Data assimilation strategies for parameter identification of elasto-plastic geomaterials and its application to geotechnical practice

Stratégie d'assimilation de données pour l'identification des paramètres de géomatériaux élastoplastiques et son applications à la pratique géotechnique

Shuku T., Nishimura S.
Graduate School of Environmental and Life Science, Okayama University, Okayama 700-8530, Japan

Murakami A., Fujisawa K.
Graduate School of Agriculture, Kyoto University, Kyoto 606-8502, Japan

ABSTRACT: The objective of this study is to demonstrate the numerical and the practical applicability of the particle filter (PF) to some geotechnical problems, i.e., the parameter identification of elasto-plastic geomaterials and the prediction of the deformation behavior of soil deposits and geotechnical structures, by applying the methodology to hypothetical experiments and an actual construction project. The results of the hypothetical experiments reveal that the parameters identified by the PF, based on the sequential importance sampling (SIS) algorithm, have converged into their true values, and that the approach presented herein can provide a highly accurate parameter identification strategy for elasto-plastic geomaterials. Moreover, the simulation results using the identified parameters are close to the actual observation data, and the ensemble-based approach produces more information about the parameters of interest than simple estimated values obtained from optimization methods. In other words, the identification comes in the form of a probability density function.

RÉSUMÉ : L'objet de cette étude est de démontrer l'applicabilité numérique et pratique du filtrage des particules (FP) pour certains problèmes géotechniques, à savoir, l'identification des paramètres de géomatériaux élastoplastiques et la prédiction du comportement en déformation de dépôts de sol et de structures géotechniques, en appliquant la méthodologie à des expériences hypothétiques et à des projets de construction existants. Les résultats des expériences à partir d'hypothèses montrent que les paramètres identifiés par le FP, basé sur l'algorithme d'échantillonnage d'importance séquentiel (SIS), ont convergé vers leurs valeurs réelles, et que l'approche présentée ici peut fournir une stratégie d'identification paramétrique très précise pour les géomatériaux élastoplastiques. En outre, les résultats de la simulation utilisant les paramètres identifiés sont proches des données d'observation réelles, et l'approche groupée produit plus d'informations sur les paramètres d'intérêt que de simples valeurs estimées obtenues à partir des méthodes d'optimisation. En d'autres termes, l'identification se présente sous la forme d'une fonction de densité de probabilité.

KEYWORDS: data assimilation, particle filter, parameter identification

1 INTRODUCTION

Inverse analyses have been successfully applied to linear elastic problems in which the deformation to be addressed is linear and depends only on the model parameters and the applied load; it does not depend on the loading history. However, the mechanical behavior of geomaterials is commonly described by an elasto-plastic model, and the deformation behavior displays strong nonlinearity and depends not only on the values of the parameters, but also to a great extent on the stress state and the history, whereby the identification of elasto-plastic parameters still remains a major challenge.

Data assimilation (DA) is available as a methodology to tackle the above difficulties (Nakamura *et al*. 2005). The estimation of the interest dynamic system via DA involves a combination of observation data and the underlying dynamical principles governing the system. The melding of data and dynamics is a powerful methodology, which makes efficient and realistic estimations possible. This approach has recently proven fruitful in earth science, e.g., geophysics, meteorology, and oceanography (e.g., Awaji *et al*. 2009).

Several kinds of powerful DA methods have been proposed. Among the existing strategies, this study focuses on the filtering techniques referred to as the particle filter (PF, Gordon *et al*. 1993), because it can be applied to nonlinear and non-Gaussian problems and can provide a simple conceptual formulation and ease of implementation.

Herein numerical and practical effectiveness of the DA strategies using the PF are examined for geotechnical problems through their applications to the numerical experiments and an actual construction project. For this purpose, first, we outline the concepts and methods of DA and refer to the PF. Second, we deal with the parameter identification of elasto-plastic parameters for geomaterials applying the PF to initial and boundary value problems in geomechanics. Finally, we investigate the applicability of the PF to a practical settlement prediction of a well-documented construction project, Kobe Airport Island, comparing the obtained simulation with the observation data, and the practical effectiveness of the DA based on the PF is discussed.

2 DA: CONCEPTS AND METHODS

DA is a versatile methodology for estimating the state of a dynamic system of interest by merging sparse observation data into a numerical model for the system. The state of the system is usually estimated with deterministic simulation models, which are subject to the uncertainty that arises due to a lack of knowledge and a poor understanding of the physical phenomena. Meanwhile, observation data, which represent the true state, but are subject to stochastic uncertainty and randomness, may occasionally be available as a function of a subset of the system variables. Based upon a prognostic model and a limited number of observations, DA attempts to provide a more comprehensive system analysis which may lead to more accurate predictions. This approach has recently proven useful in earth science (Awaji *et al*. 2009).

Novel sequential data assimilation methods include the Ensemble Kalman Filter (EnKF, Evensen 1994) and the PF which are categorized into nonlinear Kalman filtering. Although the EnKF can be applied to nonlinear systems, it basically assumes a linear relationship between a state and the observation data in calculating a Kalman gain. Therefore, the

EnKF cannot produce satisfactory estimates if its linear approximation is invalid. This means that its application to geomaterials is difficult, because the materials display strong nonlinearity. On the other hand, as the PF does not require assumptions of linearity or Gaussianity, it is applicable to general problems. Therefore, the PF has higher potential for application to geotechnical engineering and can obtain meaningful outcomes. Brief description of the PF is summarized below.

The PF approximates probability density functions (PDFs) via a set of realizations called an ensemble that has weights, and each realization is referred to as a 'particle' or a 'sample'. For example, a filtered distribution at time t–1, $p(x_{t-1}|y_{1:t-1})$, where $y_{1:t-1}$ denotes $\{y_1, y_2, \ldots, y_{t-1}\}$, is approximated with ensemble $\{x_{t-1|t-1}^{(1)}, x_{t-1|t-1}^{(2)}, \ldots, x_{t-1|t-1}^{(N)}\}$ and weights $\{w_{t-1}^{(1)}, w_{t-1}^{(2)}, \ldots, w_{t-1}^{(N)}\}$ by the following equation:

$$p(x_{t-1}|y_{1:t-1}) \approx \frac{1}{N}\sum_{i=1}^{N} w_{t-1}^{(i)} \delta(x_{t-1} - x_{t-1|t-1}^{(i)}) \tag{1}$$

where N is the number of particles and δ is the Dirac delta function. $w_{t-1}^{(i)}$ is the weight attached to particles $x_{t-1|t-1}^{(i)}$ and should suffice $w_{t-1}^{(i)} \geq 1$ and $\Sigma w_{t-1}^{(i)} = 1$.

A general approach for filtering is known as sequential importance sampling (SIS) (Doucet *et al*. 2000). The SIS algorithm is based on using the importance sampling to estimate the expectations of functions of the state variables. The algorithm of SIS is summarized as follows:

1. *Initialization:*
   Generate an ensemble (set of particles) $\{x_0^{(1)}, x_0^{(2)}, \ldots, x_0^{(N)}\}$ from the initial distribution $p(x_0)$.
2. *Prediction:*
   Each particle $x_{t-1}^{(i)}$ evolves according to the numerical dynamic model given by a numerical simulation method such as FEM.
3. *Filtering:*
   After obtaining measurement data y_t, calculate weight $w_t^{(i)}$, which expresses the "fitness" of the prior particles to the observation data, and assign a weight, $w_t^{(i)}$, to each $x_{t-1}^{(i)}$.
4. *Weight update:*
   The set of weighted particles $\{x_t^{(i)}\}$ results in an ensemble approximation of filtered distribution $p(x_t|y_{1:t})$.
   Set $t = t + 1$ and go back to Step 2.

Figure 1 shows the algorithm of the PF based on the SIS.

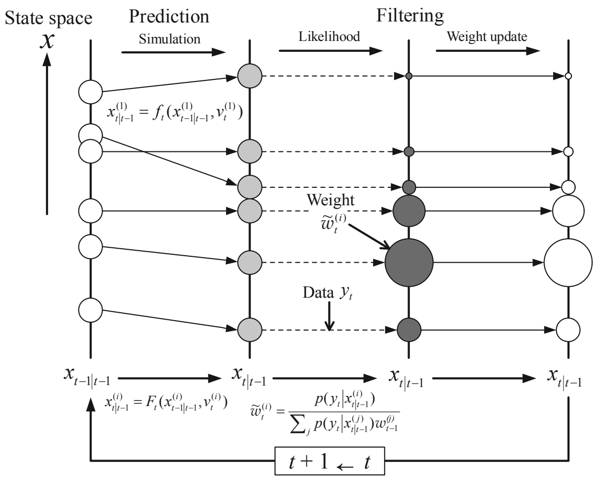

Figure 1. Algorithm of the PF based on SIS.

3 PARAMETER IDENTIFICATION OF CAM-CLAY MODEL USING THE PF

This chapter focuses on the soil-water coupled behavior of a clay foundation under monotonic loading, where the numerical simulation for hypothetical soil deposit under embankment is implemented to study the efficiency of the PF as a parameter identification method.

The soil-water coupled finite element analysis using the Cam-clay model were used in this example. The finite element mesh and the loading history are shown in Figures 2 and 3, respectively. Table 1 lists the parameters of the clay foundation. The placement of the observation points is also shown in Figure 2; the vertical displacements and the horizontal displacements are located at S1-S3 and at L1-L3, respectively. Some of the parameters are chosen to be identified and their values are called 'true values' as listed in Table 2, and we carried out 100 Monte Carlo Simulations using the sets of particles which were generated with uniform random numbers in the range shown in Table 2.

Figure 4 shows the time evolution of the identified parameters (λ, κ, and M). Identified parameters are computed as the weighted mean value of the particles computed by

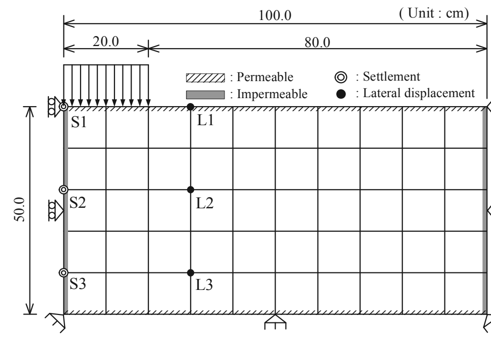

Figure 2. Finite element mesh.

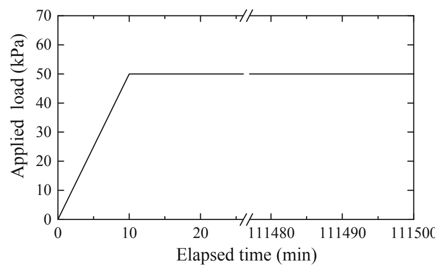

Figure 3. Loading history.

Table 1. Cam-clay parameters of the model foundation.

λ	κ	e_0	M
0.190	0.065	0.992	1.154

Table 2. True values of the parameters to be identified and range of particle generation.

Parameter	True value	Range
λ	0.239	0.090 ~ 0.290
κ	0.091	0.015 ~ 0.115
M	1.084	0.854 ~ 1.454

$$\bar{\phi}_t = \sum_{i=1}^{N} w_t^{(i)} \phi_t^{(i)} \tag{2}$$

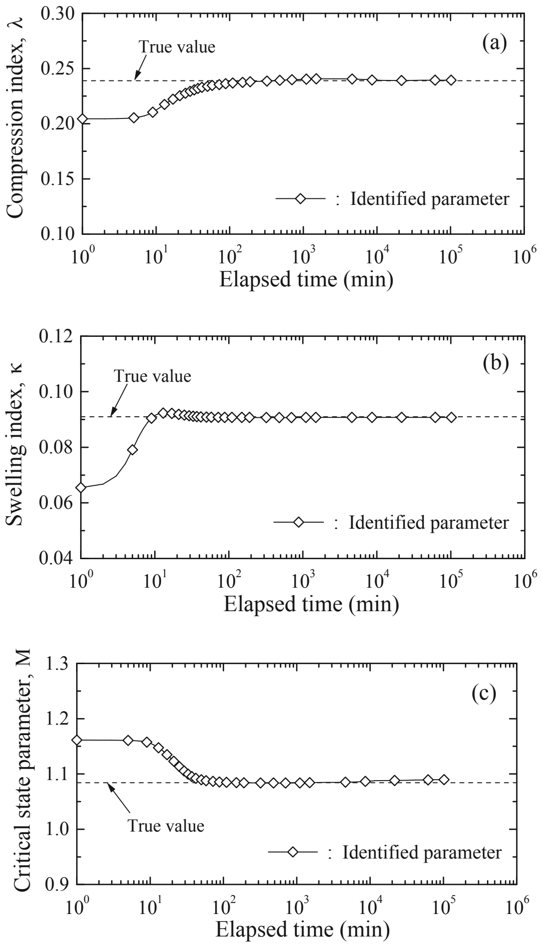

Figure 4. Time evolution of identified parameters.

where $\bar{\phi}_t$ and $\phi_t^{(i)}$ indicate the identified parameter at time step t and the parameter of particle number (i) at time step t, respectively.

The parameter identification of unknown parameters approaches the true values, although the identification starts with an incorrect $\bar{\phi}_0$ in all cases purposefully. These results verify the effectiveness of the PF for the parameter identification of the elasto-plastic model, which presents strong nonlinear behavior.

4 APPLICATION OF THE PF TO SETTLEMENT BEHAVIOR OF KOBE AIRPORT ISLAND CONSTRUCTED ON RECLAIMED LAND

The objective of this chapter is to investigate the applicability of the PF to an actual settlement prediction of a well-documented geotechnical construction project, Kobe Airport Island. To accomplish this objective, firstly, the settlements of the island are evaluated using a soil-water coupled finite element analysis with the Cam-clay model. Then, the parameters are identified using the PF. Finally, comparing the recomputed simulation using identified parameters with the observation data, the practical effectiveness of the methodology based on the PF is discussed. Some outcomes obtained from this application example were reported in Murakami *et al*. (2012).

Kobe Airport was constructed on an artificially reclaimed island just off the coast of Kobe. Figure 5 shows the cross section of the construction site. Vertical sand drains were installed in the soft clay layer in order to accelerate the settlement and increase the strength (e.g. Yamamoto *et al*. 2010).

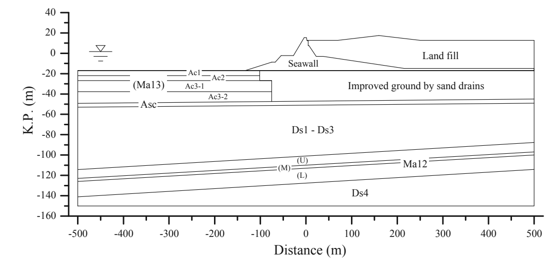

Figure 5. Cross section of the construction site (Yamamoto *et al*. 2010).

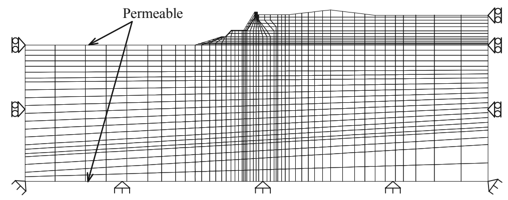

Figure 6. Finite element mesh (Murakami *et al*. 2012).

The soil-water coupled finite element analysis with the Cam-clay model was adopted for analyzing the deformation behavior of the seawall and the foundation subjected to the construction and reclamation work. Figure 6 shows the finite element mesh. In the model ground, the top surface, bottom surface and the sides of sand/gravel layers were assumed to have permeable boundary conditions, whereas the sides of clay layers were assumed to have impermeable boundary conditions. The sand layers and reclaimed ground were assumed to be linear elastic, and the clay foundations were represented by the Cam-clay model.

The *mass permeability* concept, which was proposed by Asaoka *et al*. (1995), was incorporated into this analysis. Mass permeability is the permeability representative of a clay foundation, which includes the effects of inhomogeneity, partial drainage, and load intensity. We also adopted the concept in the same sense. The analysis in this chapter focuses on the settlement behavior of only the improved alluvial clay foundation, because the soil layers which are just below the improved ground, called Ds1-Ds3, are thick, have high rigidity (the N-value obtained from SPT is more than 100), and do not significantly affect the settlement of the island.

Firstly, we considered the improved ground to be homogeneous by incorporating the mass permeability concept. Then, using the PF, some parameters of the treated ground, the so-called *mass parameter* were identified to simulate settlement of the ground under the airport island. Although the some parameters affect settlement of the ground, the compression index λ and the permeability k were treated as the only parameters to be identified, because these two parameters directly govern consolidation behavior of clay grounds. Finally, the simulations were implemented using the identified mass parameters and observation data were compared to evaluate the practical usability of the PF.

The representative parameters of the improved grounds, referred to as mass parameter (P_{mass}) in this study, are determined here by using equation (3) for simplicity.

$$P_{\mathrm{mass}} = \frac{P_1 h_1 + P_2 h_3 + \cdots + P_n h_n}{h_1 + h_2 + \cdots + h_n} \qquad (i = 1, 2, \cdots, n) \tag{3}$$

where P_i, h_i, and n are the parameters, the thickness of each layer, and the number of soil layers, respectively.

We conducted Monte Carlo simulations with 200 particles over the feasible space listed as follows:

$$0.30 \leq \lambda \leq 0.60,\ \ 1\times10^{-0} \leq k \leq 1\times10^{-3}. \tag{4}$$

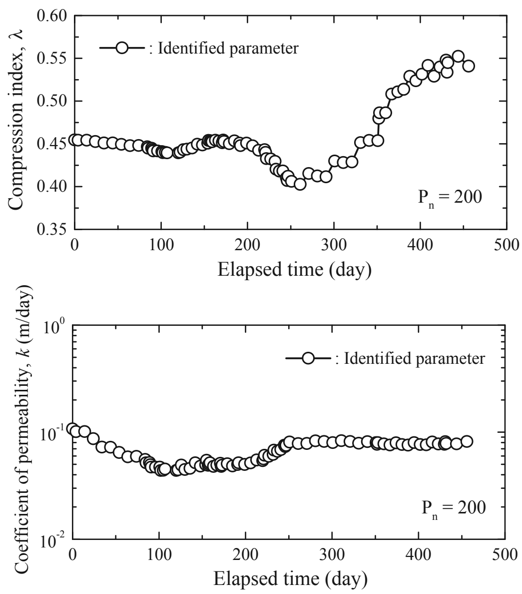

Figure 7. Time evolution of identified parameters.

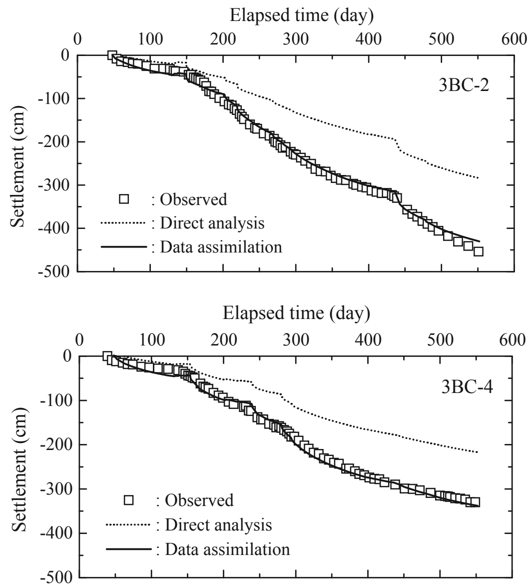

Figure 9. Simulation results using the identified parameters.

Each parameter was assumed to follow uniform randomly and was generated independently. All 200 simulations were conducted up to 676 days after the construction was started. Only the settlement values observed on the seabed were used for parameter identification.

Figure 7 shows the time evolution of the identified parameters. In the figure, the estimates for λ hardly change through the assimilation. In particular, after the 300th day, the path changes dramatically. On the other hand, in the result of *k*, the identified parameter shows almost constant value through the assimilation.

Figure 8 shows filtered PDFs of a settlement value at the 148 days after construction began. In this figure, the vertical axis represents the weight of the particle, while the horizontal axis represents settlement value. It can be seen from the Figure 8 that the distribution of the weight approximately follows the normal distribution which has sharp peak around -3.5m. From the result, we can see that the use of a large number of particles contributes to the accurate estimation of the arbitrary PDFs for settlements. This is the remarkable advantage of the PF.

The simulation results for the time-settlement relationship at observation points 3BC-2 and 3BC-4, which were placed on seabed, via the identified parameters are shown in Figure 9. The identified parameters mean the values at the end of the identification process, that is, $t = 456$ days. In the figures, dotted line represents the result of direct analysis. Although the results of direct analysis underestimate the observation data, the simulations using the identified parameters yielded predictions with high accuracy.

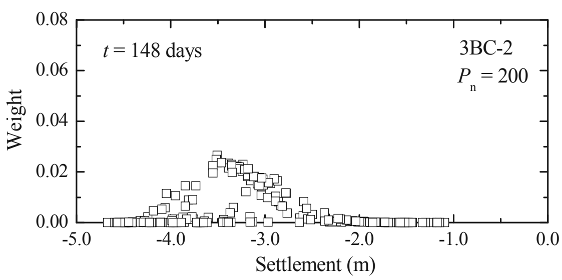

Figure 8. Filtered PDF of a settlement value.

5 CONCLUSIONS

In this study, we have investigated the numerical and the practical effectiveness of the DA strategies using the PF for geotechnical problems through their applications to the hypothetical experiment and the actual construction project.

The parameters identified by the PF have converged into their true values, and the presented approach has shown effective parameter-identification method for elasto-plastic geomaterials. Moreover, the simulated time-settlement behavior using the identified mass parameters has provided a good agreement with the actual observation.

In conclusion, the DA using the PF has been proven a powerful strategy for identifying elasto-plastic parameters of geomaterials and more accurate predictions of the mechanical behavior of geotechnical structures.

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