Calculation method of optimization the soil-cement mass dimensions to reduce the enclosure displacements in deep excavation

Calcul des dimensions optimales du massif du sol-ciment pour réduire les déplacements de fouilles profondes

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ABSTRACT: In the article basic condition to the calculation the soil-cement massif optimum dimensions, which ensure the assigned value of deep excavation wall displacement are presented. Problem statement and it's solution with the application of a Winkler-bed model, standard procedures for analysis of massive retaining walls and theory of optimum design are described. Some results of calculations and benchmark of these results with the use of PLAXIS 2D software package are given.

RÉSUMÉ : Le procédé de calcul est décrit pour la détermination des dimensions d'une masse de sol-ciment en utilisant un modèle de "Winkler-bed", des procédures standard pour l'analyse des murs de soutènement poids, et la théorie de la conception optimale. Les résultats des calculs et leur évaluation à l'aide du logiciel PLAXIS 2D sont présentés.

KEYWORDS: diaphragm wall, optimal design, Winkler-bed model, coefficient of stiffness.

1 INTRODUCTION

Acceleration of work execution for underground space constructing in dense urban conditions with the minimum influence upon surrounding buildings is one of the main problems of underground structures design today. The given paper presents the design method of optimization of the raised problem design decision. The method of excavation "top-down" is used as technological scheme only with upper floor installation and as optimized (variable) parameter, the dimensions of soil-cement mass (SCM) combined with enclosure (diaphragm wall) that provides the excavation without intermediate strutting system and minimization of wall displacement (Fig. 1).

So the main task is to determine the minimum volume of the SCM with the condition that the horizontal displacements of the enclosure during excavation of the pit do not exceed the assigned value $S^v_{max} \leq S_{pred}$.

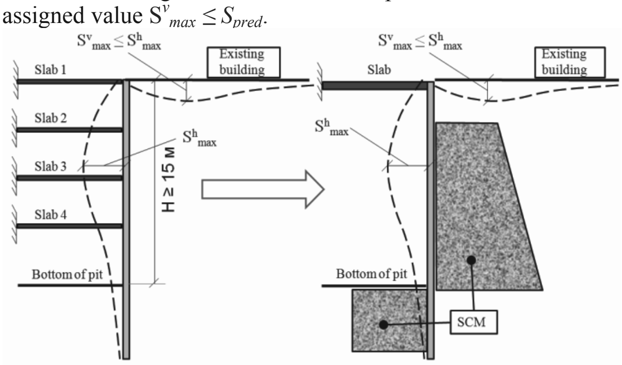

Figure 1. Problem condition

1 PROBLEM STATEMENT

The computational model of the optimized "wall-SCM-soil" system can be represented as a beam on an elastic Winkler bed, where the beam simulates the pit enclosure, and the SCM working in consort with the soil of the elastic bed. The stiffness coefficient of the bed, which varies over the height of the enclosure, is the only parameter of the model used to determine the dimensions of the SCM. The stiffness coefficient of the elastic bed, and the development of a procedure to determine the SCM dimensions corresponding to the optimal solution will therefore be the subject of optimization problem.

In its initial state of rest, the enclosure is treated as a beam affixed on two sides by prestressed springs that describe the "SCM-soil" system (Fig. 2, a, b), whereupon the prestress corresponds to the lateral pressure of the soil q_{01} in a state of rest (Fig. 2, c, d). As the pit is excavated, the prestressed springs (soil) then disappears on one side together with the pressure, which they have created, and the system is taken out of equilibrium. To attain equilibrium, the enclosure is displaced within the pit (Fig. 2, e). Here, the springs below the bottom of the pit are mutually disturbed on the inside, but are undisturbed on the outside, altering the pressure of the springs against the enclosure (Fig. 2, f). During compression, this pressure is increased, and is

$$q_2 = q_{02} + kz, \quad (1)$$

but is diminished on release

$$q_1 = q_{01} - kz, \quad (2)$$

whereupon the change in pressure will depend on the coefficient of stiffness k of each spring, and the displacement z of the enclosure at the corresponding point.

During excavation, therefore, the SCM as a component part of the soil, which is simulated by the springs (Fig. 2, g) is displaced inside the pit, and the pressure against the enclosure is changed in conformity with (1) and (2), and the pressure against the SCM remains as before q_{02} and q_{01} on the side of the soil, when no Coulomb active or passive pressure is formed.

Considering that the displacement of the soil and beam below the bottom of the pit is less than that within the bounds of the pit over its height, let us simplify the computational diagram. The work of the springs on the outside of the pit below its bottom can be neglected (Fig. 2, h), and replaced by a constant pressure of the soil in a state of rest. If these springs are eliminated from the computational diagram of the beam on the outside of the pit, the pressure of the soil against the enclosure below the bottom of pit will be $q_0 = q_{01} - q_{02}$, and the springs on the inside beneath its bottom will be under no prestress.

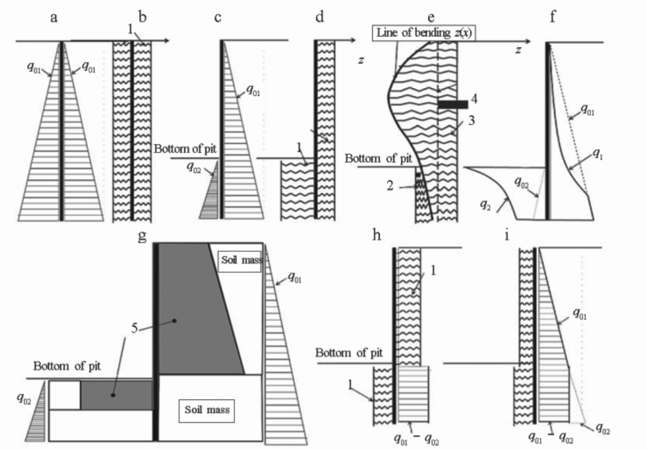

Figure 2. Steps in formulation of computational diagram: 1) elastic prestressed springs; 2) and 3) compression and release of springs; 4) position of enclosure prior to excavation of pit; 5) SCM.

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To convert to the standard diagram of a beam on an elastic bed, let us point out that the amount k*z by which the pressure of the springs is reduced against the enclosure during release in (2) is the reaction of ordinary springs with no prestress, which are positioned on the opposite side when acted upon by force q_{01}. The work of the presstressed springs is than equivalent to the work of stress-free springs under a load equal to the prestress.

Consequently, the problem reduces to one of a beam on an elastic bed (Fig. 2, i), where the beam represents the pit enclosure, the external load is the lateral pressure of the soil in a state of rest $q_0 = q_{01} - q_{02}$, and the coefficients of the elastic bed are the stiffness coefficients k of the "SCM- soil" system, which depend on the dimensions and shape of the SCM, as well on the strength and deformation characteristics of the soil. Considering the above, let is determine the following for further analysis:

1) the active pressure against the "enclosure-SCM-soil" system − the pressure q_0 of the soil at rest;

2) the contact pressure on the enclosure, which can be transmitted through the SCM

$$q = q_0 - kz; \tag{3}$$

3) the reactive pressure of the SCM − that portion of the active pressure against the system, which can be taken up by the SCM as the system transitions to a new equilibrium position after excavation of the pit

$$r = kz. \tag{4}$$

Depending on the displacements of the enclosure, the soil mass may function in an elastic stage, which can be described by the work of elastic springs, or a plastic stage when the contact pressure against the enclosure attains the active or passive pressure of the soil. Since the basic purpose of SCM use is to ensure preservation of surrounding development, the stress-strain state (SSS) of the surrounding soil mass should not go over into a plastic state, and the displacements of the enclosure during excavation should be so small that Coulomb's limiting equilibrium will not be realized. In the analyses, therefore, only the elastic work of the system, where the SCM ensures that the soil, and, accordingly, the "SCM-soil" system, will function elastically, is ensured. This enables us to perform the calculations in a linear statement, appreciably simplifying solution of the problem of the optimal design of the system under consideration.

In conformity with the physical essence of the problem, the contact pressure against the enclosure above the bottom of the pit cannot be less than zero (cannot act in the opposite direction due to excavation of the pit). For stiffness coefficients above the bottom of the pit should be satisfied the condition

$$q = q_0 - kz \geq 0. \tag{5}$$

Using (5) the "enclosure-SCM-soil" system, therefore, it is possible to use a finite-element formulation of a beam on an elastic bed, where the lateral pressure of the soil in a state of rest is the load.

Let us borrow terminology from (Hogg and Arora 1983) to construct the mathematical optimal-design model. Terms of the theory of matrix calculus can also be used when necessary.

In the computational model selected:

− the *equation of state* is the matrix equation of the finite-element method (FEM)

$$K(k)Z - Q = 0, \tag{6}$$

where $K(k)$ is the global stiffness matrix of the system, the elements of which will depend on k, and Q and Z are the vectors of the nodal loads and displacements, respectively;

− the *state variables* are the displacements Z at the nodes of the finite-element diagram, which describe the behavior of the system in question under load. The transposed form of the vector

$$Z = [Z_1, Z_2, Z_3, ..., Z_m] , \tag{7}$$

where m is the number of degrees of freedom and state variables $m = 2n + 2$, and n is the number of finite elements. After exclusion the angle of rotation from vector Z the vector z of horizontal displacements expressed as follows

$$z = [Z_1, Z_3, ..., Z_{m-1}]= [z_1, z_2, z_3, ..., z_{n+1}]; \tag{7a}$$

− the *design variables* are contained in the set of coefficients k, which describes the system itself, but not its behavior.

$$k = [k_1, k_2, k_3, ..., k_n] . \tag{8}$$

For the computational diagram under consideration, let us write in the terms of the problem statement of an optimal finite-dimensional design in state space.

It is required to determine the set of design variables k, which will minimize the efficiency function, as determined by the total stiffness coefficient with respect to all finite elements of the model

$$\psi_0 = \psi_0 (k) = \sum k_i \rightarrow \min, i = 1...n \tag{9}$$

when state equations

$$h(k, z) = K(k)Z - Q = 0 \tag{10}$$

and constraints

$$\psi(k, z) = [\psi_1(Z), \psi_2(k), \psi_3(k), \psi_4(k, Z)]^T \leq 0 \tag{11}$$

exist, where $\psi(k, Z)$ is the set of itch type of constraints, i = 1-4.

Let us define the types of bounded functions.

1. Bounded-function vector $\psi_1(Z)$ is determined from the conditions of the problem for which limits should be placed on horizontal displacements z (7, a) at all $n + 1$ nodes of the enclosure, which is broken down into n elements. The expressions $\psi_1(Z)$ are derived proceeding from the inequalities

$$z_1 \leq S_{\max}, i = 1...n + 1 \tag{12}$$

2. The vector function $\psi_2(k)$ reflects the limits placed on the design variables k. Expressions for $\psi_2(k)$ are derived, proceeding from the inequalities

$$k_i \leq k_{\max}; k_i \geq k_{\min}; i = 1...e, \tag{13}$$

where e is the number of the last element situated above the base of the pit, kmax is the value defining the upper limit of the variation of a variable, and $k_{min} = 0$ is the lower limit of variation in the absence of an SCM.

3. The vector function $\psi_3(k)$ reflects the limits placed on the design variables k below the bottom of the pit. The expressions for $\psi_3(k)$ are derived from the inequalities

$$k_i < k_{max};\ k_i > k_{min};\ i = e + 1...n, \quad (14)$$

where k_{min} is the lower limit of variation, and $k_{min} = k_{so}$ corresponds to the stiffness coefficient of the soil when functioning elastically.

4. The vector function ψ_4 (k, Z) reflects the constraint that ensures fulfillment of (5) − a reduction in pressure on the elements residing above the base of the pit, and cannot exceed the active horizontal load of the soil at rest on the corresponding element. The expressions for ψ_4(k, Z) are derived from the inequalities

$$r_i = k_i * z_i < q;\ i = 1...e + 1 \quad (15)$$

where r_i is the reduction in pressure, or the reaction of the elastic bed at the itch node, k_i is the coefficient of the elastic bed at the itch node, and z_i is the horizontal displacement z (7, a) at the itch node.

In the problem under consideration, the limits are represented by the following type of set:

$$\psi(k, z) = [\psi_1(Z), \psi_2(k), \psi_3(k), \psi_4(k, z)] \quad (16)$$

Using (9), (10), and (16), the structure and analytical form of the components of the mathematical optimal-design model are entirely defined and ready for the solution.

2 PROBLEM SOLUTION

The search algorithm for optimization based on the method of gradient projection, where that variation in design variables, for which the efficiency function is decreased, and the limits are not violated, is determined in each interval, is compiled for the problem's solution, and is implemented in the software package MATLAB v.7.9.0. The gradients of the efficiency and bounded functions with respect to design variables are required for construction of the algorithm are determined from analysis of the sensitivity of the design in state space (Hogg and Arora 1983).

The search strategies consist in the plotting of a succession of k^p points calculated in accordance with the rule

$$k^{p+1} = k^p + \delta k^p,\ p = 0, 1, ..., \quad (17)$$

where p is the number of iterations, k^p is a vector in the form of (8), and δk^p is the vector of variation in the design variables, which decreases the efficiency function as determined for each p using the gradients obtained from sensitivity analysis of the design.

Figure 3 shows a geometric interpretation of the performance of the algorithm for a two-dimensional space The resultant vector of the variation in the design δk_p is obtained as a result and the iteration process of the search for the conditional extremum acquires the form shown in Fig. 3

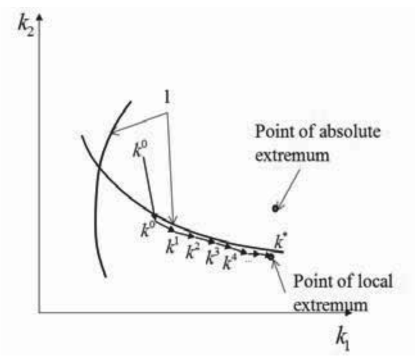

Figure 3. Geometric interpretation of algorithm performance:
1) curve of bounded function;

After running the algorithm, the succession of points converges on the optimal value of the efficiency function, i.e., the optimal distribution of k^*. As a result of running the algorithm, the sequence of points converges to the optimal value of the efficiency function, i.e., to the optimal distribution of the coefficient k^* of the bed's stiffness.

The dimensions of the SCM corresponding to this stiffness are required for determination of the optimal distribution of the coefficient k. The following are basic initial data for determination of the optimal dimensions of the SCM:

− the minimum height h of the SCM, as determined above and below the bottom of the pit based on the distribution curves of k and the reactive pressure R;

− the total reactive pressure of the SCM over the height h, which should be taken up by the SCM as a component part of the soil in order that the "SCM-soil" system correspond to the elastic behavior

$$R = \sum r_i, \quad (18)$$

where i is the number of nodes over the height h, and the point of application of the force R resides at the level of the center of gravity r of the plot; and,

− the average displacement z_{avg} of the enclosure over height h, which corresponds to the displacement of the SCM under the action of R

$$z_{avg} = \sum z_i / v, \quad (19)$$

where v and i are the number of nodes, and the numbers of the nodes over height h.

Use of R and z_{avg} enable us to convert to the assumption that as a component part of the soil, the SCM functions as a solid body, and is not calculated for individual elements over the height.

It is therefore required to determine the minimum dimensions of the SCM for which it will experience a horizontal displacement z_{avg} under the horizontal force R.

Let us examine the application of the above-indicated computational principles in an example of the calculation of optimal SCM dimensions for the excavation of a pit in sand with the following initial data: specific weight γ = 20 kN/m^3, overall compression modulus E = 25,000 kPa, angle of internal friction in shear φ = 30°, cohesion c = 1.0 kPa, pit depth of 20 m, enclosure depth of 30 m, a "diaphragm wall" enclosure with a thickness of 800 mm, and an upper thrust bracing consisting of a reinforced concrete span with a thickness of 500 mm.

Solution of the optimal-design problem (Fig. 4) includes curves of enclosure displacements (Fig. 4, a), distributions of the optimal stiffness coefficient over the height of the enclosure (Fig. 4, b), the reactive pressure (Fig. 4, c), and the contact pressure (Fig. 4, d).

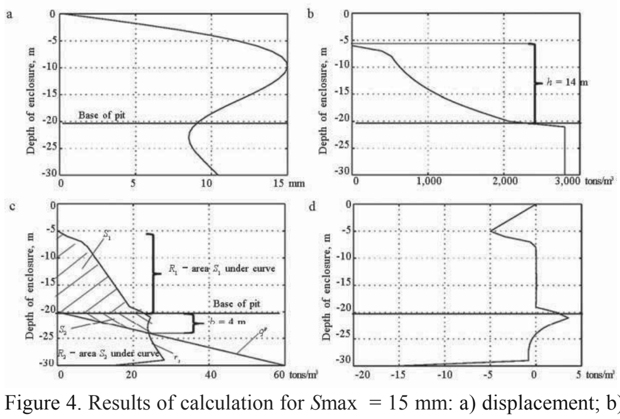

Figure 4. Results of calculation for Smax = 15 mm: a) displacement; b) stiffness coefficient diagram; c) reactive pressure; d) contact pressure

To determine the optimal dimensions, let us examine the SCM above and below the bottom of the pit.

According to the plot (Fig. 4, b), the SCM above the bottom of the pit extends to the point along the height where k = 0 and h = 14 m. For the enclosure above the bottom of the pit where k = 0, no soil cement is required. Let us consider the SCM a massive retaining wall 14 m high, the upper and lower bases a and b of which should be determined from calculation of (18) for a horizontal load R_1 corresponding to area S_1 under the reactive-pressure curve in Fig. 4, c and the displacements z_{avg} (19).

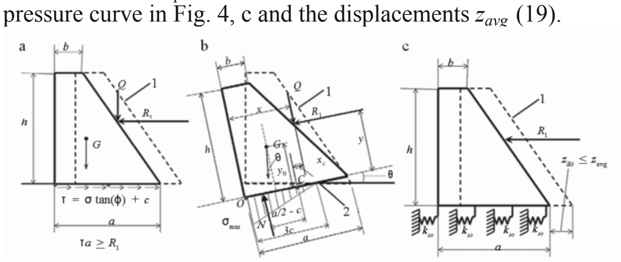

Figure 5. Computational diagrams for SCM above bottom of pit: 1) initial position; 2) turning point.

The optimal dimensions of the SCM on the outside of the pit were found by solving the other optimization problem which minimize cross sectional area of the massive retaining wall (trapezoid)

$$\psi_0(a, b) = (a+b)h/2 \rightarrow \min \quad (20)$$

for limits ensuring observance of the following conditions:

1) stability against shear (Fig. 5, a)

$$(Q(a, b) + G(a, b))\tan(\varphi) + ca \geq R_1, \quad (21)$$

where Q is the weight of the soil in the SCM benches;

2) stability against overturning about a certain point (Fig. 5, b) with consideration of the deforming bed in accordance with Klein's procedure (Klein 1964)

$$M_{re}(a, b) = N(a/2 - c) \geq M_a(a, b) = R_1 y + G(a/2 - x) - Qx_c; \quad (22)$$

3) the displacement z_{sc} of the SCM under the action of R_1 should not exceed the displacement z_{avg} averaged over the height h (Fig. 5, c).

$$z_{сц} = \frac{R_1}{k_{гр} a \cdot tg\varphi} \leq z_{ср};$$

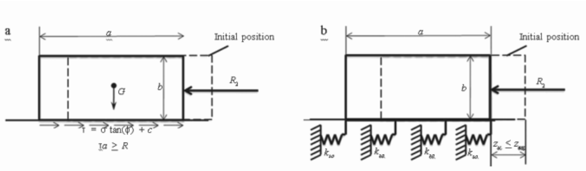

Figure 6. Computational diagram for SCM below bottom of pit

If the first two stability conditions are observed, the displacement of the SCM is determined only by the horizontal deformation of the soil in the bed, which is described by the stiffness coefficient k_{so} of the soil when the rotation of the mass is disregarded.

In order for the system to function in the elastic stage, the dimensions of the SCM below the bottom of the pit are determined with consideration of the fact the soil takes up the reactive pressure, and the SCM the remaining portion. Above the point of intersection, the reactive pressure exceeds the passive pressure of the soil q_p (see Fig. 4, c), but is lower than the passive pressure below the point; the minimal height of the SCM will therefore correspond to the distance from the bottom of the pit to the point of intersection of the r and q_p curves. Here, the design load on the SCM is calculated based on area S_2 (see Fig. 4, c)

The optimal dimensions of the SCM under the bottom of the pit are determined by solving the optimization problem which minimize the value of the efficiency function corresponding to the cross-sectional area of the mass

$$\psi_0(a, b) = ab \rightarrow \min \quad (25)$$

with limits ensuring observance of the following conditions:

− the stability of the SCM against shear (Fig. 6, a)

$$R_2 \leq F_n = G\tan(\varphi) + ca; \quad (26)$$

− the displacement z_{sc} of the SCM under load R_2 is no greater than the displacement z_{avg} averaged over height h (Fig. 6, b)

$$z_{sc} = \frac{R_2}{k_{so} a \cdot tg\varphi} \leq z_{avg} \quad (27)$$

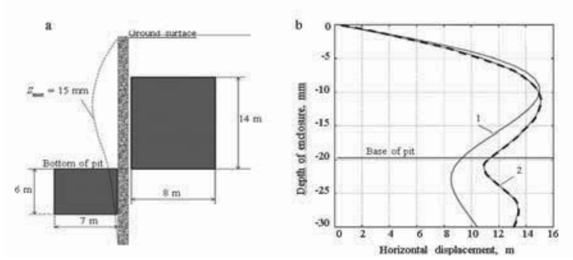

Figure 7. Results of problem solution: a) optimal SCM dimensions when S_{max} = 15 mm; b) results of comparison: 1) elastic bed; 2) PLAXIS 2D.

The optimal dimensions are determined for the allowable displacement of the enclosure S_{max} = 15 mm as a result of solution of the problems described (Fig. 7, a).

The numerical modeling was performed in the software package PLAXIS 2D with consideration of the SCM dimensions obtained, and plots of the horizontal displacements were compared (Fig. 7, b). The discrepancy between the maximum horizontal displacements was insignificant, although the pattern of the curves differed somewhat. Similar calculations and comparison with PLAXIS were conducted for the same initial data, but with limits placed on the displacements S_{max} = 30 and 60 mm. It was established that the SCM dimensions obtained with use of the proposed method yielded a high safety factor with respect to displacements with increasing tolerances. Application of the method in question is therefore restricted to the region of small enclosure displacements; this does not contradict the goals of the stated problem.

3 CONCLUSIONS

A computational method is developed for determination of the optimal dimensions of a soil-cement mass (SCM) that reduces the displacements of an enclosure to required values. The method includes:

− a computerized-search algorithm for optimal engineering of the coefficient of stiffness of an elastic bed; and,

− calculation of optimal SCM dimensions corresponding to the optimal stiffness using standard procedures for analysis of massive retaining walls.

The computational method makes it possible to determine optimal SCM dimensions for the excavation of deep pits in a dense urban setting, when it is necessary to shorten considerably the construction time, and ensure a minimal effect of excavation on surrounding development.

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