

# Evaluation of void ratio and elastic modulus of unsaturated soil using elastic waves

## Évaluation de l'indice des vides et du module élastique d'un sol non saturé en utilisant les ondes élastiques

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**ABSTRACT:** The void ratio and elastic modulus of a soil influence the soil's behavior. In unsaturated soils, soil parameters can be easily affected by the matric suction and specimen component. The object of this study is to evaluate the elastic moduli and void ratios of unsaturated soils using compression and shear waves. The analytical equation for the void ratio in unsaturated soils is derived from the elastic wave velocities. The volumetric pressure plate extractor (VPPE) is improved for the application of axial load. The compression and shear waves are measured using piezo disk elements and bender elements, respectively, installed on the confining cell. For the preparation of the specimens, sands are mixed with silts of two different volume fractions ( $v_f$ ), 0 and 20%. The specimens are subjected to the axial stresses of 100 or 350 kPa to control the matric suction. The experimental results show that the shear wave velocities in the specimens with  $v_f = 20\%$  increase with increasing matric suction, while the shear wave velocities in the sandy soils ( $v_f = 0\%$ ) remained almost constant. The compression wave velocities increase with increasing axial load, while the compression wave velocities remain constant with changes in the matric suction. The void ratio values determined from the elastic wave velocities agree well with void ratio values determined on the basis of the volume of the unsaturated soils. This study suggests that elastic waves can be effectively used to estimate the void ratios and elastic moduli of unsaturated soils.

**RÉSUMÉ :** L'indice des vides et le module élastique d'un sol influent sur le comportement du sol. Dans les sols non saturés, les paramètres du sol peuvent être facilement affectés par la succion matricielle et la nature du sol. L'objectif de cette étude est d'évaluer les modules élastiques et les indices des vides des sols non saturés en utilisant des ondes de compression et de cisaillement. L'équation analytique pour l'indice des vides dans les sols non saturés est dérivée de la vitesse des ondes élastiques. L'Extracteur de Plaque de Pression Volumétrique (EPPV) est amélioré pour l'application de la charge axiale. Les ondes de compression et de cisaillement sont mesurées en utilisant des éléments des disques piézoélectriques et des éléments piézocéramiques ("bender elements"), respectivement, installés sur la cellule de confinement. Pour la préparation des éprouvettes, le sable est mélangé avec le limon avec deux différentes fractions volumiques ( $f_v$ ), 0 et 20%. Les éprouvettes sont soumises à des contraintes axiales de 100 ou 350 kPa pour contrôler la succion matricielle. Les résultats expérimentaux montrent que les vitesses des ondes de cisaillement dans les échantillons de  $f_v = 20\%$  augmentent avec la succion matricielle croissante, tandis que les vitesses des ondes de cisaillement dans les sols sableux ( $f_v = 0\%$ ) sont restées presque constantes. Les vitesses des ondes de compression augmentent avec l'augmentation de la charge axiale, tandis que les vitesses des ondes de compression restent constantes avec les variations de la succion matricielle. Les valeurs de l'indice de vides déterminées par les vitesses des ondes élastiques sont en bon accord avec les valeurs de l'indice des vides déterminées sur la base du volume des sols non saturés. Cette étude suggère que les ondes élastiques peuvent être efficacement utilisées pour estimer les indices des vides et des modules élastiques des sols non saturés.

**KEYWORDS:** Degree of saturation, Elastic wave velocity, Matric suction, Unsaturated soils, Void ratio.

## 1 INTRODUCTION

Soils are commonly regarded as fully saturated or completely dry. Experimental studies conducted with unsaturated soils require significantly longer than studies conducted with fully saturated or completely dry soils. Soils are commonly unsaturated near the surface. Soils used in dams or bridge foundations are mostly unsaturated. The stiffness of soils is affected by the amount of air and water that they contain. Since the 1970's, studies of unsaturated soil mechanics have been conducted (Fredlund et al. 1978, Fredlund and Xing 1994, Vanapalli et al. 1996). As unsaturated soil dries out, the volumetric water content decreases and the matric suction increases (Fredlund and Rahardjo 1993).

The goal of this study was to characterize unsaturated soils using elastic waves, including compression and shear waves. The elastic waves were measured using bender elements and piezo disk elements. Bender elements and piezo disk elements

can change electrical energy into mechanical energy. Both transducers are commonly installed in consolidation cells and field penetration devices for use in seismic investigations (Lee et al. 2008, Lee et al. 2010, Yoon et al. 2010)

In this study, compression and shear wave transducers were installed on the walls of rectangular unsaturated soil characterization cells. This paper presents the background theory related to unsaturated soils, elastic wave velocities, the volumetric pressure plate extractor (VPPE) system, the test procedure, tests results, analyses, and summary and conclusions.

## 2 BACKGROUND THEORY

### 2.1 Volumetric change

Matyas (1968) reported that the volume of an unsaturated soils is affected by  $u_a - u_w$  and  $\sigma - u_a$ , where  $u_a$ ,  $u_w$ , and  $\sigma$  are the pore air pressure, the pore water pressure and the normal stress, respectively. The term  $u_a - u_w$  is the matric suction, and the term

$\sigma - u_a$  is defined as the net normal stress. When the matric suction decreases or the net normal stress increases, the volume of the unsaturated soil decreases.

The volume change of an unsaturated soil can be represented by the net normal stress and the matric suction. The constitutive relationship that takes into consideration of net normal stress and the matric suction is the following (Fredlund and Rahardjo 1993):

$$d\epsilon_v = 3 \left( \frac{1-2\mu}{E} \right) d(\sigma_{mean} - u_a) + \frac{3}{H} d(u_a - u_w) \quad (1)$$

where  $\epsilon_v$  is the volumetric strain,  $d\epsilon_v$  is the volumetric strain change for each increment,  $\mu$  is Poisson's ratio,  $E$  is the modulus of elasticity and  $H$  is the modulus of elasticity for the soil structure with respect to a change in matric suction.

### 2.2 Elastic wave velocities

Physical characterization of soil using shear waves and compression waves is commonly used in the geotechnical field. The physical characteristics of the soil structure can be represented by Young's modulus ( $E$ ) and the shear modulus ( $G$ ). The shear modulus can be determined from the shear wave velocity, using the following equation:

$$G = \rho \times V_s^2 \quad (2)$$

where  $\rho$  is the density of the soil and  $V_s$  is the shear wave velocity. Young's modulus,  $E$ , can be determined from the shear modulus,  $G$ :

$$E = 2 \cdot G \cdot (1 + \mu) \quad (3)$$

where  $\mu$  is Poisson's ratio. Poisson's ratio can be determined from the shear wave velocity ( $V_s$ ) and the compression wave velocity ( $V_p$ ).

$$\mu = \frac{\frac{1}{2} \left( \frac{V_p}{V_s} \right)^2 - 1}{\left( \frac{V_p}{V_s} \right)^2 - 1} \quad (4)$$

The change in the volume of the soil can be predicted using the modulus of elasticity, with the shear modulus of elasticity and Poisson's ratio substituted into Equation (1).

$$\Delta e = \frac{3(u_a - u_w)}{H} + 3 \frac{\left[ 1 - 2 \frac{\left( \frac{0.5V_p^2}{V_s^2} - 1 \right)}{\frac{V_p^2}{V_s^2} - 1} \right]}{2\rho V_s^2 \left[ 1 + \frac{\left( \frac{0.5V_p^2}{V_s^2} - 1 \right)}{\frac{V_p^2}{V_s^2} - 1} \right]} \times (\sigma_{mean} - u_a) \quad (5)$$

where the  $H$  value is  $E/0.17$ , as suggested by Fredlund and Rahardjo (1993). Thus, the void ratio change can be estimated from the compression and shear wave velocities.

## 3 LABORATORY TESTING

### 3.1 VPPE system

In this study, a modified volumetric pressure plate extractor (VPPE) was used. A schematic diagram of the modified system and its peripheral electronics is shown in Figure 1. The modified VPPE system was used to apply axial stresses and measure elastic waves. Axial stresses were applied through the center rod, as shown in Figure 1.

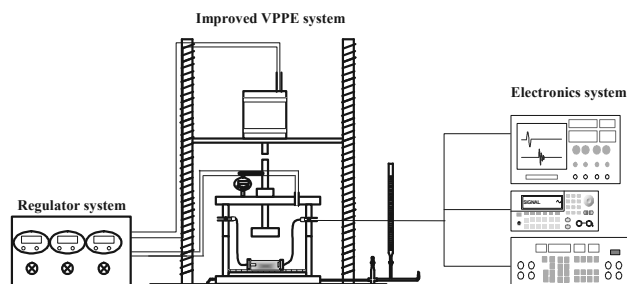


Figure 1. Modified VPPE system and peripheral electronics

The regulator system on the left side of Figure 1 controls the matric suction and axial stress. The regulator system consists of three regulators. Two of them are used to control the matric suction, and the third controls the axial stress. The settlement was measured by a digital dial gauge attached to the rod that is used to apply axial stress.

The VPPE system also included a load cell to calculate the axial stress. Elastic waves are generated and detected using bender elements and piezo disk elements. The electronics on the right side of Figure 1 are a function generator, a filter amplifier and an oscilloscope, used to generate and detect shear and compression waves. Single sine waves were used for generation and detection of shear and compression waves (Lee and Santamarina 2005, Lee and Santamarina 2006).

### 3.2 Test procedures

Two types of soil specimens were used for this study. First, a uniform-grain-sized sand with a mean particle diameter of 0.45 mm were used. Second, a sand-silt mixture with a silt volume fraction of 20% was used. The physical properties of the specimens are summarized in Table 1. After partially saturated specimens were placed into the rectangular cell, which was placed on the ceramic plate of the VPPE, the VPPE was closed and matric suction was applied

Table 1. Physical properties of the soil samples.

Soil type	$G_s$	$e_{max}$	$e_{min}$
Sand	2.62	0.82	0.56
Sand-silt	2.62	0.80	0.42

As matric suction is applied, the degree of saturation of the an unsaturated soil changes. Note that vertical axial stress was applied to control the mean normal stress. Elastic waves were continuously measured based on the matric suction. After the matric suction was applied, the soil specimen being tested gradually reached equilibrium. The elastic waves were measured at 1, 4, 9, 16 and 25 minutes and 1, 2, 4, 6 and 24 hours after the application of matric suction. The shear and compression wave velocities were obtained at each degree of saturation. After the maximum matric suction was applied, the matric suction was gradually decreased.

4 RESULTS

The matric suction versus volumetric water content, which is called the soil water characteristic curve (SWCC), is plotted in Figure 2 for the sand specimen.

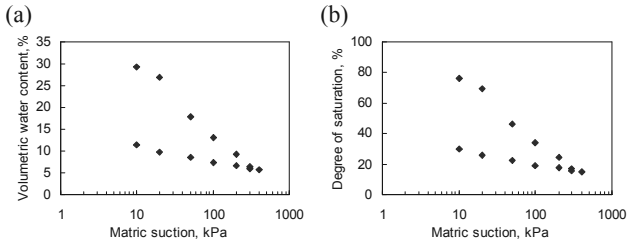


Figure 2. Sand specimen (a) volumetric water content versus matric suction; (b) degree of saturation versus matric suction.

Figure 2 shows that both the volumetric water content and the degree of saturation gradually decrease as the matric suction increases.

Elastic waves for the sand–silt mixture are plotted in Figure 3. Similar results were obtained for the sand specimen. The elastic wave velocities determined from the measured waves are plotted against the degree of saturation in Figure 4 for the sand specimen and Figure 5 for the sand–silt specimen.

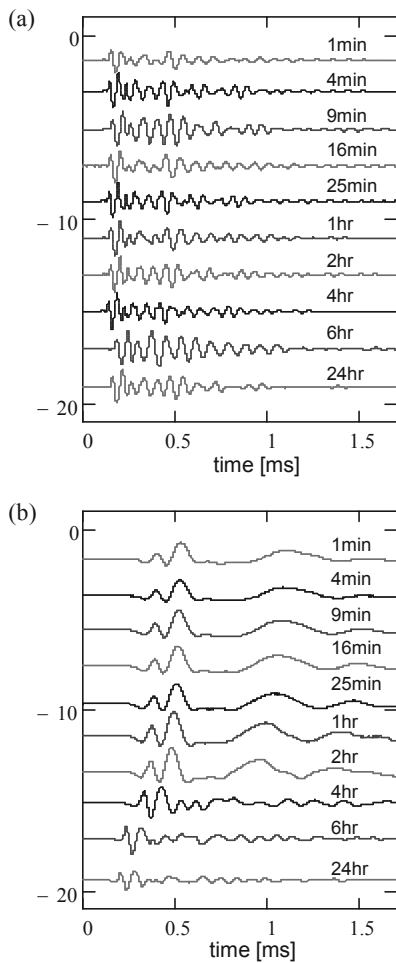


Figure 3. Measured elastic waves for the sand–silt mixture: (a) compression waves; (b) shear waves.

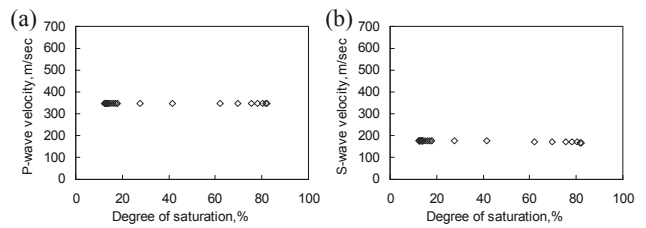


Figure 4. Elastic wave velocities versus degree of saturation for the sand specimen: (a) compression wave velocity; (b) shear wave velocity.

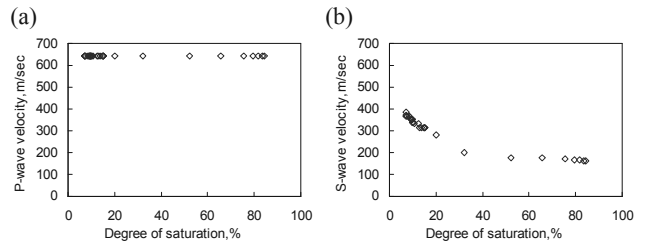


Figure 5. Elastic wave velocity versus degree of saturation for the sand–silt mixture specimen: (a) compression wave velocity; (b) shear wave velocity.

The compression and shear waves for the sand specimen remained almost constant for degrees of saturation from 15 to 85%, as shown in Figure 4. For the sand–silt mixture specimen, the compression wave velocity also remained constant as the degree of saturation decreased from 85% to 7%. The shear wave velocity of the sand–silt mixture, however, increased as the degree of saturation decreased, as shown in Figure 5(b).

5 ANALYSES

The elastic modulus and the shear modulus of each specimen can be estimated using the measured elastic wave velocities. The calculated elastic moduli based on Equations (2) and (3) are plotted in Figure 6 for the sand–silt mixture specimen. From the measured elastic wave velocities and Equation (5), the porosity can be estimated. The calculated wave-based void ratio versus volumetric void ratio is plotted in Figure 7. Figure 7 shows that the wave-based void ratio is similar to the volumetric void ratio.

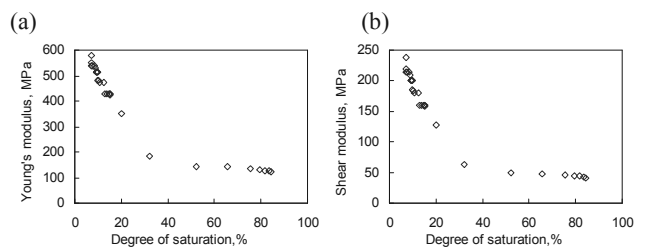


Figure 6. Elastic moduli versus degree of saturation for sand–silt mixture specimen: (a) Young's modulus; (b) shear modulus.

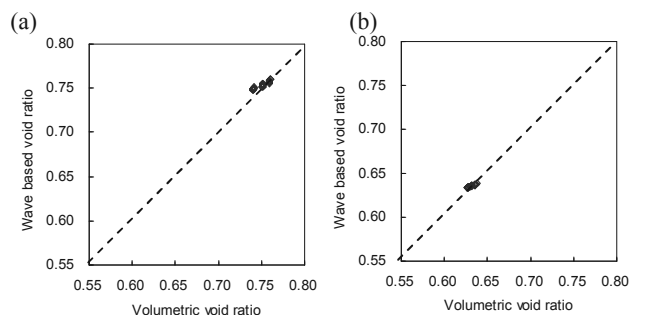


Figure 7. Wave-based void ratio versus volumetric void ratio: (a) sand specimen; (b) sand–silt mixture specimen.

## 6 SUMMARY AND CONCLUSIONS

The matric suction applied in the tests described in this paper was controlled using an improved volumetric pressure plate extractor (VPPE) system to simulate unsaturated conditions near the soil surface. The VPPE was improved for the application of the axial load. The elastic moduli and void ratios of the two unsaturated soils tested were determined using compression and shear waves. The compression waves were continuously measured by piezo disk elements and the shear waves were monitored by bender elements. The piezo disk elements and bender elements were installed on the wall of the confining cell. Two types of specimens were used: a sand specimen and a sand–silt mixture specimen with a silt volume fraction of 20%. Axial stresses of 100 and 350 kPa were applied to the sand specimen and the sand–silt specimen, respectively.

The compression and shear wave velocities of the sand specimen remained almost constant for degrees of saturation from 15 to 80%. The compression wave velocity for the sand–silt mixture specimen also remained constant, but the shear wave velocity increased as the degree of saturation decreased. As the applied axial stress increased, the compression wave velocity increased. The void ratios determined from the elastic wave velocities agreed well with the volume-based void ratios. The results of this study suggest that elastic waves can be effectively used to estimate the void ratio and elastic moduli of unsaturated soils.

## 7 ACKNOWLEDGEMENTS

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