# On the stability of a trap door evaluated by upper bound method

## Sur la stabilité d'une trappe évaluée par la méthode de borne supérieure

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ABSTRACT: It is crucial to evaluate critical configurations of underground structures, such as a width of a structure under various soil conditions and various depths, and a minimum support force to stabilize underground structures. In this article, for the sake of simplicity, it is assumed that a trap door with width *D* buried at a certain depth in a modified Cam-clay soil is supported by upward uniform pressure *q*. A critical pressure *q** which is necessary to sustain a trap door is evaluated by upper bound method. As a failure mode of a ground, it is assumed that a parabola-shaped loosened soil zone just above a trap door will fall down vertically. Calculated results are summarized in charts which will be useful for practical design.

RÉSUMÉ : Il est essentiel d'évaluer les configurations critiques des ouvrages souterrains, tels que la largeur d'une structure dans des conditions de sol et de profondeur différentes, et une force de soutien minimum pour stabiliser les structures souterraines. Dans cet article, pour des raisons de simplicité, on suppose que la trappe avec largeur *D* enterré à une certaine profondeur dans une version modifiée du Cam-Clay argileux est soutenue par la hausse de pression uniforme *q*. Une pression critique *q** qui est nécessaire pour maintenir une trappe est évaluée par la méthode de limite supérieure. En tant que mode de défaillance d'un sol, on suppose que la zone en forme de parabole sol ameubli juste au-dessus d'une trappe va tomber verticalement. Les résultats calculés sont résumés dans les tableaux qui seront utiles pour la conception pratique.

KEYWORDS: stability, trap door, underground structure, limit analysis, upper bound method, modified Cam-clay model

## 1 INTRODUCTION

It is crucial to evaluate critical configurations of underground structures, such as a width of a structure under various soil conditions and various depths, and a minimum support force to stabilize underground structures. To this end, in this article, upper bound analysis to evaluate the critical supporting pressures of a underground structure is proposed. It will be demonstrated the importance of a ground arch for the stability of underground structures, because critical supporting pressures are depending on the width of a trap door.

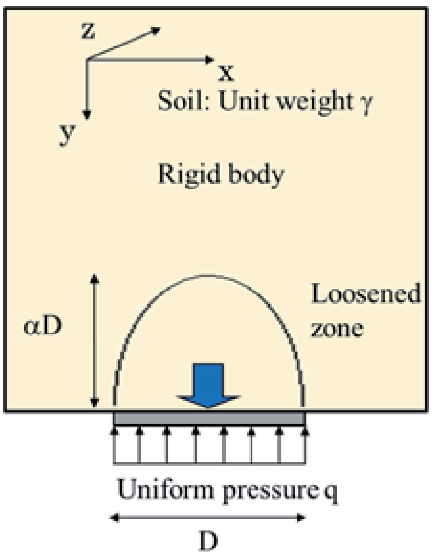


Figure 1. Assumed failure mode and a loosened soil zone

## 2 DISCRIPTION ANE FORMULATION OF A PROBLEM

For the simplicity of a problem, a stability of a underground structure at its crown will be focused on. In other words, a possible failure mode such that a crown of a underground structure and its surrounding soil may fall down, is investigated. Other members of a underground structure, such as side walls and an invert, are assumed to have enough strengths to neglect structural failures at these members.

A soil is modeled as rigid-plastic material obeying Modefied Cam-clay model and the associated flow rule. It is also assumed that a soil is under a drained condition and generation and dissipation of pore water pressures due to deformation of a soil are neglected.

As a crown of a underground structure, it is simply assumed that a trap door with width D buried at a certain depth in a soil. To stabilize a trap door and a surrounding soil, a uniform upward pressure *q* is applied on the surface of a trap door. A target problem of this study is schematically illustrated in Figure 1.

### 2.1 *Size and shape of a loosened zone of a soil*

A failure mode is assumed in upper bound analysis. Generally speaking, it is difficult to find precise shape and size of a failure zone of a soil at the instant of failure. Many case studies show only shape and size of a failure zone after the event. Therefore, in this study, a failure zone at the instant of failure is assumed to be a parabolic shape which crosses at the both end of a trap door. A vertical coordinate of this parabola is denoted as $y_0$ in Figure 2. This $y_0$ stands for the frontier of rigid and plastic zones. It is also assumed that a soil moves only vertically and its distributions are linear both in the vertical and horizontal directions within a parabola as is in Eq. (1).

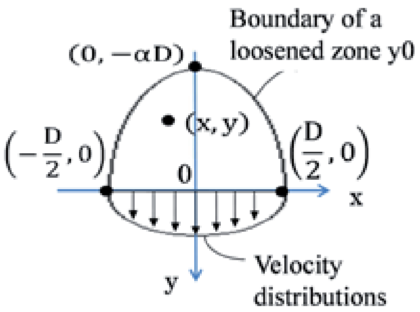


Figure 2. Assumed failure mode and velocity distributions

$$\dot{u}_x = 0, \dot{u}_y = \beta(y - y_0), \dot{u}_z = 0,$$
$$y_0 = \frac{4\alpha}{D}\left(x - \frac{D}{2}\right)\left(x + \frac{D}{2}\right) \quad (1)$$

A parameter α stands for a size of a loosened zone with respect to the width D. Another parameter β is a magnitude of a velocity which can be negligible due to the first order of homogenuity in upper bound calculations.

### 2.2 Calculation of dissipation rate

Internal dissipation rate per unit volume can be expressed as in Eq. (2) in terms of mean effective stress *p'* and deviatoric stress *q*.

$$\dot{w}_{int} = \sigma_{ij} \cdot \dot{\varepsilon}_{ij}^{p} = p' \cdot \dot{\varepsilon}_v^{p} + q \cdot \dot{\varepsilon}_d^{p} \quad (2)$$

For a case of modified Cam-clay model (Figure 3), internal dissipation rate should be derived from its yielding function (Eq. 3) and associated flow rule (Eq. 4) :

$$f = p' \cdot \frac{M^2 + \left(\frac{q}{p'}\right)^2}{M^2} - p'_y = 0 \quad (3)$$
$$\dot{\varepsilon}_v^{p} = \lambda \cdot \frac{\partial f}{\partial p'} \; \dot{\varepsilon}_d^{p} = \lambda \cdot \frac{\partial f}{\partial q} \quad (4)$$

Concrete form of internal dissipation rate for modified Cam-clay soil is as follows,

$$\dot{w}_{int} = \left\{ \frac{\left(\dot{\varepsilon}_v^{p} + \sqrt{\dot{\varepsilon}_v^{p2} + M^2 \dot{\varepsilon}_d^{p2}}\right)\left(\dot{\varepsilon}_v^{p2} + \dot{\varepsilon}_v^{p}\sqrt{\dot{\varepsilon}_v^{p2} + M^2 \dot{\varepsilon}_d^{p2}} + M^2 \dot{\varepsilon}_d^{p2}\right)}{\left(\dot{\varepsilon}_v^{p} + \sqrt{\dot{\varepsilon}_v^{p2} + M^2 \dot{\varepsilon}_d^{p2}}\right)^2 + M^2 \dot{\varepsilon}_d^{p2}} \right\} p_y' \quad (5)$$

where $p'_y$ and *M* stand for a maximum consolidation stress and a gradient of a critical state line which is can be uniquely determined by tri-axial compression tests.

Volumetric plastic strain rates $\dot{\varepsilon}_v^{p}$ and deviatoric plastic strain rates $\dot{\varepsilon}_d^{p}$ for assumed failure modes are as follows,

$$\dot{\varepsilon}_v^{p} = \dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} + \dot{\varepsilon}_{zz} = -\beta \quad (6)$$
$$\dot{\varepsilon}_d^{p} = \sqrt{\frac{2}{3}\dot{e}_{ij}^{p}\dot{e}_{ij}^{p}} = \beta\sqrt{\frac{4}{9} + \frac{64}{3} \cdot \frac{\alpha^2\beta^2}{D^2}} \quad (7)$$

By substituting Eqs. (6) and (7) into Eq. (5), internal dissipation rate per unit volume is shown as

$$\dot{w}_{int} = \frac{\beta\left\{\sqrt{(4D^2 + 192\alpha^2x^2)M^2 + 9D^2} - 3D\right\}}{6D} \cdot p_y' \quad (8)$$

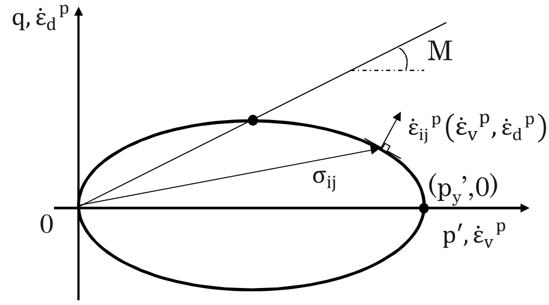


Figure 3. Modified Cam-clay model and associated flow rule

Total internal dissipation rate is then calculated by integrating Eq. (8) over a volume,

$$\dot{W}_{int} = \int_v \dot{w}_{int}(x) \cdot dv = 1 \cdot \int_{x=-\frac{D}{2}}^{\frac{D}{2}} \int_{y=y_0}^{0} \dot{w}_{int} dx\, dy$$
$$= \frac{p_y'\beta D^2}{55296\alpha^3 M^3}\left[-18432\alpha^3 M^3 + A - \sqrt{3}k(4M^2 + 9)(192\alpha^2 M^2 + 4M^2 + 9)\right] \quad (9a)$$

Where *A* and k are as follows,

$$A = \alpha M\sqrt{(48\alpha^2 + 4)M^2 + 9}\;\{(2304\alpha^2 - 96)M^2 - 216\} \quad (9b)$$

$$k = \log\left(\frac{\sqrt{3}\sqrt{(48a^2+4)M^2+9} - 12\alpha M}{\sqrt{3}\sqrt{(48a^2+4)M^2+9} + 12\alpha M}\right) \quad (9c)$$

In this formulation, a closed-form solution of $\dot{W}_{int}$ can be obtained.

### 2.3 Calculation of external plastic work rate

External plastic work rate consists of two terms; $\dot{W}_{ext_1}$ done by self-weight of a soil in a loosened zone and $\dot{W}_{ext_2}$ due to uniform upward pressure q.

$$\dot{W}_{ext_1} = \int_v \gamma\, \dot{u} dv = \int_v \gamma \cdot \beta(y - y_0)\, dv$$
$$= \int_{x=-\frac{D}{2}}^{\frac{D}{2}} \int_{y=y_0}^{0} \gamma \cdot \beta(y - y_0)\, dy\, dx = \frac{4}{15}\gamma\alpha^2\beta D^3 \quad (10)$$

$$\dot{W}_{ext_2} = \int_{x=-\frac{D}{2}}^{\frac{D}{2}} -q \cdot \dot{u}_y \; dx = \int_{x=-\frac{D}{2}}^{\frac{D}{2}} -q \cdot \beta y_0 \; dx$$
$$= \int_{x=-\frac{D}{2}}^{\frac{D}{2}} -q \cdot \left\{-\frac{4\alpha\beta}{D}\left(x - \frac{D}{2}\right)\left(x + \frac{D}{2}\right)\right\} dx = -\frac{2\alpha\beta D^2}{3} \quad (11)$$

External plastic work rate $\dot{W}_{ext}$ is then calculated as follows,

$$\dot{W}_{ext} = \dot{W}_{ext_1} + \dot{W}_{ext_2} = \frac{1}{15}\alpha\beta D^2(4\gamma\,\alpha D - 10q) \quad (12)$$

### 2.4 Evaluation of uniform pressure q to stabilize a trap door

According to the upper bound theorem, a following inequality holds if a structure is stable for the assumed failure mechanism:

$$\dot{W}_{ext} \leq \dot{W}_{int} \quad (13)$$

Therefore, by equilibrating internal dissipation rate and external plastic work rate in eq. (13) and arranging them with respect to a uniform upward pressure q, a following equation can be derived,

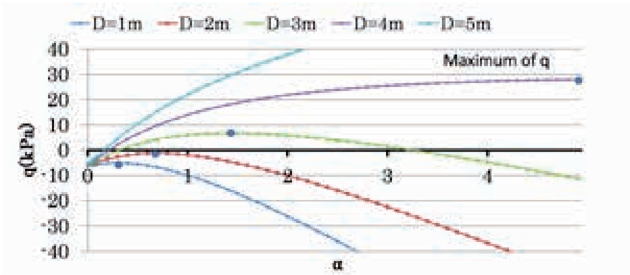


Figure 4.Relation of uniform supporting pressure q and a size parameter of a loosened zone α
(M=0.772 [ϕ=20 deg.], $p_y$' = 100kPa)

$$q = \frac{2}{5}\gamma\,\alpha D - \frac{p_y'}{36864\alpha^3 M^3} \times \left[-18432\alpha^3 M^3 + A - \sqrt{3}k(4M^2+9)(192\alpha^2 M^2 + 4M^2 + 9)\right] \quad (14)$$

where *A* and *k* are as defined in equations 9b and 9c, respectively. By substituting values of parameters in Eq. 14, it is possible to calculate a uniform upward pressure *q* to get a trap door stable for a provided failure mode.

### 2.5 *Estimation of a most critical configuration of a loosened zone and supporting pressure q**

To estimate a most critical size of a loosened zone, a maximum value of *q* with respect to a shape parameter α should be calculated. This calculated $q_{max}$ is a necessary uniform upward pressure to support a trap door q*, provided that a failure mechanism of the above soil is assumed to be a parabola. In this paper, these calculations of q* are carried out for various combinations of soil parameters.

Finally, calculated quantities such as a most critical size parameter α and a most critical uniform upward pressure q* are summarized as a function of soil parameters M , $p_y'$ and a width of a trap door D.

## 3 ANALYTICAL RESULTS AND DISCUSSIONS

Figure 4 illustrates a relation of a loosened zone size parameter a and a uniform upward pressure q to support a trap door with various trap door width D cases for a given soil condition. A largest (critical) value of q for each case is denoted as solid circle. For cases of a trap door width D=1m and D=2m, critical values of q* are negative. This indicates that a trap door is stable without a upward supporting pressure. In opposite to this, for cases of D=3m and 4m cases, upward supporting pressures are necessary to keep a trap door stable. In addition to this, for a case of D=5m, a largest value of q cannot be found within a range of a between 0 and 5. There results directly means that stability of a trap door is deteriorated if a width of a trap door is larger. It is commonly observed in tunneling engineering that a pilot tunnel or a bench excavation is adopted prior to a full cross section excavation.

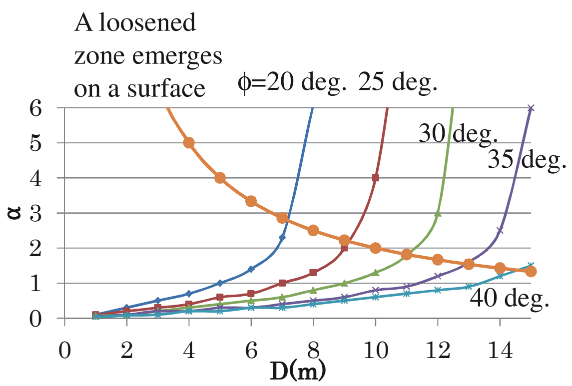


Figure 5.Relation of a trap door width D and a size parameter of a loosened zone α ($p_y$' = 200kPa, buried depth 20m[assumption])

Figure 5 depicts a relation of a size of a loosened soil α and a trap door width D at a critical supporting pressure q*. In this example, soil parameters are assumed that a maximum consolidation stress is 200 kPa and various internal friction angles. In this figure, a limit of the assumed failure modes is also drawn such that d = α * D under the assumption that a depth of a soil layer is d=20m. This limit line coincides with cases when a loosened zone reaches the surface of a ground. For such cases, a ground arch action cannot be expected. Figure 6 is also a relation of a size parameter and a trap door width for $p_y$'=300 kPa cases.

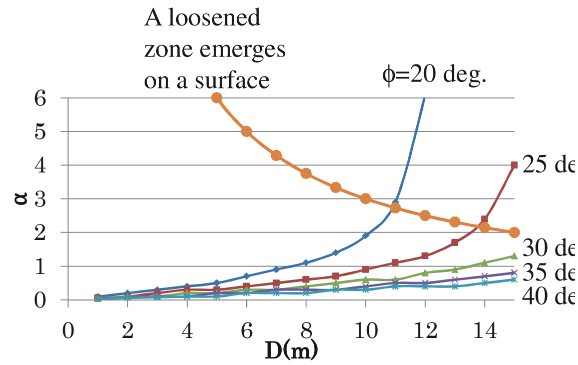


Figure 6.Relation of a trap door width D and a size parameter of a loosened zone α ($p_y$' = 300kPa, buried depth 20m [assumption])

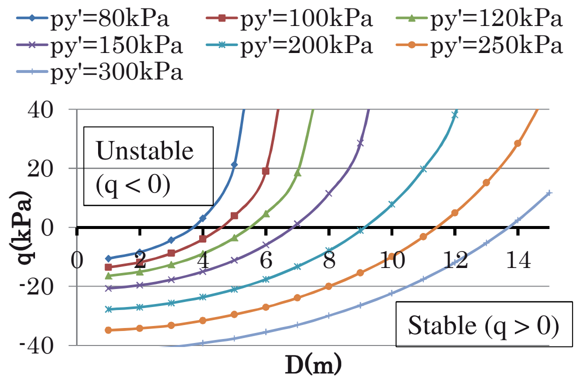


Figure 7.Relation of a trap door width D and a upward supporting pressure q for various consolidation stress cases ($p_y$' = 80-300kPa, internal friction angle ϕ=30 deg.)

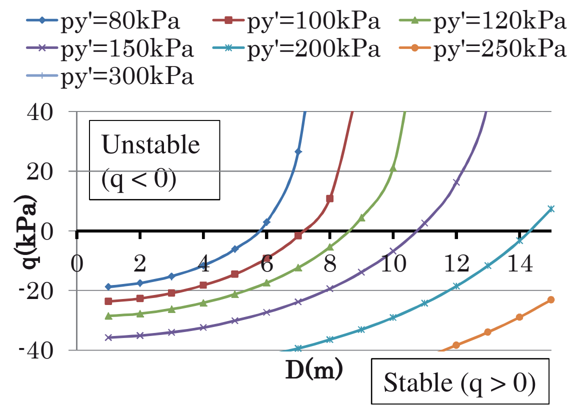


Figure 8.Relation of a trap door width D and a upward supporting

Relations of a most critical supporting pressure q* and a trap door width D are shown in Figures 7 and 8 for various consolidation stresses and internal friction angles. If q* is positive, a trap door is stable without upward supporting pressures. It is well observed that a trap door is more stable if a consolidation stress $p_y'$ and/or an internal friction angle ϕ are larger. In other words, a critical trap door width without upward supporting pressures is larger, if a consolidation stress and/or an internal friction angle are larger.

Same results can be arranged as relations of a size of a loosened soil α and a trap door width D shown in Figures 9 and 10. As a width of a trap door increases, a size parameter a also increases but much drastically. This indicates that a ground arch action is less effective for larger width cases.

It should be noted that a merit and a limit of rigid-plastic analysis in practical engineering. In this article, calculated critical pressures q* are at the instant of failure. After the occurrence of a failure, soil strength may change due to a subsequent behavior, such as plastic hardening or softening with volumetric change. However, such post failure behavior is out of scope in rigid-plastic calculations. If such evolutionary behavior is crucial, it is not recommended to use rigid-plastic calculations. However, a short term stability accompanied by unloading is dominant for a stability problem of a trap door, because subsequent behavior of soils is not an engineering interest. For this kind of problems, rigid-plastic calculations are advantageous for engineering practice, because it is easy to conduct parametric studies with less numbers of parameters of models.

## 4 CONCLUSIONS

In this article, stability of a trap door buried in a modified Cam-clay soil supported by a uniform upward pressure is investigated. Conclusions of this study are summarized as follows,

Rigid-plastic calculation is proposed to evaluate necessary upward pressures to support a trap door. A failure mode of a soil above a trap door is modeled as a parabolic shape with linearly distributed velocities. Modified Cam-clay model with the associated flow rule is adopted. Because it is focused on the initiation of failures, no evolutionary behavior such as strain hardening / softening is considered in the analysis.

Numerical results show that critical upward pressures q* for the stability of a trap door are a function of material properties of a soil ($p_y$' and M or ϕ) and a width of a trap door D. If q* is negative for a certain case, it means that a trap door is stable without an upward supporting pressure. In general, if a trap door buried in same soils, it is less stable for larger width cases. Relation of a trap door width D and a size parameter α which is linked to a size of a loosened soil zone above a trap door is also quantitatively evaluated for various soil conditions. A parameter α drastically increases with the increase of a trap door width D, which reflects a ground arch action around a trap door.

A proposed method is simple, but also based on the rigorous theoretical background. As this method requires less computational time and cost, it might be promising for parametric studies necessary for practical engineering, such as preliminary design.

## 5 ACKNOWLEDGEMENTS

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