Strain Response Envelopes for low cycle loading processes

Enveloppe de réponse d´allongement pour chargements cycliques de basse intensité

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ABSTRACT: To look onto the stress-path-dependent strain behaviour at low-cycle loading, drained, stress-controlled triaxial-tests have been carried out. Here the focus was to investigate strain-response envelopes which result from applying relatively small stress increments of ≤ 50 kN/m², to the soil-specimen. It is found that quasi-elastic behaviour can already occur at low numbers of cycles. The shapes of the obtained strain-response-envelopes are similar to symmetrical ellipses. It can be observed, that the size of the ellipses decreases with increasing mean pressure p. The major axis of the ellipses rotates depending on the initial stress state $\eta=q/p$, indicating a stress-induced anisotropy. Preloading seems to have little effect on the stiffness or the directions of the quasi-elastic strains.

RÉSUMÉ : Afin d´analyser le comportement de dilatation résultant des chemins de contrainte pour un nombre réduit de cycles de chargement, des essais triaxiaux drainés ont été réalisés. Le thème central est l´analyse des enveloppes des réponses d´allongement , en employant des incréments de contrainte relativement petits de ≤ 50 kN/m². Il s´est avéré, que le sable manifeste un comportement quasi élastique après un nombre de cycles réduit. La forme des enveloppes de réponse d´allongement est celle des ellipses symétriques. Les diamètres des ellipses se réduisent, lorsqu´on augmente la pression moyenne. Les axes majeurs des ellipses changent d´inclination en fonction de la contrainte initiale $\eta=p/q$, indiquant une anisotropie causée par la contrainte. Il apparait qu´un chargement préliminaire a peu d´influence sur la rigidité et les directions des allongements quasi-élastiques.

KEYWORDS: low cycle loading processes, triaxial tests, strain response envelopes

1 INTRODUCTION

Due to quasi-static loading with cyclic progression there are plastic, i.e. irreversible, and elastic, i.e. reversible deformations in the soil, without reaching fully elastic behaviour. In the quasi-elastic regime the material behaves asympotically elastic. Goldscheider (1978) describes this behaviour as “material shakedown”.

Considering the number of cycles one can distinguish between high cycle and low cycle loading processes.

Effects of wind load on foundations of wind energy plants, vehicle crossing on foundation constructions, vibrating of foundation-elements e.g. retaining-wall-elements or grouted piles can be related to high cycle loading. The number of cycles N during these processes is very high (N >> 50). Due to the accumulation of numerical errors and a high computing time an implicit calculation of displacements, where the deformations during one cycle are calculated separately and accumulated, is not adequate. Instead, deformations due to high cycle loading are calculated by using explicit models. Here the calculation of irreversible strains can be treated similar to creep deformations under constant loads (Wichtmann et al., 2005).

Low cycle loading processes can be defined for a lower number of cycles with N ≤ 50, Danne & Hettler (2011). Deformations in this case are usually calculated implicitly, i.e. for each cycle separately and then accumulated.

Subject of this article are low cycle loading processes, where it is assumed that inertial forces are negligible (Hettler, 1981). Un- and reloading for example, occurring during the construction phase of multiple braced excavation walls, produce stress paths quite similar to those of cyclically loaded systems at the first cycles before reaching shakedown. Therefore, these processes are also included within the scope of low cycle loading.

An external cyclic load on a foundation for example does not lead to cyclic behaviour right from the beginning. This is the case only after a certain number of cycles.

Some examples for low cycle loading processes and related un- and reloading processes are:

- construction stages of multiple braced or anchored excavation walls
- braced excavation with force-controlled struts (to control deformations)
- temperature exposure of struts
- filling and emptying of locks or silos during first utilisation phase
- summer-/winter position of abutments of integral bridges due to temperature differences

The simplified consideration of a soil element behind a strutted retaining wall shows, that monotonous stress-paths as well as repeated low cycle loading process with various directions can occur (figure 1).

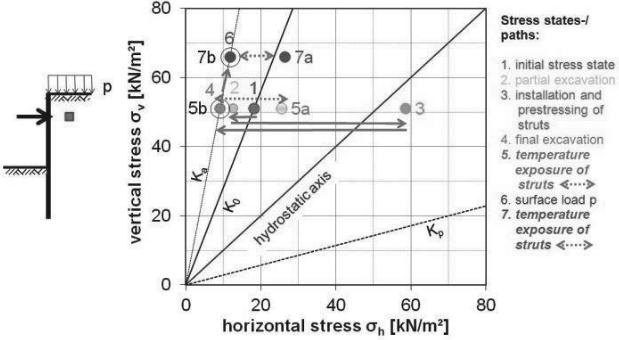

Figure 1: Typical stress-paths in a soil-element behind an excavation-wall

In front of the embedded part of the wall stress-paths are similar, but extension may be important instead of compression.

Element tests investigating the stress-strain behaviour of soils must therefore take into account any stress-path and repeated un- and reloading processes when contributing successfully to the development of new or further developed constitutive equations. It is also obvious, that stress states in compression as well as in extension region have to be considered.

2 RESPONSE ENVELOPES

2.1 *Concept*

New or improved constitutive models need to be validated and calibrated. This is often done with the aid of numerical element tests, for example triaxial tests or oedometer tests.

So called response envelopes are a useful tool for calibrating, validating and comparing constitutive equations (Sibille 2011, Doanh 2000, Kolymbas 2000, Tamagnini 2006).

First basics of response-envelopes were presented in the 1970s by Lewin & Burland (1970). A few years later Gudehus (1979) used this concept in context with the development of constitutive equations.

To obtain a response-envelope, a soil element is subjected to a certain stress- or strain-increment. The corresponding "response" of the soil in form of either strain or stress is determined and described graphically. The direction of the implied stress- or strain increment with a constant absolute value is then varied and leads to different stress- or strain responses, endpoints of which are connected to a response-envelope.

In figure 2 the strain-responses due to a constant stress increment Δσ applied in 8 different directions α_σ are shown. Keeping the absolute value of $\Delta\sigma = \sqrt{\Delta\sigma_1^2 + 2\Delta\sigma_2^2}$ constant for all directions α_σ, one gets a circle in the Rendulic-plane with the axes $\sqrt{2}\,\Delta\sigma_2$ and $\Delta\sigma_1$, figure 2a

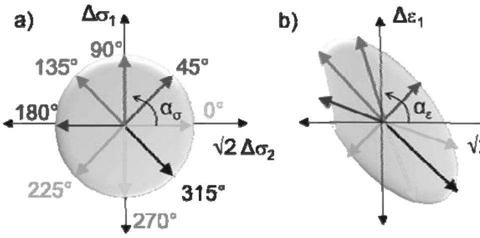

Figure 2: Concept of strain response envelopes
a) applied stress increments b) resulting strains

The $\alpha_\sigma = 90°$-stress-path in figure 2a for example is equivalent to pure triaxial compression, in the same figure, stress-path $\alpha_\sigma = 180°$ stands for pure radial extension. The strains are also plotted in the Rendulic-diagram (figure 2b), where the resulting strain-increment is $\Delta\varepsilon = \sqrt{\Delta\varepsilon_1^2 + 2\Delta\varepsilon_2^2}$.

The concept of response-envelopes is a convenient tool to investigate the incremental stress-strain behaviour during first loading as well as during un- and reloading-processes.

In this paper the quasi-elastic part of the strains, i.e. the strains due to un- and reloading is investigated and evaluated by means of strain-response-envelopes.

2.2 *Literature*

Only few experimental tests to obtain stress or strain response envelopes can be found in literature.

Anandarajah et al. (1995) performed a series of stress-probe experiments on dense and medium dense Ottawa sand to investigate the dependence of magnitude and direction of incremental plastic strain on direction of incremental stress. 6 different initial stress-states in compression were chosen and stress increments from Δσ = 9 to 52 kPa in up to 10 different directions were applied on triaxial specimens. The focus was set on plastic strains, which were evaluated by subtracting the elastic strains from the total strains. The elastic strains again were either calculated "by using suitable elastic properties" or determined by applying a stress cycle and measuring the elastic strains during reversal.

Doanh (2000) for example describes tests producing strain-response-envelopes at 3 different initial stress-states for dense Hostun sand. The considered stress increment was Δσ = 10 kN/m². For each direction, one soil-sample was used, so that the determined strain increments can be interpreted as total or elastoplastic strains after first loading. Quasi-elastic strains were not determined separately.

Costanzo et al. (2006) performed several triaxial tests to obtain strain-response-envelopes on a silty clay at 2 different initial stress-states. The strains were investigated and plotted for stress-increments between Δσ = 20 to 90 kN/m². Quasi-elastic strains were not considered explicitly either.

There is hardly any literature where "quasi-elastic" strain-response-envelopes due to low cycle loading are presented. There are quite some articles though, where quasi-elastic stress-strain-behaviour is investigated after applying very small axial or radial stress- or strain amplitudes (Ezaoui & Di Benedetto 2009, Hoque & Tatsuoka 1998, Kuwano et al. 2002).

3 RESULTS

3.1 *Experimental fundamentals*

The triaxial device used for the presented experiments is equipped with high-resolution measurement- and control-technology. The confining pressure as well as the axial force can be controlled independently, so that any possible stress-path from any initial stress-state can be followed. Height and diameter of the soil specimen are 10 cm.

The tested soil is a fine grained sand with a low uniformity-index (C_U = 1,25 mm, d_{50} = 0,15 mm). It could be shown by different criterions (Nicholson et. al., 1993), that by using this kind of sand, errors from bedding-effects and membrane-penetration can be reduced significantly compared e.g. to Karlsruhe middle-sand.

3.2 *Testing procedure*

Before running the triaxial tests the dry sand is pluviated to obtain the soil sample and then the sample is saturated with deaerated water. The specimen-preparation-method was kept constant for all tests. The relative density varied between I_D = 0,6…0,7.

After saturating the soil sample, an initial stress state is reached by first increasing the isotropic stress. Depending on the position of the initial stress-state, either the vertical stress (for stress-states in compression) or the horizontal stress (for stress-states in extension) is then increased. Not only the mean pressure p is varied, but also the deviator-stress q or the stress-ratio η = q/p respectively.

Then stress cycles of relatively small stress increments of Δσ ≤ 50 kN/m² are applied in a certain direction α_σ. To avoid pore water pressure the frequency of the cycles is kept low.

The cyclic load in the first direction is repeated until the measured strains are practically reversible or rather quasi-elastic. The definition of "quasi-elasticity" implies that during one cycle the plastic strains are less than 1…3 % of the total strains, see Danne & Hettler (2011). It turns out that quasi-elastic behaviour can occur after a low number of cycles. The strain response of the last cycle is evaluated and plotted. After that, the test is continued with the same stress increment Δσ, but in a different direction α_σ in the stress-space (figure 3a) until quasi-elastic behaviour occurs again. The corresponding strains of the last cycle are plotted in a diagram, figure 3b.

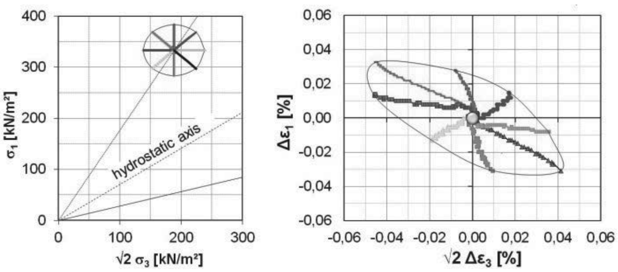

Figure 3: Construction of the strain response envelope

a) application of $\Delta\sigma = 50$ kN/m² in 8 different directions

b) strain responses for all 8 directions

The envelope of the strain-responses tends to have the shape of an ellipse. The highest absolute values of quasi-elastic strains always occur in the directions $\alpha_\sigma = 135°$ and $\alpha_\sigma = 315°$; the smallest absolute values result from directions $\alpha_\sigma = 45°$ and $\alpha_\sigma = 225°$.

The investigation of the influence of the sequence of the directions, the mean pressure p, a monotonous isotropic prestress and the stress-ration η on the shape, size and inclination of the response-envelope, i.e. the direction-dependent quasi-elastic stiffness, is described in the following.

3.3 *Different sequence of stress paths*

To investigate the influence of the sequence of the applied stress-paths on the quasi-elastic strain-responses, the testing procedure described in section 3.2 was applied for different sequences of directions α_σ. The rotational direction was also varied and carried out clockwise and counter clockwise. It is found, that – for the investigated initial stress-states – neither the sequence nor the rotational direction of the applied stress-paths leads to a substantial influence on the strain response envelopes, figure 4.

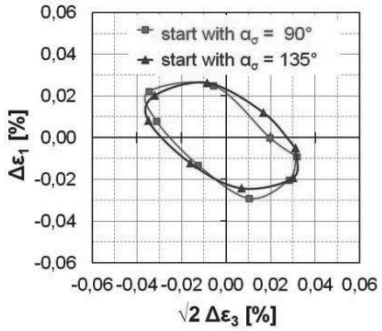

Figure 4: Response envelopes due to $\Delta\sigma = 50$ kN/m² for 2 different sequences of stress-paths from the same initial stress-state

Further test are carried out which seem to confirm these results.

3.4 *Stress-dependent stiffness*

To investigate the stress-dependency of the quasi-elastic stiffness at low cycle loading tests at 3 different initial stress-states with a constant stress-ratio η and varying mean pressure p were performed. The quasi-elastic strains due to a stress increment $\Delta\sigma = 50$ kN/m² were determined and plotted by means of response-envelopes, figure 5.

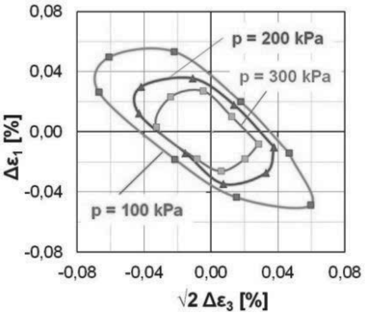

Figure 5: Comparison of response-envelopes due to $\Delta\sigma = 50$ kN/m² for 3 different mean pressures p and constant initial stress-ratio $\eta = 0{,}75$

As shown in figure 5 the size of the ellipses decreases with increasing mean pressure p. This means that the stiffness increases. This is especially evident at the stress-paths $\alpha_\sigma = 135°$ and $\alpha_\sigma = 315°$. The influence of p on the elastic moduli at the directions $\alpha_\sigma = 45°$ und $\alpha_\sigma = 225°$ is much lower.

3.5 *Isotropic prestress*

To examine the influence of a static isotropic preloading on the size and shape of the quasi-elastic response-envelopes, different tests were carried out starting at the same stress point with and without preloading.

It seems that the influence of an isotropic preloading is negligible, figure 6.

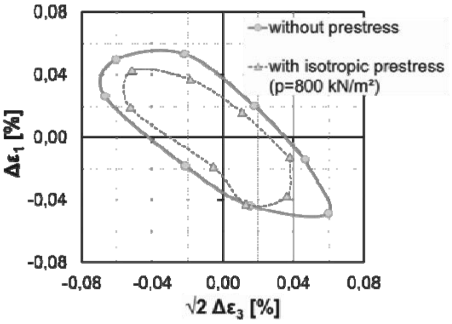

Figure 6: Response-envelope due to $\Delta\sigma = 50$ kN/m² with and without prestress

Similar observations were made when applying an anisotropic preloading.

3.6 *Anisotropy*

To investigate anisotropic material properties, tests were carried out for p = const. and different initial stress-ratios η. The resulting envelopes are plotted in the p-q-plane, figure 7.

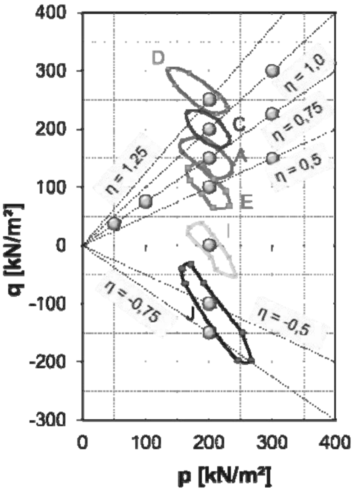

Figure 7: Rotation of axes of the response-envelopes depending on the stress-ration η

Obviously, there is a rotation of the main axes of the response-envelopes. That means that the ratios of quasi-elastic moduli depend on the stress ratio η.

This influence can be quantified. Figure 8 shows the ratio E_v/E_h of the vertical stiffness $E_v = \Delta\sigma_v/\Delta\varepsilon_v$ and the horizontal stiffness $E_h = \Delta\sigma_h/\Delta\varepsilon_h$ as a function of the stress-ratio η. For this purpose data were analysed for stress-paths $\alpha_\sigma = 90°$ and 270°, (axial compression and extension) and $\alpha_\sigma = 0°$ and 180° (radial compression and extension).

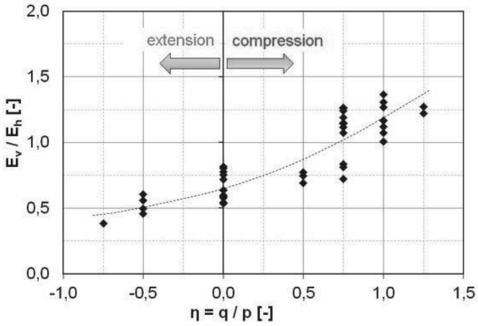

Figure 8: Ratio E_v/E_h depending on the initial stress-ratio η

The dependence of the ratio E_v/E_h on the initial stress-ratio η can be interpreted as a **stress-induced anisotropy**. Similar observations are also made when investigating much smaller stress- or strain-cycles, e.g. Ezaoui and Di Benedetto (2009) or Hoque and Tatsuoka (1998). The coarser the sand, the more distinctive is the difference between E_v and E_h, i.e. the ratio E_v/E_h increases Hoque and Tatsuoka (1998). A detailed analysis shows a stronger influence of the stress-ratio η on the vertical than on the horizontal stiffness, see Bellotti, et al. (1996).

Figure 8 does not only show a stress induced anisotropy. At the isotropic stress state with η = 0 the ratio E_v/E_h is $\neq 1$. This means, that there are no isotropic properties at an initial isotropic stress state, i.e. there also is an **inherent anisotropy**. Most authors come to similar conclusions. While Hoque and Tatsuoka (1998) find out $E_v/E_h \geq 1$ for all tested sands at isotropic stress-states, Di Benedetto (2010) also finds ratios $E_v/E_h < 1$ for the preparation-methods pluviation and vibration and thus demonstrates a dependency of this ratio on the specimen preparation-method. These discrepancies seem to be due to several factors e.g. the grain-size distribution, the shape of the specimen and the preparation-method.

4 SUMMARY AND FURTHER HINTS

Producing experimental or numerical response envelopes is a convenient tool to investigate a soil's incremental stress-strain behaviour and to test or compare constitutive equations.

The investigation of the incremental stress-strain behaviour of sand at low cycle loading procedures in triaxial testing shows, that for stress-increments $\Delta\sigma \leq 50$ kPa quasi-elastic behaviour can occur after a low number of cycles. While the influence of the sequence of the stress-paths on the quasi-elastic strains seems to be negligible, a strong influence of the mean pressure p on the size of the strain-response-envelopes is observed.

For low number of cycles, the influence of an isotropic prestress on the quasi-elastic strains seems to be negligible so far. There is a stress-induced anisotropy, which can be shown by the rotation of the axes of the ellipses depending on the initial stress-ratio η.

Further triaxial tests are necessary in order to investigate e.g. the influence of the void ratio and of a K_0-preloading. Because of the role of triaxial extension, further tests in extension region will also be carried out. In addition plastic strains due to low cycle loading will also be investigated; first results are already available.

It is known, that some common constitutive models show deficits when predicting deformations due to high and low cycle loading processes, e.g. ratcheting in hypoplasticity, elastic behaviour after the first un- and reloading in elastoplastic constitutive models, missing anisotropy.

It is intended to use the results presented in this paper together with future tests as a basis for calibrating and validating more complex constitutive equations especially developed for low cycle loading processes (Ehlers and Avci, 2011, Niemunis, et al., 2011).

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