Seismic bearing capacity of strip footings near cohesive slopes using lower bound limit analysis

Capacité portante séismique des fondations superficielles en bord des talus purement cohérents ; une évaluation par défaut suivant la méthode du calcul à la rupture

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ABSTRACT: A finite element lower bound method together with linear programming technique are used to determine the seismic bearing capacity of strip footings adjacent to purely cohesive slopes. The pseudo-static approach is utilized and the earthquake forces consist of a horizontal load applied to the foundation and the inertia of the soil mass. It is assumed that the soil obeys the associated flow rule and undrained behavior of soil under seismic condition can be modeled by Tresca yield criterion. The normalized seismic bearing capacity of footing is considered as a function of dimensionless parameters which affect the stability of footing-on-slope system. The effect of seismic coefficient, k_h, on bearing capacity of so-called system is investigated and design charts are presented for a rational range of parameters.

RÉSUMÉ : L'approche par défaut de la théorie du calcul à la rupture, basée sur un modèle des éléments finis et la technique de la programmation linéaire, a été utilisée pour évaluer la capacité portante sismique des semelles filantes en bord des talus. Le sol est purement cohérent, obéissant au critère de plasticité de Tresca. Les sollicitations sismiques, étant par hypothèses pseudo-statiques, sont les suivantes; une composante tangentielle s'ajoutant à la force normale de fondation et une composante horizontale de force volumique appliquée au sol. La capacité portante normalisée , en fonction des paramètres du problème à été calculée et sa réduction en fonction du coefficient séismique, k_h, a été présentée sous forme des diagrammes sans dimensions pour de larges gammes des paramètres concernés .

KEYWORDS:Seismic bearing capacity, Strip footing, Slope, Lower bound, Linear programming.

1 INTRODUCTION

Some of engineering structures require to be built near a slope, particularly in mountainous and hilly regions, so, stability of these structures is one of the important problems of geotechnical engineering. Bearing capacity of strip footings near slopes has been investigated by numerous researchers but most of these researches include the static condition and there are only a few studies about stability of strip footings near slopes under earthquake condition.

Available analytical limit state solutions for seismic bearing capacity of strip footings near slopes include upper bound method (Sawada et al. 1994, Askari and Farzaneh 2003), limit equilibrium method (Sarma and Chen 1996, Sarma 1999, Kumar and Kumar 2003, Choudhury and SubbaRao 2006) and method of characteristics (Kumar and Rao 2003, Jahanandish and Arvin 2008). Limit analysis methods (i.e. upper and lower bounds) are direct approaches of classical plasticity theory for calculation of collapse load in stability problems. Static and kinematic approach of limit analysis lead to lower and upper estimation of true collapse load respectively.

As lower bound solution gives a load which is below the exact ultimate load, it is at safe side and therefore more appealing. As there was no lower bound solution in the literature, the current paper is presented to estimate seismic bearing capacity of strip footings near cohesive slopes by lower bound method. A brief theory and formulation of finite element lower bound method is presented and more details can be found in relevant references and won't be covered here. It should be mentioned that a MATLAB code has been provided by the authors for determination of seismic bearing capacity of strip footings near slopes.

2 PROBLEM DEFINITION

The problem of seismic bearing capacity of a strip footing adjacent to a purely cohesive slope is shown in Figure 1. Geometric parameters include the slope angle β, distance of footing from the slope a, footing width B and height of the slope H. It is assumed that the soil obeys the associated flow rule and Tresca yield criterion and has the undrained shear strength of c_u and unit weight of γ.The base of the foundation is assumed rough. The horizontal acceleration coefficient of earthquake (k_h) is considered outward the slope which is more critical and the vertical acceleration of earthquake is ignored in analyses.

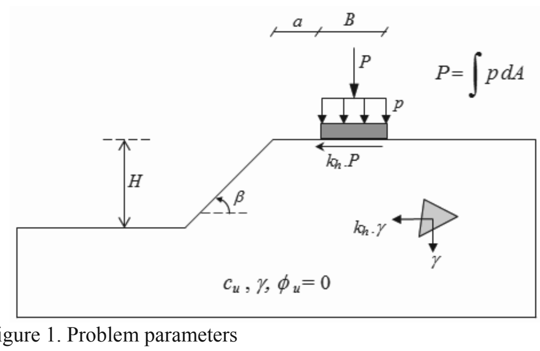

Figure 1. Problem parameters

The approach of this paper is to consider the normalized limit pressure as a function of dimensionless parameters affecting the stability of the footing-on-slope system and can be stated as:

$$\frac{p}{\gamma B} = f(\beta, \frac{a}{B}, \frac{H}{B}, \frac{c_u}{\gamma B}, k_h) \quad (1)$$

in which p is the average vertical limit pressure under the footing base. The effects of these parameters on bearing capacity of so-called system will be discussed in the following sections.

3 FINITE ELEMENT FORM OF LOWERBOUND LIMITANALYSIS

The lower bound limit theory (Drucker et al. 1952) may be stated as: "If all changes in geometry occurring during collapse are neglected, a load obtained from a statically admissible stress field is less than or equal to the exact collapse load."

A statically admissible stress field is one which satisfies equilibrium and boundary conditions and nowhere violates the yield criterion.

The formulation used in this paper follows that of Sloan (1988) in which the linear finite element method is applied and the domain of problem is discretized by 3-noded triangular elements. Unknowns of the problem are nodal stresses (σ_x, σ_y, τ_{xy}). Figure 2 shows the typical elements used in lower bound analysis. Extension elements are used to extend the statically admissible stress field into a semi-infinite domain and thus lead to a rigorous lower bound solution.

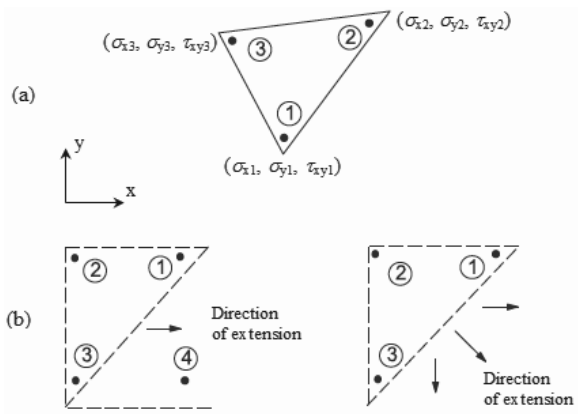

Figure 2.Typical linear triangular element (a) and extension elements (b) used in lower bound analysis (Shiauet al. 2003)

Using linear finite element method, the stresses vary linearly throughout each element according to:

$$\sigma_x = \sum_{l=1}^{3} N_l\, \sigma_x^l \;;\; \sigma_y = \sum_{l=1}^{3} N_l\, \sigma_y^l \;;\; \tau_{xy} = \sum_{l=1}^{3} N_l\, \tau_{xy}^l \qquad (2)$$

where σ_x^l, σ_y^l and τ_{xy}^l are nodal stress components and N_l are linear shape functions.

For present problem where the pseudo-static force acts outward the slope and the vertical acceleration of the earthquake is ignored, equilibrium equations for a right-handed x-y coordinate system can be stated as:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = k_h.\gamma \;;\; \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = \gamma \qquad (3)$$

where $k_h=a_h/g$ is the horizontal acceleration coefficient of earthquake. Combination of equations (2) and (3) leads to a matrix form of element equilibrium equations.

The main difference between a lower bound mesh and a classic finite element one is that some nodes may have the same co-ordinate. Thus, the statically admissible stress discontinuities can occur at shared edges of adjacent elements (Figure3).

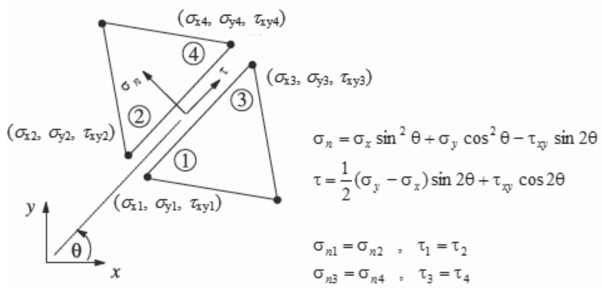

Figure 3. Statically admissible stress discontinuity

The undrained behavior of clays (ϕ_u=0) can be modeled by Tresca yield criterion and can be expressed as following nonlinear equation in plain strain condition:

$$F = (\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2 - (2c_u)^2 = 0 \qquad (4)$$

in which tensile stresses are taken as positive (sloan 1988).

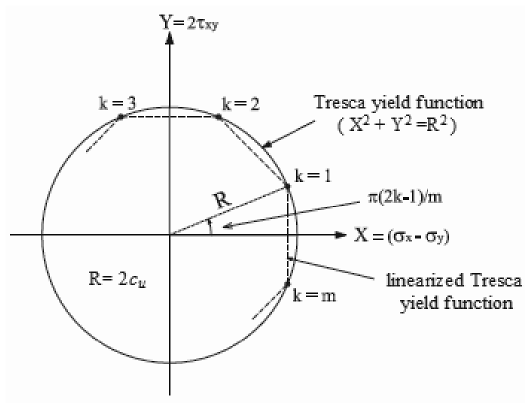

Figure 4. Linearized Tresca yield function

In order to use linear programming technique, the linearized Tresca yield function should be utilized in formulation of lower bound theory. For this purpose, the nonlinear yield criterion is approximated by an interior polygon in lower bound limit analysis (Figure 4).

By assembling all equalities and inequalities, a discrete formulation of the lower bound theory leads to the following constrained optimization problem:

$$\text{maximize}: \quad \mathrm{c}^{\mathrm{T}}\boldsymbol{\sigma}$$
$$\text{subject to}: \quad \begin{cases} \mathrm{A}_1\boldsymbol{\sigma} = \mathrm{b}_1 \\ \mathrm{A}_2\boldsymbol{\sigma} \le \mathrm{b}_2 \end{cases} \qquad (5)$$

where cis a vector of objective function coefficients, $\boldsymbol{\sigma}$ is the vector of problem unknowns, $A_1$is an overall matrix of equality constraint coefficients which derives from elements equilibrium, discontinuities equilibrium and boundary conditions, b_2 is a right-hand vector of equality coefficients, A_2 is an overall matrix of inequality constraint coefficients which derives from yield criterion and b_2 is the corresponding right-hand vector. Using linear finite elements and linearized yield function, the lower estimation of true collapse load can be obtained through linear programming techniques. In current study, the "active-set" algorithm is used for optimization of lower bound limit load.

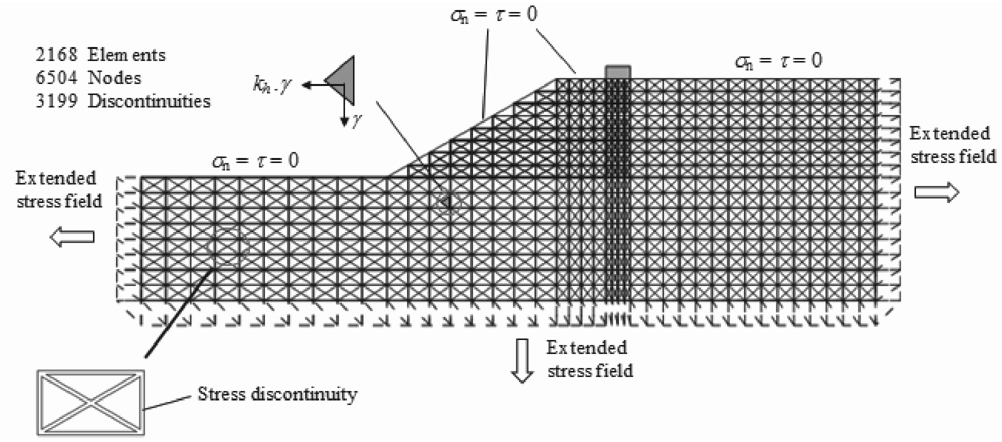

Figure 5. Typical finite element mesh used in lower bound analysis (β=30˚, a/B=2)

4 DETAILS OF ANALYSES

A typical finite element model used in lower bound analysis for a problem with β=30˚and a/B=2 is illustrated in Figure 5 which consists of 2168 elements, 6504 nodes and 3199 stress discontinuities. It should be mentioned that in all of the finite element models of current study, statically admissible stress discontinuities are considered at all shared edges of adjacent elements.

For a footing-on-slope system, the ultimate bearing capacity of the footing may be governed by either the foundation bearing capacity or the overall stability of the slope. The combination of these two factors makes the problem difficult to solve (Shiau et al. 2011).Typical mechanisms of these two failure modes are illustrated in Figure 6.In current paper, the bearing capacity mode is investigated in analyses.

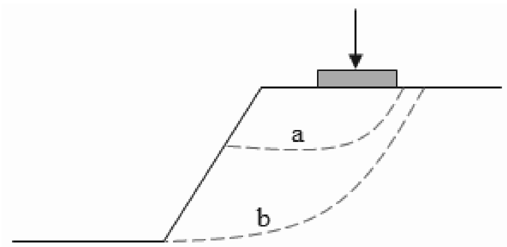

Figure 6. Typical failure modes for footing-on-slope system: bearing capacity mode (a) and overall slope failure (b).

A wide range of parameters has been examined in analyses. Three slope angles β=30˚,60˚,90˚,horizontal acceleration coefficients of k_h=0, 0.1, 0.15, 0.2, 0.25,0.3 and various ratios of a/B and $c_u/\gamma B$ were considered. According to results, the value of H/B=4insures that the bearing capacity mode (mode a) will occur for all values of k_h mentioned above. So, H/B=4was considered in all models.

5 RESULTS AND DISCUSSION

The effects of various parameters on seismic bearing capacity of strip footings near cohesive slopes were examined. Figure 7 shows the variation of dimensionless bearing capacity ($p/\gamma B$) with the seismic coefficient (k_h) for a problem with $c_u/\gamma B$=2 and a/B=0.

As can be observed, the bearing capacity decreases as the horizontal acceleration increases. The bearing capacity reduces more steeply from k_h=0 to 0.1with respect to k_h=0.1 to 0.3. It is also seen that the decline of bearing capacity for the second part of diagrams (k_h≥0.1) intensifies as the slope angle reduces. For example, the dimensionless bearing capacity of a slope with β=30˚ changes from 6.88 to 5.16 (roughly 25% reduction) as k_h increases from 0.1 to 0.3. This reduction is about 21.5 % for β=60˚ and 19% forβ=90˚.

The influence of relative distance of the footing (a/B) on seismic bearing capacity of a footing-on-slope problem is illustrated in Figure 8 for $c_u/\gamma B$=5, k_h= 0.2 and various slope angles.

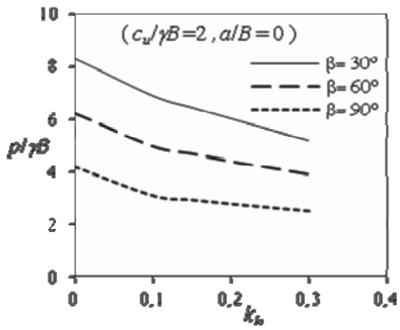

Figure 7. Variation of dimensionless seismic bearing capacity with seismic coefficient k_h

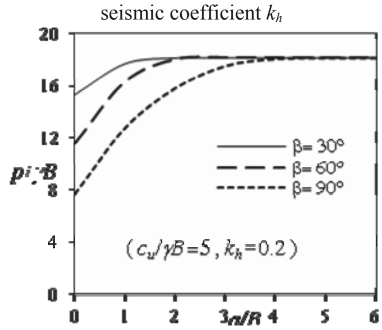

Figure 8. Variation of seismic bearing capacity with a/B

The bearing capacity increases when footing distance from the slope crest increases and from a definite value of a/B the bearing capacity remains constant. This means that the effect of slope diminishes as the foundation goes far from the slope crest and the bearing capacity approaches the bearing capacity of a footing on level ground. It can also be observed that the footing which rests on a slope with lower angle reaches to this constant value more rapidly. The other parameter which has a great effect on seismic stability of strip footings near slopes is the dimensionless strength of $c_u/\gamma B$. Results show that the variation of $p/\gamma B$ is linear for a definite range of this parameter (Figure 9).

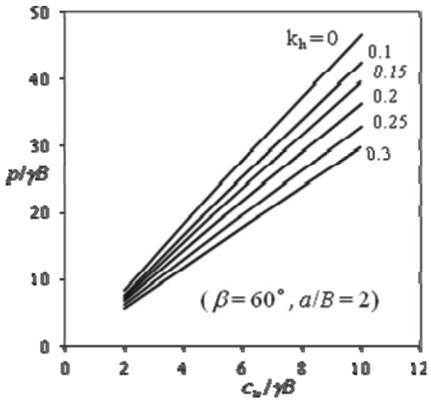

Figure 9. Variation of seismic bearing capacity with $c_u/\gamma B$

As (Shiau et al. 2011) discussed elaborately, for small values of $c_u/\gamma B$, diagrams of bearing capacity versus dimensionless cohesion parameter ($c_u/\gamma B$) becomes nonlinear and the slope stability reduces rapidly and the overall slope failure (mode b in Figure 5) takes place. In current study, the situation is more critical due to the lateral effect of earthquake. So, the range of nonlinear part in bearing capacity diagrams is more extended than that of static condition. The beginning point of linear portion of the diagrams depends on various parameters such as β and k_h. For higher values of β and k_h, linear portion begins at greater ratio of $c_u/\gamma B$. In current paper, the nonlinear parts of diagrams are not displayed and design charts are presented for $c_u/\gamma B \geq 2$ where all diagrams are linear and bearing capacity mode (mode a of Figure 5) will occur.

6 DESIGN CHARTS

In this section, design charts for lower bound estimation of seismic bearing capacity of strip footings near cohesive slopes are presented. These charts covers the range of parameters as β =30˚, 60˚, 90˚, k_h=0, 0.1, 0.15, 0.2, 0.25, 0.3, a/B=0, 1 and2≤ $c_u/\gamma B$≤10which are presented in Figures 10 and 11.

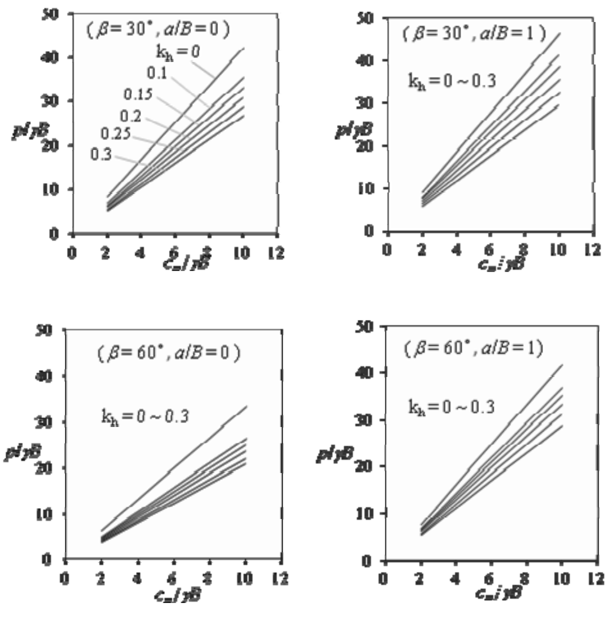

Figure 10. Design charts for seismic bearing capacity of strip footings near cohesive slopes(β=30˚, 60˚and a/B =0,1)

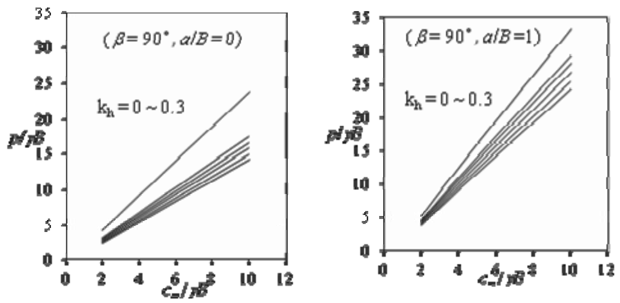

Figure 11. Design charts for seismic bearing capacity of strip footings near cohesive slopes (β=90˚and a/B =0, 1)

7 CONCLUSION

Finite element lower bound method and pseudo-static approach facilitated the investigation of seismic bearing capacity of strip footings adjacent to slopes. Earthquake force has a decreasing effect on bearing capacity of footing-on-slope system which was examined through various analyses and relevant charts. It was observed that the slopes with smaller angles have a slightly steeper reduction in seismic bearing capacity. Presented design charts can be used for estimation of seismic bearing capacity of strip footings near slopes for a wide range of horizontal seismic coefficients.

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