# Earth Pressure from Strip Footings on an Anchored Sheet Pile Wall

## Poussée des terres provenant de semelles filantes sur un mur de palplanches ancré

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ABSTRACT : A strip footing is frequently situated near a sheet pile wall. Assessment of the extra pressure on the wall generated by a footing causes theoretical problems for the designer. The distribution of this pressure depends in fact on many parameters. Besides the location and magnitude of the load a characterzation of the soil and the wall is neccesary for a rational design. Furthermore, the movement of the wall has a significant impact on the pressure. In this paper an anchored wall is investigated where the movement in failure is a rotation about the anchor point. The problem is solved by means of different analytical methods compared with solutions by finite element modeling applied to a number of representative examples. These comprise different strengths for the cohesion-less soil and different load scenarios. After a discussion of the results a simple calculation procedures is proposed.

RÉSUMÉ : Une semelle filante est souvent située à proximité d'un mur de palplanches. L'évaluation de la pression supplémentaire sur la paroi générée par la semelle provoque des problèmes théoriques pour le concepteur. La répartition de cette pression dépend en fait de nombreux paramètres. Outre l'emplacement et l'ampleur de la charge, une caractérisation du sol et du mur est nécessaire pour une conception rationnelle. De plus, tout deplacement de la paroi a un impact significatif sur la pression. Dans cet article, une paroi ancrée est étudiée lorsque le déplacement amenant à une défaillance consiste en une inclinaison autour du point d'ancrage. Le problème est résolu par le biais de différentes méthodes d'analyse que l'on compare aux solutions de modélisation d'éléments finis, appliquées à de nombreux exemples représentatifs. Celles-ci comprennent différentes forces pour un sol sans cohésion ainsi que différentes configurations de charge. Une simple procédure de calcul est proposée après la discussion des résultats.

KEYWORDS: Sheet pile wall, continuous footing, earth pressure, finite element method, sand, stress distribution.

## 1 INTRODUCTION

Sheet pile wall design methods in Europe generally rely on simplified earth pressure theories where the failure mechanism of the soil is in fact not compatible with the wall deflections. The Danish design method of sheet pile walls is based on Brinch Hansen's earth pressure theory, which assumes plastic behaviour for the wall and the soil. The computer program SPOOKS, which is a product from GEO-Danish Geotechnical Institute, is successfully used for sheet pile wall design in Denmark and abroad. The program calculates the required driving depth, the maximum bending moment and the anchor force for a user defined failure mode of the wall and the adjacent soil. The wall may be either anchored or free and either hinged or fixed in an anchor point. In the limit state a yield hinge in the wall with an ultimate positive moment may develop below the anchor level.

When excavating close to an existing building the effect of the partial distributed loads, from for example strip foundations (two dimensional (2D) conditions), or plate foundations (three dimensional (3D) conditions), are usually implemented in the sheet pile wall plastic design by means of the elasticity theory and the principle of superposition, where the extra earth pressure simply is added to the plastic solution. It is however not correct in the plastic design to separately calculate the active earth pressures from partial distributed loads without taking into account the active pressure from the unit weight of the soil.

The objective of the present paper is to supplement an earlier investigation for a free wall, Denver & Kellezi (2011), with establishment of an empirical relationship to estimate the extra earth pressure on an anchored sheet pile wall from a strip load behind the wall. This relationship is compared with solutions from finite elements (FE) results. The additional pressure is found as the difference between the combined pressure from self weight of the soil and the strip footing and the pressure from only the self weight. In an attempt to assess the additional pressure on the wall, different approaches are investigated:

- Analytical calculations by the theory of plasticity on a suitable rupture figure.
- Empirical solutions inspired by Coulomb's theory.
- Numerical modelling by the FE method.

## 2 GENERAL

The earth pressure calculation on a wall is here illustrated by the Danish method denoted as Earth Pressure Calculation. This method has been proposed by J. B. Hansen (1953) and is extensively used in Denmark. The pressure on the wall ($e$) is calculated as a sum of three terms as given in equation (1).

$$e = \gamma' d K_\gamma + q K_p + (c K_c) \qquad (1)$$

These terms and the other parameters used in the calculation are: $\gamma'$ the effective unit weight of the soil; $K$ the earth pressure coefficient (different for the three terms); $c$ the cohesion of the soil; $p$ the surface load behind the wall, and $d$ the depth along the wall from the soil surface. The last term is enclosed in parenthesis as this paper deals only with frictional soil.

In the Danish method the wall is considered composed of several rigid parts interconnected by yield hinges. Each part is assumed to rotate about a point and the earth pressure coefficients are functions of the position of this point and the direction of rotation (besides the friction angle of the soil, $\varphi$). Examples of anchored walls with yield hinges are shown in Figure 1, and examples of rupture figures used for calculation of $K$ are shown in Figure 3. The result of each calculation is the total force on the wall and the point of application. The normal component of this force ($E$) is distributed along the wall. A part of $E$ is applied near the top as a Prandtl rupture zone.

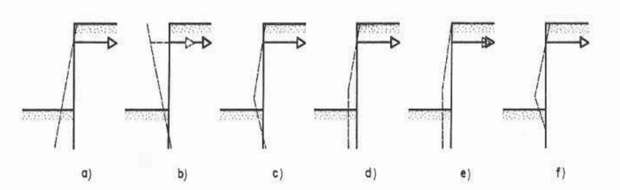


Figure 1, Anchored wall in failure composed of one or more rigid segments connected by yield hinges in failure. This paper deals with failure mode a) marked with rectangular.

A pressure jump near the top is then applied to ensure that the effect of the distribution (in terms of total force and moment) corresponds with the rupture figure The method has been described in detail by Mortensen & Steenfelt (2001) and results of calculated examples are compared with FE calculations.

## 3 COMPUTER PROGRAM 'SPOOKS'

Although J. B. Hansen has developed a complete set of diagrams to find the values of $K$, the earth pressure calculation for a specific design situation is rather time consuming. To this end GEO-Danish Geotechnical Institute has made a commercially available computer program named 'SPOOKS'.

Here, apart from the geometry of the excavation, the soil conditions and water tables, only a selection of the total wall movements (as shown in Figure 1) is necessary as input. The results are a distribution of both earth and water pressures, curve of bending moments along the wall, tip level, and anchor force. All together ready for the final design of the sheet pile wall profile and anchor. However, this program has no facility to include a partial surface load.

## 4 THEORY OF PLASTICITY

A method to assess the extra soil pressure caused by a partial load has been introduced by J.S. Steenfelt and B. Hansen (1984). The Danish method to calculate the earth pressure coefficient from a relevant rupture line has been adopted. A circular rupture line is used as an appropriate choice for a rotation about a point at the anchor level. The stresses from the rupture line are determined by the Kötter's differential equation. The total force is found by integration of this equation presented by Brinch Hansen (1953) and shown as the resulting force ($F_o$) and moment ($M_o$) about the centre of the circle as shown in Figure 2 where the significance of the variables is indicated.

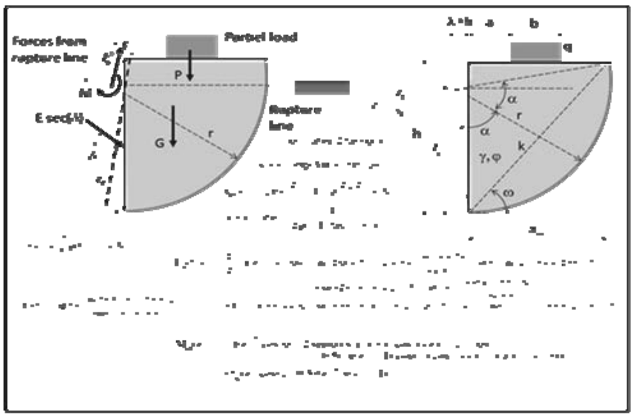


Figure 2, Analytical method where circular rupture figure is applied. Negative values of $\varphi$ and $\delta$ shall be applied as the rupture is active.

It should be mentioned that $t_c = \sqrt{\sigma^2 + \tau^2}$ where $t_c$ refers to the starting point of the integration where the rupture circle meets the soil surface and $\sigma$ and $\tau$ are coordinates to the yield point in the Mohr's circle. The function $q(\lambda)$ refers to the value of $q$ in the point where the circle meets the surface. ($q$ beneath the load and 0 otherwise). The three unknowns ($\lambda$, $E$, and $z_p$) are finally found by the three equilibrium equations.

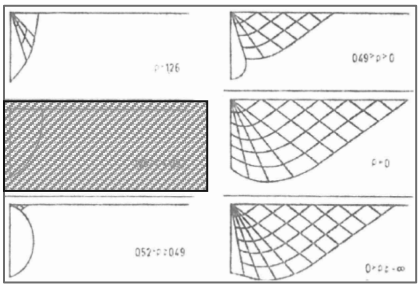

Figure 3, Rupture figures with different rotation points (ρ: relative height from the bottom of the wall). The figures are drawn for $\varphi = 30°$, $c = 0$ and rough wall rotating clockwise. The pure line rupture is investigated analytically in this paper (shaded in the figure).

This method (Figure 3 shaded) is in detail introduced and discussed by the authors and the results of a large number of load scenarios are presented in their paper.

## 5 EMPIRICAL METHOD

It is usual practice to apply a soil pressure derived from the distribution for the uniformly loaded surface. A minor part of this distribution is then used situated to a depth interval defined by inclined lines through the soil.

In Figure 4 a method of this kind often used in Denmark is shown. However, a tail below the lower line has been in this method proposed by K. Mortensen (1973) who has pointed out the complexity of the problem assuming a smooth wall that rotates anti-clockwise about a point below the tip of the wall. Consequently, the upper part with the even distribution is given by an active Rankine rupture figure. The tail is probably inspired by calculations by Coulomb's method where the lower part is more dependent of other parameters than $a$ and $b$.

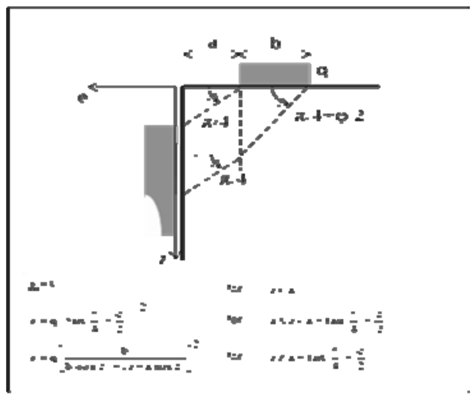

Figure 4, Empirical method based partly on the Coulomb's earth pressure theory.

As this method is often used also for other movements, en lieu of other procedures it is adopted here as an example of an empirical solution.

## 6 ELASTIC SOLUTION

An elastic solution developed by Boussinesq (1885) is often used because of its simplicity as shown in Figure 5. Besides the theory of elasticity a smooth vertical wall without any movement is assumed. This method is often questioned as the resulting distribution is too large and situated much too high on the wall with respect to results from model tests and calculations based on the Coulomb's method.

This is also the authors experience when the movement of the wall is anti clockwise about a low point in the wall. However, if the movement is a clockwise rotation about the anchor (as in this paper) the assumptions for an elastic solution are more justified.

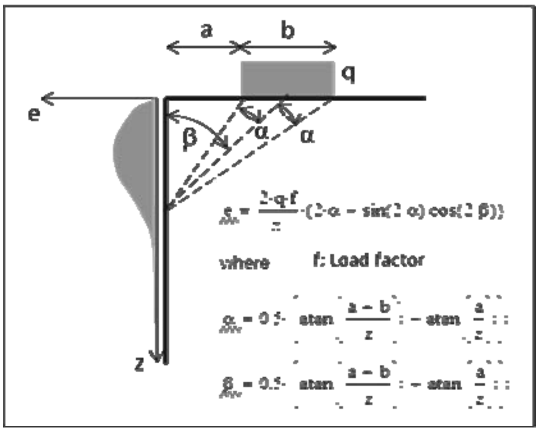


Figure 5, Elastic solution by Boussinesq (1885)

## 7 FINITE ELEMENT METHOD

In order to validate the method a number of load scenarios have been calculated by the FE program Plaxis 2011.

A 2D mesh pattern has been generated using triangular finite elements (15-noded). Sand is modeled in drained conditions using the Mohr-Coulomb constitutive model. The sheet pile wall is assumed weightless and with a large stiffness to prevent any interaction of stresses caused by deformation of the wall. The initial geostatic conditions are calculated first. Mesh sensitivity analyses have been carried out and an optimal mesh pattern with respect to element size and obtained accuracy has been chosen for the final analyses.

Plaxis plastic analyses (small deformation theory) and Updated Mesh (large deformation theory) have been applied to estimate the effect of the wall movement on the results. The calculations are carried out in different ways considering the impact the staged construction (excavating after, before or at the same time with the load application) has on the results.

Some different load scenarios are modeled and calculated to illustrate the problem. The loads / pressures applied over the foundations are chosen in such a way that the foundation bearing capacity is satisfied. The load scenarios are shown in Table 1.

Table 1, Load scenarios calculated

| No | φ | a | b | q | No | φ | a | b | q |
|---|---|---|---|---|---|---|---|---|---|
| | (deg) | (m) | (m) | (kPa) | | (deg) | (m) | (m) | (kPa) |
| 1 | 30 | 1 | 2.5 | 125 | 6 | 40 | 1 | 2.5 | 713 |
| 2 | 30 | 1 | 1 | 50 | 7 | 40 | 1 | 1 | 285 |
| 3 | 30 | 2.5 | 1 | 50 | 8 | 40 | 2.5 | 1 | 285 |
| 4 | 30 | 5 | 1 | 50 | 9 | 40 | 5 | 1 | 285 |
| 5 | 30 | 0 | ∞ | 50 | 10 | 40 | 0 | ∞ | 285 |
| h = 12 m | | γ = 14 kN/m$^3$ | | | c = 0 kPa | rough wall | | | |
| Height to rotation point: | | | | | $h_p$ = 9.6 m | | | | |

A unit weight has been applied to the soil to provide a realistic stress distribution near the top of the wall. Interface elements are applied along the wall. However, the soil strength at the interface has not been reduced as a rough wall is considered. The influence of the load on the wall has thus been derived as the difference between results of calculations of the wall with load and unit weight and with unit weight alone.

A rigid anchor is applied at a depth corresponding to 0.8*h referring to the bottom of excavation or the height of the wall h. The anchor point ensures a rotation around this point during failure (Figure 6).

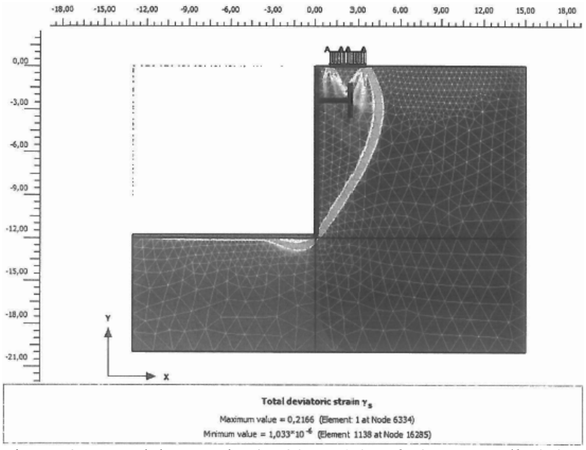


Figure 6 FE model example, ($\varphi$=30◦ $a$=1.0 m $b$=2.5 m or $a/b$=0.4 $p$=125 kPa).

A complete presentation of the results is not included due to lack of space. The normal pressure ($e$) on wall from the soil, and the soil plus load, and the additional pressure from the load derived as their difference (Delta $e$) are derived from the interface zone as given in Figure 7 and 8 for both soil types considered. The FE results are used as benchmarks for the accuracy of the other methods and shown relative to those in the discussion.

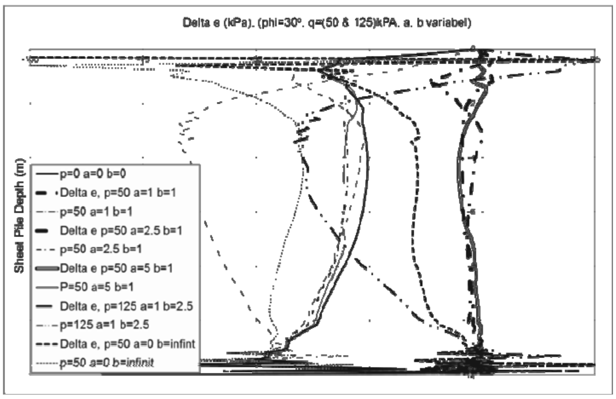


Figure 7 FE models results ($\varphi$=30◦)

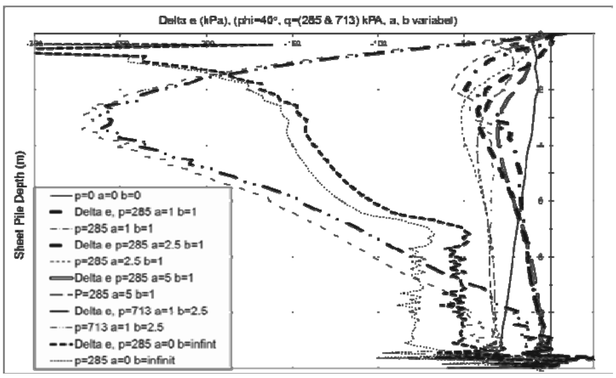


Figure 8 FE models results ($\varphi$=40◦)

## 8 DISCUSSION OF CALCULATIONS

It was expected that a study of the theory of plasticity would yield a deeper insight into the problem and provide useful results. However, our calculations have produced rather scattered results. The calculations presented by Steenfelt & Hansen do by no means suggest simple relations to the input parameters. Their recommendation is also to make a computer code using the detailed described procedure and solve the problem in question explicitly. The resulting force from the integration along a rupture line consists together with simple zone ruptures the backbone of the Danish earth pressure theory, and should by no means be questioned here.

It should be mentioned that the procedure involves two calculations: (i) a calculation with both $P$ and $G$ (Figure 2), and (ii) a calculation with $G$ alone. The influence of $P$ is found by a subtraction of the two vectors. As $G$ is great compared to $P$ the latter is poorly determined and also problems with the validity of supposition as assumed here will distort the result.

Another problem is connected with the integration of the Kötter's equation. The only contribution to a change of the ambient stress condition is caused by the unit weight. However, the rupture line will pass through domains in the soil much differently affected by the partial loaded surface.

In every case the method will not provide the distribution of the pressure which is imperative especially to determine the moment in the wall in the anchor level.

## 9 PROPOSED PROCEDURE

When a procedure to assess the influence of a partial loaded surface it should be taken into consideration that the proposed distribution should converge to the distribution usually applied for a fully loaded surface.

The procedure proposed is:

- Calculate the elastic distribution ($e_e(z)$) using the equations in Figure 5.
- Calculate the distribution usually used for a fully loaded soil surface. Use only the part of this distribution corresponding to the uniform part of the distribution ($e_p(z)$) shown in Figure 4.
- The final distribution is: $e(z) = W{*}e_p + (1\text{-}W){*}e_e(z)$, where $W$ is a weight function $W = 1.5{*}(F\text{-}0.167)\text{-}2{*}(F\text{-}0.5)^3$ and $F = 0.8{*}b/h$.

## 10 VERIFICATION

The benchmark for the verification is chosen as the results of the FE calculations. As before mentioned it is difficult to characterize the distributions by simple means. We have here focused on the usage of the distribution: (i) to calculate the anchor force ($A$), and (ii) to calculate the moment in the wall in the anchor level ($M$).

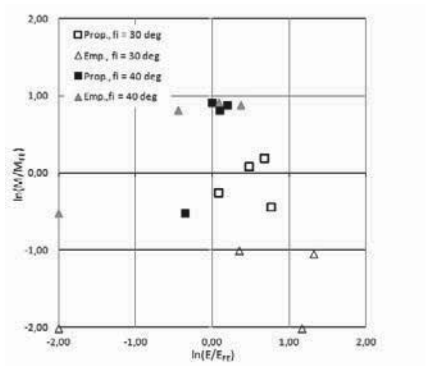


Figure 9, Accuracy of the methods (Empirical, Proposed vs. FE).

The anchor force is estimated as the part of the distribution above the depth $z$ equal to double the height of the wall above the anchor. This procedure excludes the results found by the theory of plasticity to be represented. The quantities $\ln(A/A_{FE})$ and $\ln(M/M_{FE})$ are made where the denominators are the results from the FE calculations. These quantities are plotted against each other in Figure 9 for load cases (1-4) and (6-9).

When a quantity is greater than zero, the predicted value is on the safe side. When a quantity is 1 the corresponding ratio is 2.7. If $A$, or $M$ is zero the quantity is minus infinity but plotted on the frame of the diagram.

A study of Figure 9 shows that the proposed procedure is superior to the empirical procedure and the fit is surprisingly accurate taking into account the complexity of the problem.

In order to offer a qualitative impression of the results a single distribution from the FE calculations is shown in Figure 10. This distribution is supplemented with distributions from two other methods (Empirical and Proposed).

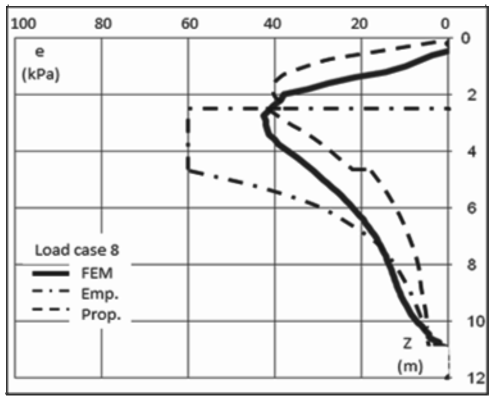


Figure 10, Normal pressure distribution (e) from FE calculations compared with other methods (Empirical, Proposed) for a single load, (Load Case 8).

## 11 CONCLUSION

A procedure to calculate the pressure distribution has been proposed and has proved an excellent fit with results from FE calculations. The procedure is based on the theory of elasticity where the assumption of an immobile wall is justified by the high rotation point. The result converges to the usually applied when the entire surface is loaded.

## 12 ACKNOWLEDGEMENT

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