b), a single anchored wall (Figure 1c) or a grouted anchored wall (Figure 1d). These three constructions differ in their topology. Some topology optimization algorithms can be found in Bendsøe (1995) or Allaire (2005).

Once a topology is chosen, the shape of the topology can be optimized with regard to the design problem. Concerning the design problem in Figure 1, the topology in Figure 1d is chosen. Figure 2 shows different possibilities to optimize the shape of the topology in Figure 1d during the category shape optimization, for example the variation of the anchor positions (b), of the anhor inclination (c) or the anchor length (d). The variations in Figure 2 b)-d) can be varied regarding an optimization task, for instance the minimization of the bending of the wall or minimization of the installation costs. The application of shape optimization algorithms in geotechnical application has been shown in Kinzler (2007) and Grabe et al. (2010, 2011).

The third category is the dimension optimization of each construction part. Within this category every cross section and dimension is determined. Neither the topology nor the shape can be changed by the dimension optimization. The dimension optimization is the most widely applied category of structural optimization in geotechnical engineering. The shape optimization of Figure 2 results in the shape shown in Figure 3. For this example, possible parameters for the dimension optimization are given in Figure 3.

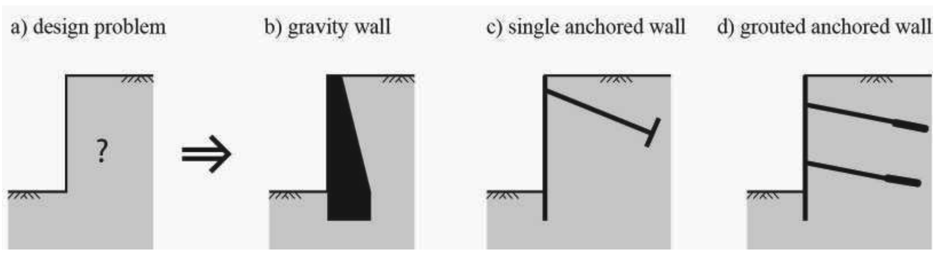

Figure 1. Different topologies for a design problem a): b) gravity wall, c) single anchored wall, d) grouted anchored wall

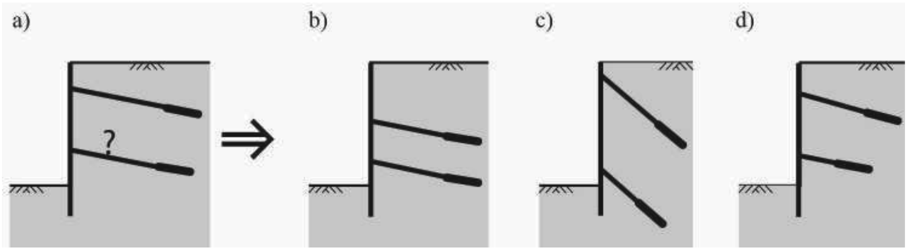

Figure 2. Shape optimization: a) main topology, b) variation 1: anchor position, c) variation 2: anchor inclination, d) variation 3: anchor length

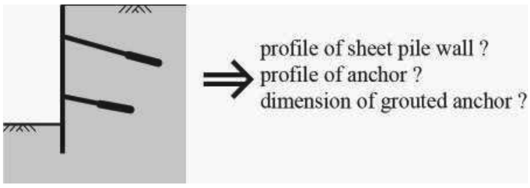

Figure 3. Dimension optimization

2 NUMERICAL OPTIMIZATION ALGORITHM

An iterative algorithm is used to solve the optimization problem numerically. First a topology is created and second its load-displacement behavior is determined using a finite-elemente analysis. In the next step, the results of the finite-element analysis are interpreted and transmitted to a topology optimization algorithm, which creates a new improved topology. Afterwards step two is performed again.

The SIMP-Method (Solid Isotropic Material with Penelization) after Sigmund (2001) is used as topology optimization algorithm. The algorithm is based on the idea, that the material of the optimized structure already exists in the design domain *Ω*, but is not optimally distributed. Therefore, the material is equally distributed in the design domain Ω at the beginning of the optimization process. The material distribution changes during the optimization process and the material compacts in areas where it is needed to achieve the optimization task.

The design domain *Ω* is descretized with finite elements. The material parameters are specified individually for each element depending on the material distribution. The virtual density ρ at a point a has to be between 0 and 1, see Equation 1.

$$\rho(a) = \begin{cases} 0 \rightarrow \text{no material} \\ 1 \rightarrow \text{material} \end{cases} \tag{1}$$

Regarding a geotechnical application, for example a foundation made of concrete, a finite-element with $\rho(a) = 0$ is a soil element and with $\rho(a) = 1$ is a concrete element.

Using the SIMP-Method, the objective function is the minimization of the compliance of the structure in the design domain *Ω*. Thus, the stiffness of the structure is maximized. The compliance of the structure can be expressed using the internal energy of the system. The internal energy c of an elastic material is defined by Equation 2.

$$c(\mathbf{x}) = U^T KU \tag{2}$$

In Equation 2 U is the global deformation tensor, K the global stiffness matrix and **x** the tensor of design parameters. The virtual density $\rho(a)$ matches the design parameters of x_i of the tensor **x** at point a.

The minimization of the compliance is restricted by two constraints. The first constraint ensures that the observed system is in equilibrium an every step of the optimization process. Using the finite-element method, this constraint is ensured by Equation 3. F is the global tensor of the external forces.

$$KU = F \tag{3}$$

The second constraint ensures that the volume of the material distributed in the design domain remains constant during the optimization process, see Equation 4. V_δ is the volume of the structure.

$$\int_{\Omega^{mat}} 1 \, \mathrm{d}\Omega = V_\delta \tag{4}$$

Additionally, the design parameters x_i are limited by an upper and an lower bound, such that the optimized material parameters lie within to the physically possible range.

The optimization task for minimal compliance design can be written using Equation 5. U_e is the element deformation tensor, K_e the element stiffness matrix, ρ_e the element virtual density, δ the volume fraction, V_δ the structure volume and V_0 is the volume of the design domain. The values of the material distribution are limited by $x_{\min}$ to avoid singularities during the finite-element analysis. Using the algorithm for geotechnical application, this limit is not necessary because the stiffness of an element belongs to the soils stiffness at $x_i = 0$ and cannot tend to zero.

$$\left.\begin{array}{ll} \text{min :} & c(x) = U^T KU = \sum_e^N (\rho_e)^p U_e^T K_e U_e \\ \text{subject to :} & KU = F \\ & V_\delta = V_0 \cdot \delta \\ & 0 < x_{\min} \leq x \leq 1 \end{array}\right\} \tag{5}$$

The penalty p controls the material change-over to ensure complete material change for example from soil to concrete, see Figure 4.

The improved topology in every iteration step is determined using the method of optimal criteria, see Equation 6 (Bendsøe 1995). A positive move-limit m and a numerical damping coefficient η = 0.50 are introduced, see Bendsøe (1995). The move-limit m limits the change of the topology in each iteration step. The sensitivity of the objective function is expressed in Equation 7. Using the Lagrangian multiplier λ, B_e is defined in Equation 8.

$$\rho_e^{new} \begin{cases} \max(\rho_{\min}, \rho_e - m), & \text{if } \rho_e B_e^{\eta} \le \max(\rho_{\min}, \rho_e - m) \\ \rho_e B_e^{\eta}, & \text{if } \max(\rho_{\min}, \rho_e - m) < \rho_e B_e^{\eta} < \max(1, \rho_e + m) \\ \min(1, \rho_e + m), & \text{if } \min(1, \rho_e + m) \le \rho_e B_e^{\eta} \end{cases} \quad (6)$$

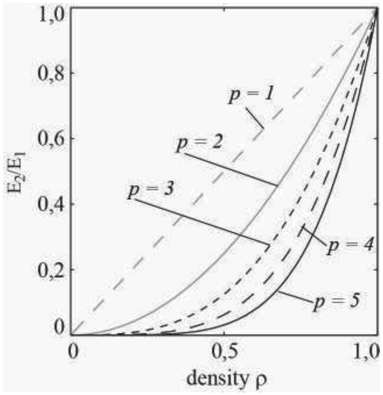

Figure 4: Change-over of the Young's modulus E of two different materials depending on different values of the penalty term p

$$\frac{\partial c}{\partial \rho_e} = -p(\rho_e)^{p-1} U_e^T K_e U_e \quad (7)$$

$$B_e = \frac{-\frac{\partial c}{\partial \rho_e}}{\lambda \frac{\partial V}{\partial \rho_e}} \quad (8)$$

3 APPLICATION

1.2 *Numerical model*

The presented topology optimization algorithm is applied to a vertically loaded strip footing foundation. The topology underneath the foundation is to be optimized. The inital width of the foundation is 2 m and the height is 1 m. The load-settlement behavior is simulated in a 2D finite-element analysis. The discretization of the model is shown in Figure 5.

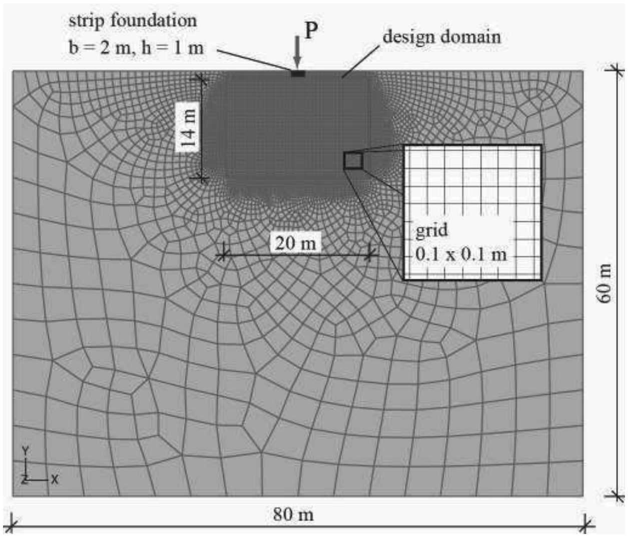

Figure 5. Discretization of the FE-model of a strip foundation with a width of 2 m and a height of 1 m, vertically loaded with 500 kN at the loading point P

The foundation is vertically loaded with 500 kN. The soil and the improved material underneath the strip foundation is modeled using the hypoplastic constitutive model after von Wolffersdorff (1996) with the extension of intergranular strain after Niemunis and Herle (1997). Detailed information can be found in Pucker and Grabe (2011).

1.3 *Optimization*

Three optimizations are performed at different material volumes. The material volume is 2%, 5% and 10% of the design domain. The design domain is 20 m of width and 14 m of height, see Figure 5.

1.4 *Results*

In Figure 7 the optimized topologies with 2%, 5% and 10% material volume are illustrated. Regarding the 2% material volume, mainly the aera at the foundation edges are compacted. Since the foundation can be considered to be rigid, the optimization results can be explained with the theory of a rigid foundation on an elastic half-space, according to which high stresses will occur at the edges of the foundation. The optimization algorithm compacts the material mainly in these areas.

Figure 6 shows the displacement of the soil underneath the foundation with the unoptimized (Figure 6 a) und the optimized (Figure 6 b) structure with a volume of 5%. The settlements can be reduced up to 50%.

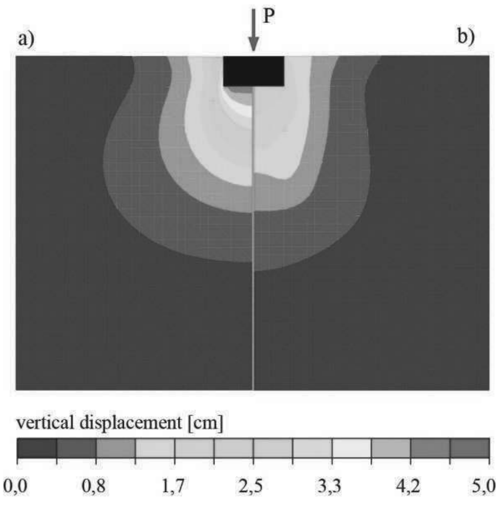

Figure 6. Vertical displacement before (left) and after (right) the topology optimization with 5% material volume

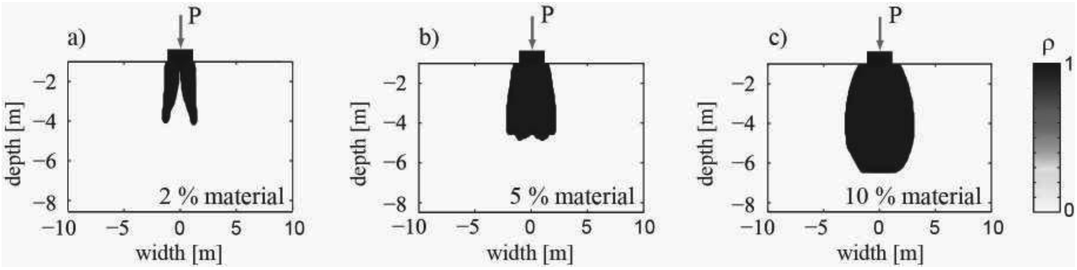

Figure 7. Optimized foundations with a) 2% material volume, b) 5% material volume, c) 10% material volume

The influence of the material volume on the improvement is shown in Figure 8. Figure 8 top shows the material volume over the vertical load P at different settlements. The applicable load at a settlement restriction about 1 cm cannot be significantly improved using more than 2% material volume. Regarding higher settlement restrictions, the increase of the material volume also increases the applicable load P.

The same results can be obtained from the load settlement curves of the improved foundations in Figure 8 bottom. The main improvement is reached with a material volume about 2%.

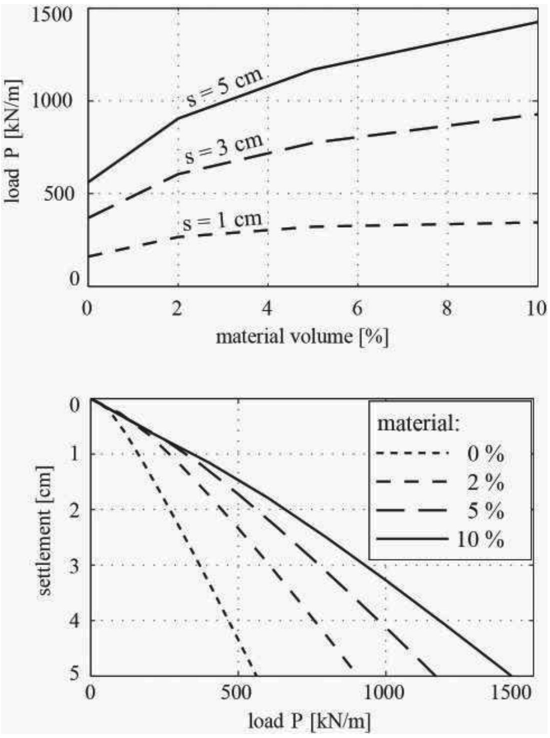

Figure 8. top: load P over material volume at different settlement restrictions; bottom: load-settlement curves of the optimized foundation topologies

1.5 *Practical realization*

The practical realization of the optimized topologies can be quite difficult. The optimized topologies of the presented example can be realized using the jet grouting method. This method allows the realization of every possible topology restricted by the minimum diameter of a jet grouting body.

Another possibility to realize such topologies is the interpretation of the topologies and the conversion of the topology into standard geotechnical construction parts. The realization of the topology with 2% material can be done using a classical strip foundation topology in combination with micro piles, see Figure 9.

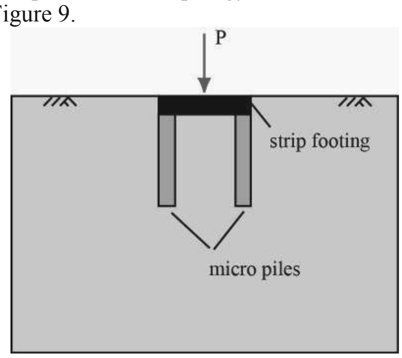

Figure 9. Possible practical realization of the topology with 2% material volume

2 CONCLUSION

The application of topology optimization in geotechnical engineering was presented. The applied SIMP-Method is suitable for geotechnical problems. In the presented example, the settlements of a strip foundation could be reduced up 66%.

Topology optimization in geotechnical engineering has a great potential and can lead to innovative and efficient designs.

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